ELSEVIER

Nuclear Physics A670 (2000) 48c–55c

NUCLEAR PHYSICS A

www.elsevier.nl/locate/npe

Relativistic Faddeev description of baryons and nucleon structure function in the NJL model

W. Bentz, H. Mineo, H. Asami, K. Yazaki [a]

[a]Department of Physics, Univ. of Tokyo
7-3-1 Hongo, Bunkyo-ku, Tokyo 113-0033, Japan

In this work we use the Nambu-Jona-Lasinio (NJL) model as an effective quark theory based on QCD to describe the structure of baryons. Based on the solutions of the relativistic 3-quark Faddeev equation in the ladder approximation, we discuss the masses of the nucleon and the delta, the static properties of the nucleon, and the quark light cone momentum distributions in the nucleon.

1. Introduction

As an effective quark theory of QCD in the low energy region, the Nambu-Jona-Lasinio model [1] provides a powerful tool to investigate the structure of hadrons. The model exhibits the spontaneous breaking of the chiral symmetry in a particularly clear manner, and gives a simple framework to describe the structure of mesons within the Bethe-Salpeter approach [2]. Baryons have been described conventionally in the mean field (soliton) approach [3], but in recent years exact numerical solutions to the relativistic 3-quark Faddeev equation [4] have been obtained in the ladder approximation [5,6]. (Attempts have also been made to combine the soliton description, which includes some effects of the meson cloud, or in other words correlations in the $q\overline{q}$ channel, with the Faddeev description, which includes the effects of correlations in the qq channel [7,8].) It has been shown [9] that the relativistic Faddeev approach gives reasonable static properties of the nucleon, and it is therefore natural to extend these investigations to the structure functions of the nucleon measured in deep inelastic lepton-nucleon scattering [10]. The spin independent nucleon structure functions have been studied recently in the mean field approach [11], and in the covariant diquark spectator model [12]. As a first step towards a full Faddeev calculation, in this work we will show results for the spin independent nucleon structure functions obtained by employing a simple approximation to the Faddeev equation [13].

2. BARYON MASSES IN THE FADDEEV APPROACH

The NJL model is characterized by a chirally symmetric 4-fermion interaction lagrangian $\mathcal{L}_I$. Examples are the original NJL form [1]

$$\mathcal{L}_I^{NJL} = g\left(\left(\overline{\psi}\psi\right)^2 - \left(\overline{\psi}(\gamma_5\boldsymbol{\tau})\psi\right)^2\right), \tag{1}$$

0375-9474/00/$ – see front matter
PII S0375-9474(00)00067-1

or the color current interaction lagrangian [14]

$$\mathcal{L}_I^{cc} = -g \sum_{c=1}^{8} \left(\overline{\psi}(\gamma_\mu \frac{\lambda_c}{2})\psi \right)^2 , \tag{2}$$

where λ_c are the color generators. Instead of choosing a particular form of $\mathcal{L}_I$, we wish to study how the results for the baryon masses depend on it. For this, we note that by using Fierz transformations [5], we can extract the effective coupling constants in the pionic $\bar{q}q$ channel (g_π), the 'scalar' ($0^+, T = 0$) qq channel (g_s), and the 'axial vector' $1^+, T = 1$ qq channel (g_a). The explicit forms of the effective interaction lagrangians in these channels can be found in ref. [5]. Introducing the ratios

$$r_s = \frac{g_s}{g_\pi}, \qquad r_a = \frac{g_a}{g_\pi}, \tag{3}$$

we have for for the color current lagrangian (2): $r_s = \frac{1}{2}$, $r_a = \frac{1}{4}$, while for the original NJL lagrangian (1) we have the much smaller values $r_s = \frac{2}{13}$, $r_a = \frac{1}{13}$. We see that if g_π is fixed by the pion mass, the strengths of the interactions in the 0^+ and 1^+ qq channels depend on the form of the interaction lagrangian. Here we treat r_s and r_a as free parameters, reflecting different possible forms of the 4-fermi interaction lagrangian. The other parameters of the model, i.e; g_π, the current quark mass and the Euclidean cut-off, are chosen such as to reproduce $m_\pi = 140MeV$ and $f_\pi = 93MeV$ in the familiar Bethe-Salpeter framework for the pion, and a constituent quark mass of $M = 400MeV$ via the gap equation.

The relativistic Faddeev equation in the NJL model can be reduced to a Bethe-Salpeter equation for a quark a 'diquark' interacting via quark exchange, as shown in fig. 1. Here by 'diquark' we simply mean a qq system in a particular channel, not necessarily bound. In the actual calculation of baryon masses we include the scalar (0^+) and axial vector (1^+) diquark channels.

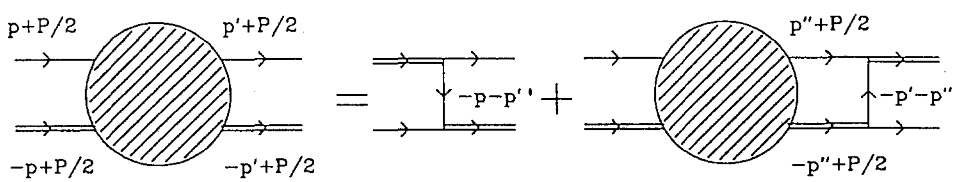

Figure 1. Graphical representation of the relativistic Faddeev equation. The solid line represents a constituent quark propagator, and double line a qq t-matrix.

As an example [5] we show in fig.2 the nucleon and delta masses as functions of r_s for different values of r_a. (The delta mass depends only on r_a.)

¿From this figure and the discussion given above we see that, while for the original NJL lagrangian (1) no 3-quark bound state exists, the color current interaction lagrangian (2) gives a reasonable nucleon mass, but the delta is unbound. One can, however, construct

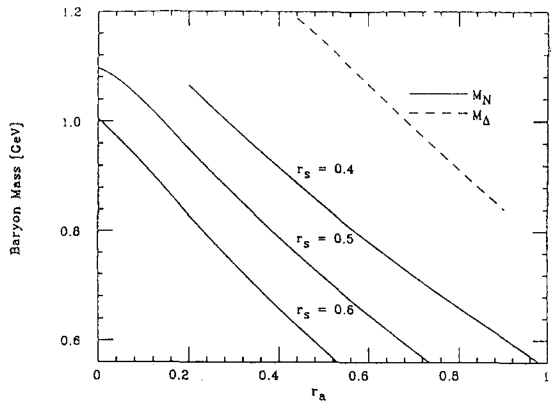

Figure 2. The solid lines show the nucleon mass as a function of r_a for three values of r_s, and the dashed line shows the delta mass as a function of r_a.

a lagrangian which can describe simultaneously the masses of the pion, the nucleon and the delta:

$$\mathcal{L}_I = -g\left[\sum_{c=1}^{8}\left(\bar{\psi}\gamma^\mu\frac{\lambda_c}{2}\psi\right)^2 + \kappa\left(\bar{\psi}\gamma^\mu\gamma_5\psi\right)^2\right]. \tag{4}$$

For $\kappa = 0.27$ ($r_s = 0.37,\ r_a = 0.44$), we obtain $M_\Delta = 1190MeV$, and $M_N = 920MeV$. The main mechanism for the delta-nucleon mass difference is the interaction in the scalar diquark channel: Increasing r_s lowers the nucleon mass but leaves the delta mass unchanged. Our numerical results can be parametrized approximately by the relation

$$M_\Delta - M_N \simeq (1.1r_s - 0.06r_a - 0.18)\,GeV. \tag{5}$$

Recently we have also estimated the effects of (composite) pion and sigma meson exchange between quarks by making a separable approximation to the meson exchange potentials and re-solving the relativistic Faddeev equation [8]. The result is that pion exchange gives a large attraction and leads to nucleon mass shifts of about 200 to 300 MeV, depending on the form of the interaction lagrangian. Before drawing conclusions, however, one should go beyond the separable approximation, and include also the effects of self energy corrections of the quarks due to the meson cloud.

3. STATIC PROPERTIES OF THE NUCLEON

The static properties of the nucleon are obtained by calculating bound state matrix elements of one-quark operators between the Faddeev wave functions. This amounts to calculating the Feynman diagrams shown in fig.3, and leads to the correct Ward-Takahashi identities [9] in the case of an external electromagnetic field. However, since we are using an Euclidean sharp cut-off, the charge conservation is violated by a small

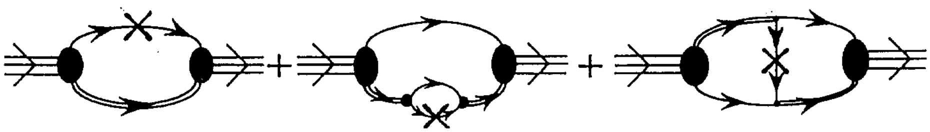

Figure 3. Feynman diagrams to calculate the expectation value of one-body operators (the cross) between 3-body wave functions.

Table 1
The mass, axial vector coupling constant, magnetic moments, and charge radii of the nucleon, as well as the pion nucleon sigma term, for three values of r_s and $r_a = 0$.

	$r_s = 1/2$	2/3	0.8	exp
M_N[MeV]	1096	934	764	939
g_A	1.23	1.33	1.40	1.25
μ_p [n.m.]	1.49	1.78	2.11	2.79
μ_n [n.m.]	-0.95	-1.34	-1.77	-1.91
$< r^2 >_p [fm^2]$	3.39	0.806	0.534	0.74
$< r^2 >_n [fm^2]$	-0.775	-0.075	-0.038	-0.12
$\Sigma_{\pi N}$ [MeV]	35.0	35.1	34.3	45 ± 7

amount (typically 3%). Some of the results are shown in table 1. Here, for simplicity, we take into account only the scalar diquark channel ($g_a = 0$). For the case $r_s = 2/3$, which reproduces the correct nucleon mass, the axial vector coupling constant, the charge radii and the pion nucleon sigma-term agree reasonably well with the experimental values. The magnetic moments are too small in magnitude, but one can expect that the interaction in the axial vector diquark channel gives further contributions.

4. QUARK LIGHT CONE MOMENTUM DISTRIBUTIONS IN THE NUCLEON

We finally discuss the quark light cone momentum distributions in the nucleon, which are related to the spin independent structure functions F_1 and F_2 [15,16]. The twist 2 quark light cone momentum distribution is defined by [17]

$$q(x) = \frac{1}{4\pi} \int dz^- e^{i(p_- x)z^-} \langle p|\overline{\psi}(0)\gamma^+\psi(z^-)|p\rangle, \tag{6}$$

where x is the Bjorken variable, $|p>$ denotes the nucleon state with momentum p, and the light cone components $a^\pm = a_\mp$ of a four vector a^μ are defined by $a^\pm = \frac{1}{\sqrt{2}}(a^0 \pm a^3)$. As has been discussed in detail in ref. [17], the quark distribution (6) has support only in the interval $-1 < x < 1$. The antiquark distribution $\overline{q}(x)$ is defined by a similar equation with ψ replaced by $\psi_c = i\gamma_2\gamma_0\overline{\psi}^T$, and can be obtained from $q(x)$ in the 'unphysical region'

$-1 < x < 0$ according to $\overline{q}(x) = -q(-x)$. Since it can be shown [17] that for connected matrix elements the operator product in eq. (6) can be replaced by the T-product, we see that the distribution (6) is nothing but the quark propagator (traced with γ^+) inside the nucleon with a fixed quark momentum $k_- = p_- x$, while the other quark momentum components are integrated out. We therefore have to evaluate the same Feynman diagrams as in fig. 3, but with the momentum component $k_- = p_- x$ of the struck quark held fixed.

As a first step, we employ here the 'static approximation' to the Faddeev equation [14], in which the momentum dependence of the quark exchange kernel (see fig. 1) is neglected. In this approximation the Faddeev equation can be solved almost analytically. It is known that this static approximation is reasonable as far as the nucleon mass is concerned [5], but the overbinding leads to a nucleon radius which is too small [18]. We can therefore expect that the momentum distributions calculated with the exact Faddeev wave functions will be somewhat softer than the distributions calculated within the static approximation. In the static approximation to the Faddeev equation, the quark exchange diagram in fig. 3 does not contribute, and the calculation of the quark distributions becomes somewhat similar to the calculations performed earlier in the quark-diquark model [19].

Since the calculation based on the diagrams fig. 3 gives $\overline{q}(x) = 0$ for $0 < x < 1$, the resulting quark distributions are valence-like, that is, $q(x) = q_v(x)$. In order to estimate also the sea quark distributions, we take into account the pionic cloud around the constituent quarks by using the familiar convolution formalism [20]. This amounts to calculating the quark and antiquark distributions within an on-shell 'parent quark' from the pion cloud diagrams shown in fig. 4, and convoluting them with the distributions of the parent quarks within the nucleon obtained from the diagrams fig. 3. In the actual calculation of

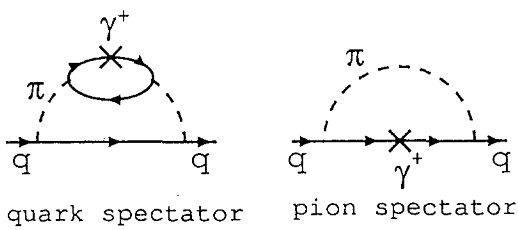

Figure 4. Feynman diagrams for the quark and antiquark distributions within a parent quark. The dashed line denotes the $q\overline{q}$ t-matrix in the pionic channel.

the quark light cone momentum distributions we include only the scalar diquark channel in the nucleon wave function, and employ the 'invariant mass regularization', which can be formulated in terms of light cone variables [21]. (For details concerning this regularization scheme, see ref. [22].) In order to compare with the empirical parametrizations of the quark distributions at $Q^2 = 4GeV^2$ based on experimental data [23], we have to evolve our calculated distributions from the NJL low energy scale Q_0^2 up to $Q^2 = 4GeV^2$. For this purpose we used the DGLAP evolution code of ref. [24] in next-to-leading order. Here we consider Q_0^2 as a parameter which is adjusted such as to reproduce the empirical valence quark distributions reasonably well.

The resulting distributions including the pionic cloud effects are shown in fig. 5 in comparison to the empirical distributions of ref. [23]. The upper two figures show the valence quark distributions, and the lower ones the sea quark distributions. We see that

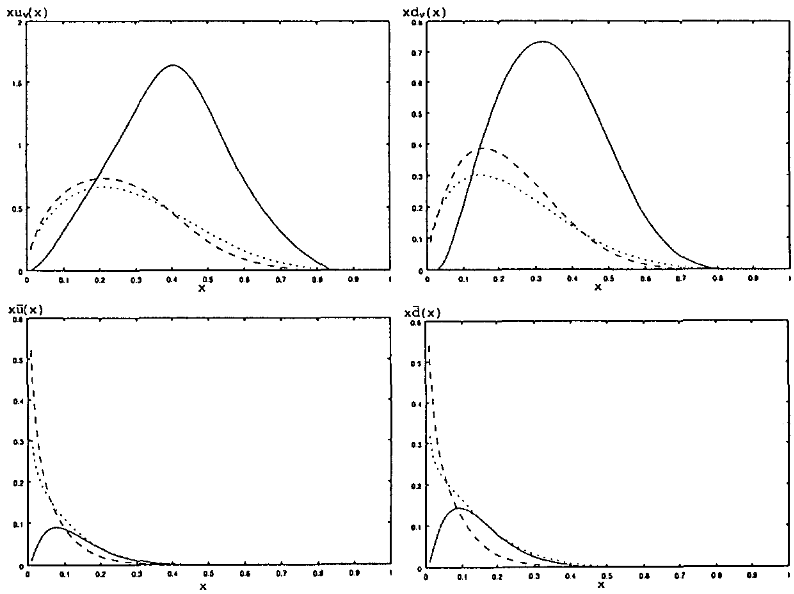

Figure 5. The valence quark (upper figures) and sea quark (lower figures) distributions in the nucleon. The solid lines show the NJL input distributions ($Q_0^2 \equiv 0.16GeV^2$), the dashed lines the distributions evolved to $Q^2 = 4GeV^2$, and the dotted lines the empirical distributions at $Q^2 = 4GeV^2$.

this simple model leads to reasonable valence quark distributions, although they show somewhat too strong variations as functions of x. (This fact is related to the invariant mass cut-off scheme, which leads to input distributions which vanish for large and small values of x.) By integrating the curves shown in fig. 5 we find that at the low energy scale ($Q_0^2 = 0.16GeV^2$) the valence quarks carry 91% of the nucleon momentum. We also see from the figure that the pionic effects lead to the well known isospin asymmetry of the sea quark distributions [16], that is, $\overline{d} > \overline{u}$. The Gottfried sum is reduced from the naive (valence quark) value $\frac{1}{3}$ to 0.262, compared to the experimental value of 0.235 ± 0.026 [25].

In fig. 6 we compare the the valence u-quark distributions without pionic effects to those including the pionic effects. Since the sea quarks in the pion take away some amount of the total light cone momentum, the valence quark distributions become somewhat softer due to the pionic effects. In spite of this, however, one still needs a rather small value of Q_0^2 in order to have a reasonable agreement with the empirical distributions at $Q^2 = 4GeV^2$. In order to get a further softening of the valence quark distributions, one usually considers the exchange of Regge poles besides the pion exchange contributions [26]. However, as we pointed out previously, the simple static approximation employed here gives probably too stiff momentum distributions. Therefore the next step is to use the full Faddeev wave functions for the calculation of the nucleon structure functions.

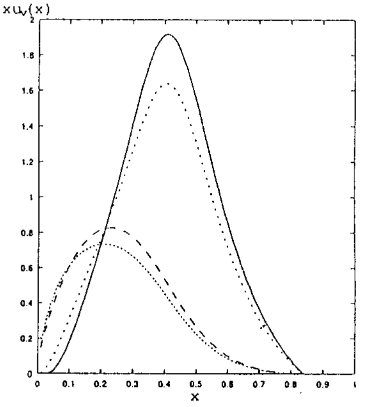

Figure 6. The upper two curves show the valence u-distributions at $Q_0^2 = 0.16GeV^2$, and the lower two curves refer to $Q^2 = 4GeV^2$. For each pair of curves, the lower one includes the pionic effects, while the upper one does not.

5. SUMMARY

In this work we calculated baryon masses, static properties of the nucleon and the quark light cone momentum distributions in the nucleon by using the relativistic Faddeev approach to the NJL model. The results for the nucleon and delta masses favor a 4-fermi interaction lagrangian of the color-current form, besides a small correction term (see eq. (4). A Faddeev calculation including only the scalar diquark channel is able to reproduce the static properties of the nucleon reasonably well, except for the magnetic moments for which the axial vector diquark contributions might be important. Concerning the spin independent nucleon structure functions, we have shown first results for the quark light cone momentum distributions in the nucleon obtained by using the simple static approximation to the Faddeev equation. The pionic cloud effects were taken into account by using the simple convolution formalism. Although we obtained distributions which agree reasonably well with the empirical ones, we should note that our value for the low energy scale Q_0 is rather low (actually equal to the constituent quark mass), and the application of the Q^2 evolution in this region might not be justified. However, since we can anticipate that the exact Faddeev wave functions will give softer quark distributions, we expect that some improvements concerning this point will be achieved by performing a full Faddeev calculation of the nucleon structure functions.

ACKNOWLEDGEMENTS
The authors express their thanks to M. Miyama and S. Kumano for the Q^2 evolution

program (ref. [24]), and to A. W. Thomas for many helpful discussions on structure functions.

REFERENCES

1. Y. Nambu and G. Jona-Lasinio, *Phys. Rev.* **122**, 345 (1960); **124**, 246 (1961).
2. R. Brockmann, W. Weise and E. Werner, Phys. Lett. **B 122** (1983) 201
   T. Kunihiro and T. Hatsuda, Phys. Lett. **B 185** (1987) 304.
3. M. Kato, W. Bentz, K. Yazaki and K. Tanaka, Nucl. Phys. **A 551** (1993) 541
   Chr.V. Christov, A.Z. Górski, K. Goeke and P.V. Pobylitsa, Nucl. Phys. **A 592** (1995) 513.
4. R.T. Cahill, C.D. Roberts and J. Praschifka, Aust. J. Phys. **42** (1989) 129.
5. N.Ishii, W. Bentz and K. Yazaki, Phys. Lett. **B 301** (1993) 165; **B 318** (1993) 26
   N.Ishii, W. Bentz and K. Yazaki, Nucl. Phys. **A 587**, 617 (1995).
6. S. Huang and J. Tjon, Phys. Rev. **C 49** (1994) 1702
   C. Hanhart and S. Krewald, Phys. Lett. **B 344** (1995) 55.
7. U. Zückert, R. Alkofer, H. Weigel and H. Reinhardt, Phys. Rev. **C 55** (1997) 2030.
8. N. Ishii, Phys. Lett. **B 431** (1998) 1.
9. H. Asami, N. Ishii, W. Bentz and K. Yazaki, Phys. Rev. **C 51**, 3388 (1995).
10. J. Speth and A. W. Thomas, Adv. Nucl. Phys. **24** (1997) 83.
11. H. Weigel, L. Gamberg and H. Reinhardt, Mod. Phys. Lett. **A 11** (1996) 3021.
12. K. Kusaka, G. Piller, A.W. Thomas and A.G. Williams, Phys. Rev. **D 55** (1997) 5299.
13. W. Bentz, H. Mineo, N. Ishii, H. Asami and K. Yazaki, 1998 YITP Workshop on QCD and Hadron Physics, to be publ. by World Scientific, Singapore (1999)
    H. Mineo, Master thesis, Univ. of Tokyo (1999), unpublished.
14. A. Buck, R. Alkofer and H. Reinhardt, *Phys. Lett.* B **286**, 29 (1992).
15. R.L. Jaffe, 1985 Los Alamos school on Relativistic Dynamics and Quark Nuclear Physics, eds. M.B. Johnson and A. Pickleseimer, Wiley, New York, 1985.
16. S. Kumano, Phys. Rep. **303** (1998) 183.
17. R.L. Jaffe, Nucl. Phys. **B 229** (1983) 205.
18. G. Hellstern and C. Weiss, Phys. Lett. **B 351** (1995) 64.
19. K. Suzuki, T. Shigetani and H. Toki, Nucl. Phys. **A 573** (1994) 5299.
20. A. Szczurek, A. Buchmann and A. Faessler, J. Phys. **G 22** (1996) 1741
    K. Suzuki and W. Weise, Nucl. Phys. **A 634** (1998) 141.
21. G.P. Lepage and S.J. Brodsky, Phys. Rev. **D 22** (1980) 2157.
22. W. Bentz, T. Hama, T. Matsuki and K. Yazaki, NJL model on the light cone and pion structure function, subm. to Nucl. Phys. **A**.
23. A. D. Martin, W. J. Stirling and R. G. Roberts, Phys. Rev. **C 50**, 6734 (1994).
24. M. Miyama and S. Kumano, Comp. Phys. Commun. **94** (1996) 185.
25. M. Arneodo et al., Phys. Rev. **D 50** (1994) R1.
26. S.A. Kulagin, W. Melnitchouk, T. Weigl and W. Weise, Nucl. Phys. **A 597** (1996) 515.