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Structure of the nucleon in a relativistic 3-body approach to the NJL model

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After a brief review of the relativistic Faddeev approach to the nucleon in the NJL model, some results for the nucleon structure functions and the nuclear matter equation of state are discussed.

1. THE RELATIVISTIC FADDEEV APPROACH TO THE NUCLEON

The relativistic Faddeev approach to baryons in the NJL model [1] is a direct extension of the successful Bethe-Salpeter approach to the mesons, and incorporates the relativistic dynamics and spontaneous breaking of the chiral symmetry. The relativistic Faddeev equation in the NJL model can be reduced to a Bethe-Salpeter equation for a quark a 'di-quark' interacting via quark exchange [2,3]. In the actual calculation of baryon masses [2] we included the scalar (0^+) and axial vector (1^+) diquark channels. For the constituent quark mass we used $M = 400MeV$, and the Euclidean sharp cut-off scheme was employed. It has been shown [2] that the results for the baryon masses favour an interaction lagrangian of the 'color current' type, $\mathcal{L}_I = -g\sum_{c=1}^{8}\left(\bar{\psi}\gamma^{\mu}\frac{\lambda_c}{2}\psi\right)^2$, supplemented by a small correction term of pseudovector color singlet type. Using such a lagrangian in the ladder approximation, the pion, the nucleon and the delta masses are reproduced reasonably well. The main mechanism for the delta-nucleon mass difference is the attraction in the scalar diquark channel, since it lowers the nucleon mass but leaves the delta mass unchanged.

Using the relativistic Faddeev wave functions, the static properties of the nucleon have been calculated [4]. If for simplicity one neglects the axial vector diquark channel, one obtains $g_A = 1.33$, $< r^2 >_p = 0.81fm^2$, $< r^2 >_n = -0.08fm^2$, $\Sigma_{\pi N} = 35.1MeV$, $\mu_p = 1.78$, and $\mu_n = -1.34$, if the parameters are adjusted to reproduce the nucleon mass. Comparing these values with the experimental ones ($g_A = 1.25$, $< r^2 >_p = 0.74fm^2$, $< r^2 >_n = -0.12fm^2$, $\Sigma_{\pi N} = 45 \pm 7MeV$, $\mu_p = 2.79$, and $\mu_n = -1.91$), one sees that the axial vector coupling constant, the charge radii and the pion nucleon sigma-term agree reasonably well with the experimental values, while the magnetic moments are too small in magnitude, leaving room for further contributions due to the axial vector diquark channel.

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2. QUARK LIGHT CONE MOMENTUM DISTRIBUTIONS

In the Bjorken limit, the spin independent structure functions of the nucleon can be expressed in terms of the twist 2 quark light cone momentum distributions [5]

$$q(x) = \frac{1}{4\pi} \int dz^- e^{i(p_- x)z^-} \langle p | T \left(\overline{\psi}(0) \gamma^+ \psi(z^-) \right) | p \rangle. \quad (1)$$

To evaluate (1) we have to consider the same Feynman diagrams as for the expectation values of local one-body operators, but the light cone momentum component $k_- = p_- x$ of the struck quark must be held fixed [6].

As a first step, we employ here the simple 'static approximation' to the Faddeev equation [7], and include only the scalar diquark channel. In order to estimate also the sea quark distributions, we take into account the (composite) pionic cloud around the constituent quarks by using the familiar convolution formalism [8]. The 'invariant mass regularization' is employed, since this scheme can be formulated in terms of light cone variables [9]. (For results obtained in other schemes, see ref. [6].)

The resulting distributions are shown in fig. 1 in comparison to the empirical parametrizations at $Q^2 = 4GeV^2$ of ref. [10]. [1] We see that this simple model leads to reasonable

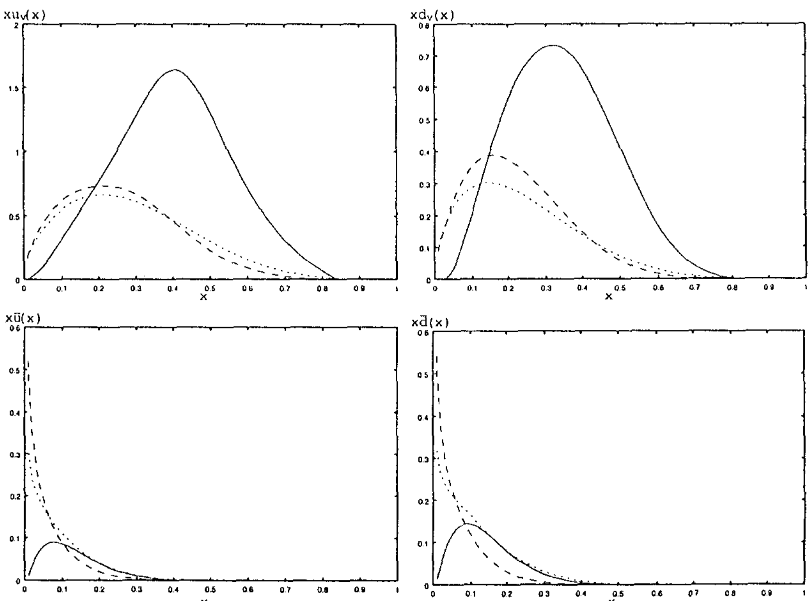

Figure 1. The valence quark (upper figures) and sea quark (lower figures) momentum distributions in the nucleon. The solid lines show the NJL input distributions ($Q_0^2 \equiv 0.16GeV^2$), the dashed lines the distributions evolved to $Q^2 = 4GeV^2$, and the dotted lines the empirical distributions at $Q^2 = 4GeV^2$.

[1] To perform the Q^2 evolution we used the DGLAP code of ref. [11]. The low energy scale Q_0^2 is adjusted such as to reproduce the empirical valence quark distributions reasonably well.

valence quark distributions, although they show somewhat too strong variations as functions of x. At the low energy scale ($Q_0^2 = 0.16 GeV^2$) the valence quarks carry 92% of the nucleon momentum. The pionic effects lead to the well known isospin asymmetry of the sea quark distributions, and the Gottfried sum is reduced from the naive (valence quark) value $\frac{1}{3}$ to 0.262, compared to the experimental value of 0.235 ± 0.026 [12].

3. NUCLEAR MATTER EQUATION OF STATE

It would be interesting to extend the calculations of nucleon properties described in the previous sections to finite density. For this purpose, however, one first has to construct an equation of state. Here we consider a kind of 'hybrid model' for nuclear matter, in which the Dirac sea is made of quarks and the Fermi sea consists of nucleons. The single nucleon is constructed as a solution of the Faddeev equation, where again we restrict ourselves for simplicity to the 'static approximation' and the scalar diquark channel. The quarks in the Dirac sea and within the nucleon couple to self consistent mean fields (quark bilinears) of σ and ω meson type. The energy density of the system then consists of the vacuum part ('Mexican hat' potential)

$$\epsilon_V = -12 \int^{\Lambda} \frac{d^3k}{(2\pi)^3} \left(\sqrt{k^2 + M^{*2}} - \sqrt{k^2 + M^2}\right) + \frac{M^{*2} - M^2}{4g_\pi}, \tag{2}$$

where $M^* = m - 2g_\pi \langle \overline{\Psi}\Psi \rangle$ is the effective quark mass and g_π the 4-fermi coupling constant in the pionic channel, a repulsive ω meson like term (ϵ_ω), and the energy due to the Fermi motion of the composite nucleons in the scalar field (ϵ_F). The resulting expression for the energy density $\epsilon = \epsilon_V + \epsilon_\omega + \epsilon_F$ is similar to the familiar linear sigma model expression [13] (with $M_\sigma = 2M$) for elementary nucleons, but (i) the vacuum polarization term (2) is smaller since the relevant scale is now the quark mass instead of the nucleon mass, and (ii) the effective nucleon mass does not behave linearly as a function of the scalar potential due to the composite nature of the nucleon [14]. Concerning (i), since the vacuum polarization term is small, it cannot prevent the well known instability due to the transition to the Lee-Wick abnormal state [15]. Concerning (ii), the sign and magnitude of the 'scalar polarizability' [14] plays a crucial role for the saturation. Our numerical results, however, show that the normal state does not saturate before the quark condensate drops to small values.

It is known that instabilities of this type in sigma or NJL type theories can be prevented by including higher order interaction terms [16]. For example, the effect of a 8-fermi interaction term of the form

$$\mathcal{L}_8 = g_8 \left[\left(\overline{\psi}\psi\right)^2 - \left(\overline{\psi}\gamma_5 \boldsymbol{\tau}\psi\right)^2 \right] \left(\overline{\psi}\gamma^\mu\psi\right)^2 \tag{3}$$

can be incorporated into an effective coupling constant in the vector channel which increases with increasing scalar field. This cures the instability and leads to a saturating equation of state. As an example of such a calculation, we show in fig. 2 the binding energy per nucleon and the effective nucleon and quark masses. The details of this calculation will be discussed elsewhere [17].

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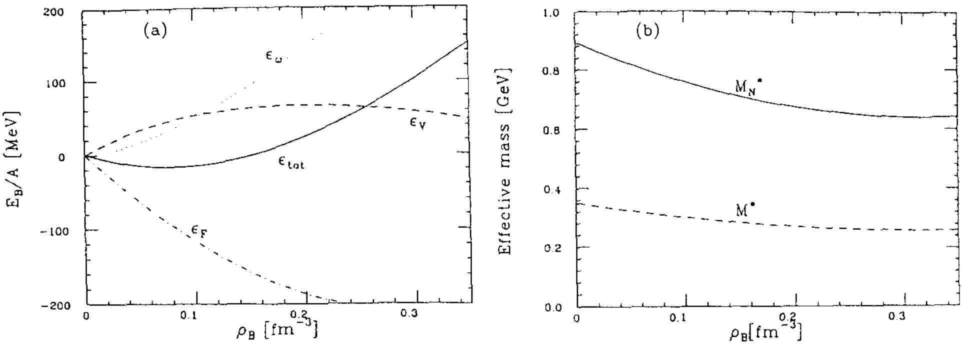

Figure 2. An example of a saturating nuclear matter equation of state based on the quark-diquark picture of the single nucleon. (a) shows the various contributions to the binding energy per nucleon, where the ω meson like repulsion ϵ_ω includes the effect of the 8-fermi term (3), and (b) shows the effective quark and nucleon masses. Details of the calculation will be reported elsewhere [17].

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