QUARK-DIQUARK STRUCTURE OF THE NUCLEON: STRUCTURE FUNCTIONS, STATIC PROPERTIES AND NUCLEAR MATTER EQUATION OF STATE

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In the first part of this work we investigate the structure functions, in particular their flavor dependencies, and static properties of the nucleon in a relativistic three-body approach to the NLJ model, using a simple quark-diquark approximation to the full Faddeev method. In the second part we consider the problem of nuclear matter stability in chiral models like the NJL model.

1 Introduction

The study of modifications of nucleon properties inside the nuclear medium is currently a very active field both experimentally and theoretically. The relativistic Faddeev approach to nucleon structure based on effective chiral quark theories [1] seems to be a powerful candidate to study these problems. For this purpose, first one should be able to account for the properties of a free nucleon, and second one should construct an equation of state of nuclear matter which gives the density dependence of the effective nucleon and quark masses. In this work we consider these two aspects in the Faddeev approach to the Nambu-Jona-Lasinio (NJL) model [2], having in mind future applications of the model to study the medium modifications. We will, however, for technical reasons limit ourselves to the "static approximation" to the full Faddeev equation [3] when describing the nucleon as a quark-diquark state.

Concerning the properties of a free nucleon, we concentrate on the spin independent structure functions [4] and static properties of the nucleon [5]. The valence (v) and sea quark distributions extracted from the experimental nucleon structure functions show an interesting flavor dependence [6]: $d_v(x)$ is 'softer' than $u_v(x)$, i.e; it is more concentrated at smaller x, while $\overline{d}(x)$ is larger than $\overline{u}(x)$ in the whole region of x. The first feature can naturally be explained [7] as an effect of quark-quark (diquark) correlations in the scalar ($J^P = 0^+$, $T = 0$) channel, while the second feature is usually attributed to the pion cloud around the valence quarks [8]. In the first part of this work we investigate how attractive quark-quark correlations in the axial vector (a.v.) channel ($J^P = 1^+$, $T = 1$) influence the flavor dependence of the valence quark distributions and the static properties of the nucleon. The aim is to extract information on the strength of the interaction in the a.v. channel [9].

In the second part we discuss a simple hybrid model for the nuclear matter equation of state in the NJL model, where the single nucleon is described as a quark-diquark state [10]. We will point out that a simple method based on the introduction of an infrared cut-off to mimic confinement effects leads to a saturating equation of state.

2 Structure functions and static properties of the nucleon

Our approach is based on the relativistic Faddeev approach to the Nambu-Jona-Lasinio (NJL) model. The relativistic Faddeev equation in the NJL model can be reduced to a Bethe-Salpeter (BS) equation for a quark and a structured 'diquark' interacting via quark exchange [1]. A very simple approximation to the full Faddeev equation arises if one neglects the momentum dependence of the quark exchange kernel. This 'static approximation' has been shown to be not unreasonable for the nucleon mass and some static properties of the nucleon [3,5]. In the actual calculation we include the scalar and a.v. diquark channels with the effective 4-fermi coupling constants g_s and g_a, respectively. Since any 4-fermi interaction of the NJL type can be decomposed into the various diquark channels by means of Fierz transformations, the ratios $r_s = g_s/g_\pi$ and $r_a = g_a/g_\pi$ of these effective coupling constants to the one in the pionic $q\overline{q}$ channel (g_π) reflect different possible forms of the original 4-fermi interaction lagrangian [1]. (For example, one has $r_s = 0.5$, $r_a = 0.25$ for the 'color current interaction lagrangian' [3].) We will use r_a as a free parameter, and adjust r_s (for each r_a) such as to reproduce the nucleon mass. We use the constituent quark mass $M = 400$ MeV, which is related to the current quark mass m via the gap equation, and employ the invariant mass regularization scheme [11], which is well suited for the use of light cone momentum variables and equivalent to the familiar 3-momentum cut-off scheme when applied for the usual momentum variables [12]. We will consider the three parameter sets I, II, III for r_s and r_a shown in Table 1. The case I (III) involves relatively strong (weak) scalar diquark correlations, while the set II describes an intermediate situation. The value of r_a used in case III reproduces also a bound state for the delta isobar with a mass of $M_\Delta = 1140$ MeV.

Table 1: Three different parameter sets for r_a and r_s, the corresponding scalar diquark masses (M_s, from the quark-quark BS equation) and weights of the scalar diquark channel in the nucleon state (W_s, from the quark-diquark BS equation), and some static properties of the nucleon in comparison to their experimental values. The a.v. diquark is unbound (continuum state) for all 3 cases.

case	I	II	III	exp.
r_a	0	0.25	0.66	
r_s	0.73	0.63	0.50	
M_s[MeV]	600	684	766	
W_s[%]	100	90	66	
μ_p	2.32	2.84	2.79	2.79
μ_n	-1.39	-1.79	-1.83	-1.91
$g_A^{(3)}$	0.66	0.77	0.81	1.26
$g_{\pi NN}$	7.5	12.68	15.25	13.2
$g_A^{(0)}$	0.60	0.41	0.31	$0.2 \sim 0.3$

The twist-2 light cone momentum distributions of the quarks (flavor q) in the proton (momentum p) are defined by [13]

$$q(x) = \frac{1}{4\pi} \int dz^- e^{i(p_- x)z^-} \langle p | T\left(\overline{\psi}(0)\gamma^+ \psi(z^-)\right) | p \rangle, \tag{1}$$

where x is the Bjorken variable, and the light cone variables are defined as $a_\pm = (a_0 \pm a_3)/\sqrt{2}$. The antiquark distributions are obtained as $\overline{q}(x) = -q(-x)$ for $0 < x < 1$, and the valence and sea quark distributions are then given by $q_v(x) = q(x) - \overline{q}(x)$ and $q_s(x) = \overline{q}(x)$. Since Eq.(1) is just the quark propagator in the proton (traced with γ^+) for fixed quark light cone momentum component $k_- = p_- x$, the evaluation of (1) amounts to a straight forward Feynman diagram calculation [4] using the quark-diquark vertex functions including the scalar and a.v. diquark channels. The effects of the pion cloud around the valence quarks are taken into account by using the standard one-dimensional convolution formalism [8]. We then perform the Q^2 evolution in the next-to-leading order [14] from a low energy scale Q_0^2 to the value of Q^2 where experimental data, or parametrizations of quark distributions [15], are available.

If we choose $Q_0^2 = 0.16$ GeV^2, the overall features of the empirical valence quark distributions can be reproduced.

In Fig.1 we plot the ratio of structure functions F_2^n/F_2^p in comparison with the experimental data [16]. Strong scalar diquark correlations (case I) lead to a strong flavor asymmetry of the valence quark distributions, i.e., $d_v(x)$ becomes softer than $u_v(x)$ and the ratio F_2^n/F_2^p becomes very small as $x \to 1$. The axial vector diquark correlations work towards more flavor symmetric distributions. We see from Fig. 1 that reasonable values of r_a are in the range between 0.0 and 0.25, i.e., to reproduce the observed flavor dependence the weight of the a.v. diquark channel should not be larger than about 10% (see Table 1).

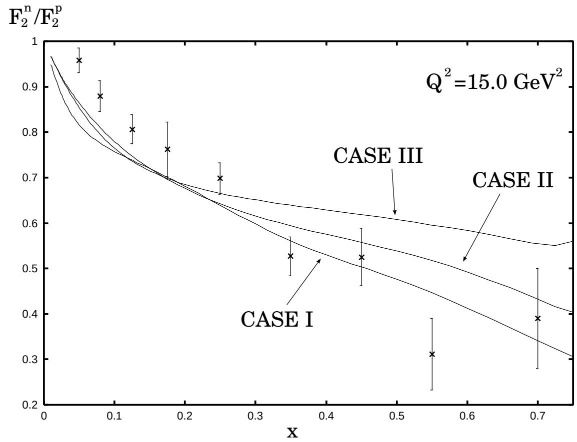

Figure 1: The ratio of structure functions F_2^n/F_2^p for $Q^2 = 15$ GeV^2 is shown for the three cases of Table 1 and compared to the experimental data of ref. [16].

The valence quark and antiquark distributions for the case II ($r_a = 0.25$) are compared to the empirical distributions of ref. [15] in Figs. 2 and 3, respectively. Due to the pion cloud effects we also obtain a flavor asymmetry of the antiquark distributions ($\overline{d} > \overline{u}$), but the simple convolution formalism used here underestimates this asymmetry for intermediate values of x, although the Gottfried sum is reproduced rather well ($S_G = 0.262$ compared to the experimental value of 0.235 ± 0.026).

In conclusion of this part we can say that the overall description of the structure functions in the present quark-diquark model is reasonable as long as the weight of the a.v. diquark component is less than about 10%.

In order to see whether this conclusion is consistent with other observables, we show some static properties of the nucleon, including corrections due to the

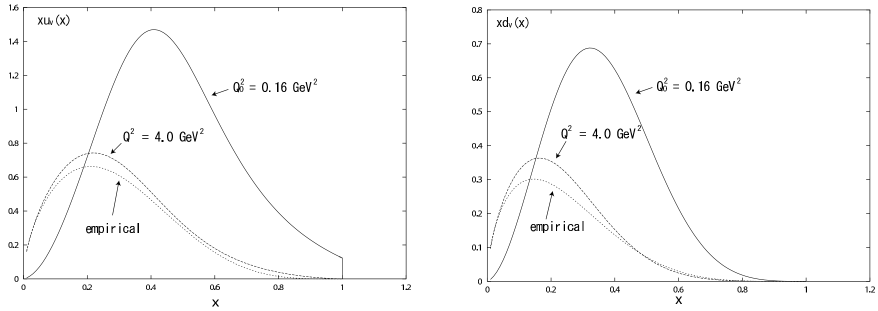

Figure 2: Valence quark distributions in the proton including pionic cloud effects for case II of Table 1. The calculated distributions (solid lines) are evolved from $Q_0^2 = 0.16$ GeV^2 to $Q^2 = 4$ GeV^2 (dashed lines) and compared to the empirical distributions of ref. [15] (dotted lines).

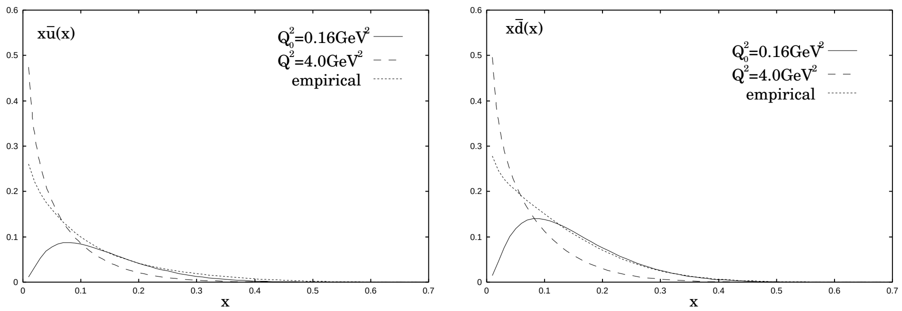

Figure 3: Same as Fig. 2 for the antiquark distributions in the proton.

pion cloud, in the lower part of Table 1. We see that the a.v. diquark channel has beneficial effects for all static properties considered here, but it is not necessary to introduce very strong (case III) correlations. (In particular, the fact that $g_A^{(3)}$ is too small cannot be improved significantly by introducing very strong a.v. correlations.) Therefore the static nucleon properties support the conclusion that $r_a \simeq 0.25$ ($W_a \simeq 10\%$) is reasonable.

3 Nuclear matter equation of state

It is well known that models based on the linear realization of chiral symmetry, like the linear σ model or the NJL model, do not lead to a saturating nuclear matter equation of state [17]. To see where the basic problem is, we note that these models are characterized by a vacuum effective potential (or energy density) $\mathcal{E}_V(\sigma)$ of the Mexican hat form. (Here σ is the nuclear matter expectation value of the scalar field.) Since the curvature of this potential decreases as one moves away from the vacuum value σ_0 towards smaller values of σ, this implies attractive contributions ('vacuum fluctuation terms') to the sigma meson mass $M_\sigma^2 \propto \frac{\partial^2 \mathcal{E}}{\partial \sigma^2}$ and therefore also to the effective NN interaction in the medium (Landau-Migdal interaction), leading to a collapse of the system for decreasing σ (increasing density).

A possible solution of this problem is provided by the internal quark structure of the nucleon: If the scalar field is coupled to the quarks inside the nucleon instead of an elementary nucleon, the nucleon mass in the scalar field $M_N(\sigma)$ can have a finite curvature $\frac{\partial^2 M_N}{\partial \sigma^2}$. If this 'scalar polarizability' [18] of the nucleon is positive and large enough, it can lead to a repulsive contribution to $\frac{\partial^2 \mathcal{E}}{\partial \sigma^2}$, i.e., the sigma meson mass, and can avoid the collapse discussed above. It is, however, very easy to see that this mechanism can work effectively only if the confinement is taken into account: As the scalar potential $\Phi \equiv \sigma_0 - \sigma$ becomes strong the quark mass tends to zero. On the other hand, a positive curvature of M_N persisting also for large scalar potentials means that $M_N(\sigma)$ should become flat, i.e., the nucleon mass should *not* tend to zero for large scalar potentials. Such a situation ($M \to 0$ but $M_N \to$ finite) is possible only if the confinement is taken into account, or, if at least there are no unphysical thresholds which force the nucleon mass to vanish as $M \to 0$. This aspect of the confinement, namely the absence of unphysical quark- diquark or 3-quark thresholds, can be achieved in the NJL model by introducing an infrared cut-off (μ) in addition to the standard ultraviolet one in the framework of the proper-time regularization scheme [19].

In order to show how the nuclear matter equation of state is stabilized by taking into account confinement effects via an infrared cut-off μ, we plot the nucleon mass in the scalar field in Fig. 4 for the cases $\mu = 0$ and $\mu = 0.1$ GeV. The positive curvature for large scalar potentials in the case $\mu = 0.1$ GeV is clearly seen. In Fig. 4 we also show the results for the binding energy per nucleon as a function of the density for $\mu = 0$ and $\mu = 0.1$ GeV. The details of this 'hybrid model' for the nuclear matter energy density, which is based on the quark-diquark description of the free nucleon discussed above and very similar in spirit to the successful model of Guichon and collaborators [20], are described in ref. [10]. The case $\mu = 0$ leads to the collapse of the system due to the reasons explained above, while the case $\mu = 0.1$ GeV gives a saturating equation of state. Some implications of this equation of state for the density dependence of the sigma mass and the πN an πNN sigma terms are discussed in ref. [10]. As an example, we show the πN sigma term $\Sigma_{\pi N} = m \frac{\mathrm{d}\epsilon_F}{\mathrm{d}m}$ and the πNN sigma term

$\Sigma_{\pi NN} = m\frac{\mathrm{d}f_0}{\mathrm{d}m}$, which was introduced in ref.[10] for the first time, as functions of the density in Fig. 5. (Here ϵ_F the Fermi energy, and f_0 the spin-isospin independent $L = 0$ Landau-Migdal parameter.) The decrease of $\Sigma_{\pi N}$ is caused by the decrease of the sigma-nucleon coupling ($\propto \frac{\partial M_N}{\partial\sigma}$) with increasing scalar potential, while the behaviour of $\Sigma_{\pi NN}$ reflects the dependence of the sigma coupling to a 2-nucleon state ($\propto \frac{\partial f_0}{\partial\sigma}$). Finally we would like to mention that the reduction of the sigma-nucleon coupling (reduction of the slope of $M_N(\sigma)$ due to the confinement effects) leads also to a suppression of the famous 'Z-graph contributions'.

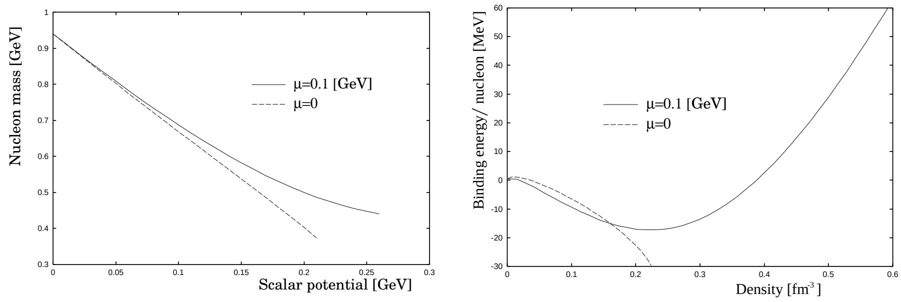

Figure 4: The nucleon mass in the quark-diquark model as a function of the scalar potential (left figure) and the binding energy per nucleon as a function of the density (right figure) for the cases $\mu = 0$ (dashed lines) and $\mu = 0.1$ GeV (solid lines).

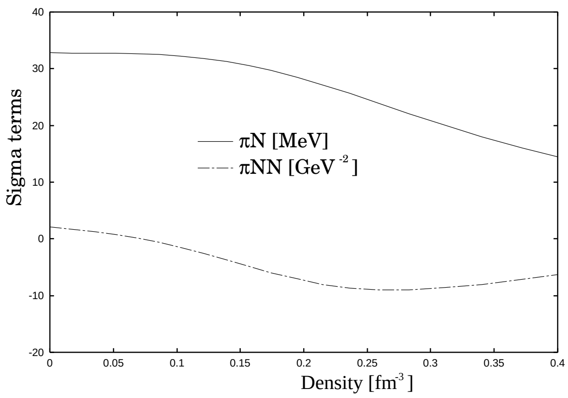

Figure 5: The πN sigma term (solid line, in [MeV]), and the πNN sigma term (dot-dashed line, in [GeV^{-2}]) for the case $\mu = 0.1$ GeV as functions of the density.

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