PERGAMON

Progress in Particle and Nuclear Physics 46 (2001) 129–134

Progress in Particle and Nuclear Physics

http://www.elsevier.nl/locate/npe

The Relation Between the E1 Sum Rule and Magnetic Moments

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Abstract

A general field theoretical derivation of the relation between the enhancement factor of the photonuclear E1 sum rule $(1+\kappa)$ and the orbital angular momentum g-factor of a bound nucleon (g_ℓ) is presented in the framework of the Landau-Migdal theory. The relation to the isovector giant dipole resonance is also discussed.

1 Introduction

About 30 years ago, Fujita and Hirata (FH) [1] used the Fermi gas model as a model for an N=Z nucleus to derive the simple relation $1+\kappa = 2g_{\ell,\mathrm{IV}}$ in lowest order perturbation theory, where $1+\kappa$ is the enhancement factor of the photonuclear E1 sum rule and $g_{\ell,\mathrm{IV}}$ the isovector (IV) orbital angular momentum g-factor of a bound nucleon [2]. Later it has been shown [3] that due to 2-particle 2-hole (2p-2h) correlations only a part of κ is related to $g_{\ell,\mathrm{IV}}$, and it has been argued [4] that this part, which was called κ_{GDR}, is related to the sum of the E1 strength in the low energy region, which is dominated by the giant dipole resonance (GDR). In recent years, this 'modified FH relation' has been used extensively to analyze the results of photo-neutron [5] and photon scattering experiments for nuclei in the lead region [6, 7, 8]. On the other hand, the relation $1+\kappa = 2g_{\ell,\mathrm{IV}}$ has also been derived by Migdal and collaborators [9] by using the 'quasiparticle gas approximation' to calculate the enhancement factor $1+\kappa$. ($g_{\ell,\mathrm{IV}}$ can be derived exactly in the Landau-Migdal theory in the nuclear matter picture.) They directly evaluated the expectation value of the double commutator (see eq. (3) below) by using an effective quasiparticle hamiltonian expressed in terms of the Landau-Migdal parameters and employing the gas approximation for the quasiparticle vacuum. In this method, the entire enhancement factor is related to the orbital g-factor via the FH relation, and no remainder of κ appears which has no connection to the orbital g-factor.

The purpose of this work (see also ref.[10]) is to present a general discussion of the $\kappa - g_{\ell,\mathrm{IV}}$ relation in the framework of the Landau-Migdal theory [11] in the nuclear matter picture. By analyzing the relevant Feynman diagrams we wish to discuss the separation of κ into one part (κ_A), which is related to the orbital g-factor, and a remainder (κ_B), which has no connection to it. Then we wish to discuss whether κ_A can be identified with the enhancement factor due to the GDR (κ_{GDR}) or not.

2 The E1 sum rule

In the long wave length limit $(e^{i\boldsymbol{q}\cdot\boldsymbol{r}} \to 1)$ the strength function for the absorption of unpolarized photons by the ground state of an N=Z nucleus is given by [1]

$$S(\omega) = 4\pi^2 \sum_n |\langle n| j_{\mathrm{IV}}(\boldsymbol{q}=0)\frac{1}{\sqrt{\omega}}|0\rangle|^2\, \delta(\omega-\omega_{n0}), \tag{1}$$

[1] In order to avoid unnecessary indices, we define for any vector $\boldsymbol{V}$: $V \equiv V^3$, and similarly for any tensor T^{ij}: $T \equiv T^{33}$. For example, D denotes the 3-component of the dipole operator, Γ the 3-component of the electromagnetic vertex, Π the 33-component of the current-current correlation function, etc. We also note that for N=Z the restriction to the IV current in eq.(1) corresponds to the effective nucleon charges $\pm\frac{1}{2}$, and is correct as long as the dipole operator $\boldsymbol{D}$ is given by the expression for point charges.

0146-6410/01/$ - see front matter
PII: S0146-6410(01)00116-8

where $\omega > 0$, $j_{IV}(\boldsymbol{q})$ is the Fourier transform of the isovector current operator in the Schrödinger picture, and $\omega_{n0} = E_n - E_0$ is the excitation energy of the excited state $|n\rangle$. Due to current conservation, the isovector current for $\boldsymbol{q} = 0$ can be expressed in terms of the isovector dipole operator D_{IV} by $j_{IV}(\boldsymbol{q} = 0) = i[H, D_{IV}]$, and we obtain for the sum rule the well known double commutator:

$$S \equiv \int_0^\infty \mathrm{d}\omega\, S(\omega) = 4\pi^2 \sum_n \frac{1}{\omega_{n0}} |\langle n|j_{IV}(\boldsymbol{q} = 0)|0\rangle|^2 \tag{2}$$

$$= -2\pi^2 \langle 0| \left[[H, D_{IV}], D_{IV}\right] |0\rangle \equiv (\mathrm{TRK})\,(1+\kappa), \tag{3}$$

where (TRK)=$\pi^2 A/2M$ is the Thomas-Reiche-Kuhn sum rule value for N=Z with M the free nucleon mass. One should note that the quantity S of eq.(2) is not directly observable, since for high photon energies the long wave length limit is not justified. It is therefore natural to ask whether there exists a separate sum rule for the region of the GDR, where this limit would be reasonable.

In terms of the Fourier transform of the correlation function of two Heisenberg IV current operators

$$\Pi_{IV}(\boldsymbol{q}, \omega) = i \int_{-\infty}^{\infty} \mathrm{d}\tau e^{i\omega\tau} \langle 0|T\left(j_{IV}(\boldsymbol{q}, t')\, j_{IV}(-\boldsymbol{q}, t)\right) 0\rangle, \tag{4}$$

where $\tau = t' - t$, the strength function and the sum rule are given by

$$S(\omega) = \frac{4\pi}{\omega} \mathrm{Im}\, \Pi_{IV}(\boldsymbol{q} = 0, \omega) \tag{5}$$

$$S = 2\pi^2\, \Pi_{IV}(\boldsymbol{q} = 0, \omega = 0)|_{\omega \to 0\,\mathrm{first}}, \tag{6}$$

where '$\omega \to 0$ first' in eq.(6) means that the limit $\omega \to 0$ should be taken before the limit $\boldsymbol{q} \to 0$. The equivalence of the expressions (5) and (6) to (1) and (2) can be seen from the spectral representation (for fixed $\boldsymbol{q}$) of the correlation function [2]

$$\Pi_{IV}(\boldsymbol{q}, \omega) = -\sum_n |\langle 0|j_{IV}(\boldsymbol{q})|n\rangle|^2 \left(\frac{1}{\omega - \omega_{n0} + i\epsilon} - \frac{1}{\omega + \omega_{n0} - i\epsilon}\right). \tag{7}$$

The correlation function can be represented generally by a bubble diagram involving the full nucleon propagators (S) and the full NNγ vertex (Γ): In symbolic notation,

$$\Pi_{IV}(\boldsymbol{q}, \omega) = i\, \mathrm{Tr}\,(\Gamma_0\, S\, S\, \Gamma), \tag{8}$$

where 'Tr' refers to the summation over the quantum numbers characterizing the intermediate NN state (4-momentum integral and spin-isospin summation in the case of an infinite system, or frequency integral and summation over discrete single particle quantum numbers in the case of a finite system.) The vertex Γ_0 is that part of Γ which is irreducible in the particle-hole channel, i.e; it is the driving term in the integral equation for the vertex [11]

$$\Gamma = \Gamma_0 + i\, \mathrm{Tr}\,(T\, S\, S\, \Gamma_0), \tag{9}$$

where T is the NN scattering matrix in the medium. Note that Γ_0 in general contains also contributions due to mesonic exchange currents.

We now follow the Landau-Migdal theory [11] and split the product of the two nucleon propagators in eq.(8) as

$$S\, S = A + B, \tag{10}$$

where A represents the product of the pole parts of a particle and a hole propagator, and B is the rest (e.g; the product of two particle propagators, etc.) It is then possible to rewrite the integral

[2] To derive also the order of the limits indicated in eq. (6) requires a bit more work, i.e; deriving the Ward-Takahashi identity for the current-current correlation function and then extracting the 33-component for zero frequency and wave number, see ref.[10]. We remind the reader that the limit '$\boldsymbol{q} \to 0$ first' is relevant for the definition of the local 'quasiparticle charges' like g_ℓ, while the limit '$\omega \to 0$ first' is relevant for the collective excitations. Therefore, from the physical standpoint, one can expect that the correlator in the sum rule (6) should be defined according to the second limit.

equations for the vertex and the T-matrix in the space of 1p-1h states, and incorporate the effects of 2p-2p or more complicated states (and, in the case of the vertex, of the meson exchange currents) into an effective vertex $\Gamma^{(\omega)}$ and an effective T-matrix (or effective particle-hole interaction) $T^{(\omega)}$. The correlation function then splits into two parts:

$$\Pi = \Pi_A + \Pi_B. \tag{11}$$

The part Π_A and the RPA-like equation for the vertex [11] are given by

$$\Pi_A = i\,\mathrm{Tr}\left(\Gamma^{(\omega)} A\,\Gamma\right) \tag{12}$$

$$\Gamma = \Gamma^{(\omega)} - i\,\mathrm{Tr}\left(T^{(\omega)} A\,\Gamma\right). \tag{13}$$

The piece Π_A contains 1p-1h intermediate states and is represented generally by the diagram fig.1, while the piece Π_B has no 1p-1h intermediate states and contains contributions like shown in fig.2.

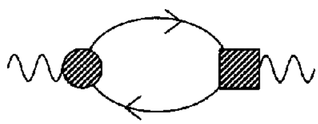

Figure 1: Graphical representation of Π_A, eq.(12). The circle represents the particle-hole irreducible vertex $\Gamma^{(\omega)}$, the square stands for the full vertex Γ, and the intermediate state is a particle-hole pair.

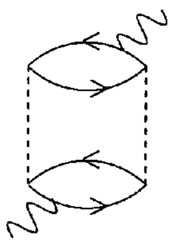

Figure 2: Example for a contribution to Π_B. The intermediate state is a 2p-2h state.

Examples for the Feynman diagrams representing the effective particle-hole vertex $\Gamma^{(\omega)}$ and the effective particle-hole interaction $T^{(\omega)}$ are shown in figs. 3 and 4. By definition these quantities are particle-hole irreducible, and are renormalized due to 2p-2h or more complicated states, and also mesonic exchange currents in the case of the vertex $\Gamma^{(\omega)}$. Applying the separation (11) to the correlation function of two IV currents (Π_{IV}), the sum rule (6) can be written as

$$S = (\mathrm{TRK})\,(1 + \kappa_A + \kappa_B)\,, \tag{14}$$

where the part $(\mathrm{TRK})(1+\kappa_A)$ comes from the piece Π_A, and the rest is due to Π_B:

$$(\mathrm{TRK})(1+\kappa_A) = 2\pi^2\,\Pi_{\mathrm{IV},A}(\boldsymbol{q}=0,\omega=0)|_{\omega\to 0\,\mathrm{first}} \tag{15}$$

$$(\mathrm{TRK})\kappa_B = 2\pi^2\,\Pi_{\mathrm{IV},B}(\boldsymbol{q}=0,\omega=0)|_{\omega\to 0\,\mathrm{first}} \tag{16}$$

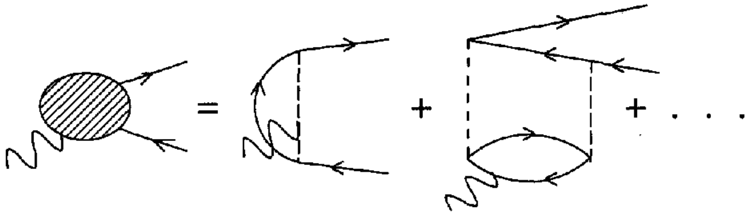

Figure 3: Examples for contributions to the particle-hole irreducible vertex $\Gamma^{(\omega)}$. The first diagram represents a meson exchange current, and the second one shows the renormalization due to 2p-2h states.

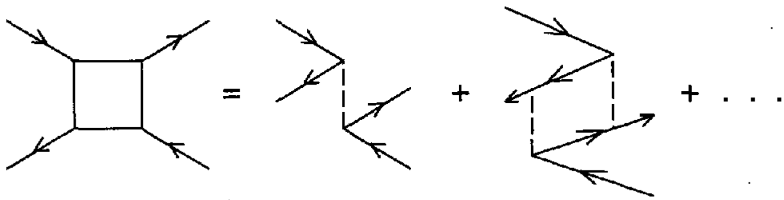

Figure 4: Examples for contributions to the particle-hole irreducible interaction $T^{(\omega)}$. The first diagram represents the bare interaction, and the second one shows the renormalization due to 2p-2h states.

3 Relation to the orbital g-factor

In the nuclear matter picture it is now easy to see that the part of the sum rule originating from $\Pi_{\mathrm{IV},A}$ (eq.15) is related to the orbital isovector g-factor $g_{\ell,\mathrm{IV}}$: It is clear from fig. 1 or eq.(12) that $\Pi_{\mathrm{IV},A}$ involves the matrix element of the 'local' quasiparticle vertex $\Gamma^{(\omega)}_{\mathrm{IV}}$ between the particle and the hole state. In the limit $\omega \to 0$ followed by $\boldsymbol{q} \to 0$, which is relevant for the sum rule (15), these particle-hole states are restricted to the Fermi surface. Since the matrix element of the vertex $\Gamma^{(\omega)}_{\mathrm{IV}}$ between the single particle states on the Fermi surface defines $g_{\ell,\mathrm{IV}}$ [12, 13], the relation follows immediately. Explicitly, in nuclear matter the quantity $\Pi_{\mathrm{IV},A}$ in the limit $\omega \to 0$ followed by $\boldsymbol{q} \to 0$ becomes

$$\Pi_{\mathrm{IV},A}(\boldsymbol{q}=0,\omega=0)|_{\omega\to 0\text{ first}} = 4V\int \frac{d^3k}{(2\pi)^3}\,\delta(k-p_F)\,\frac{1}{v_F}\,\left(j^{(\omega)}_{\mathrm{IV}}(k)\,j^{(st)}_{\mathrm{IV}}(k)\right), \tag{17}$$

where V is the volume of the system, v_F is the Fermi velocity, and the currents are defined by $j^{(\alpha)}_{\mathrm{IV}}(k) = Z_2\,\Gamma^{(\alpha)}_{\mathrm{IV}}(k,k)$ $(\alpha = st, \omega)$, where the arguments of the vertex are the incoming and outgoing quasiparticle momenta, Z_2 is the residue of the nucleon propagator at the quasiparticle pole on the Fermi surface, and the 'static limit' (st) [11] is defined as $\omega \to 0$ followed by $\boldsymbol{q} \to 0$, while the 'ω-limit' is defined as before by $\boldsymbol{q} \to 0$ followed by $\omega \to 0$. These currents are determined by gauge invariance (the Ward identity) as follows [11, 12, 13]:

$$j^{(st)}_{\mathrm{IV}}(k) = \frac{1}{2}v_F \tag{18}$$

$$j^{(\omega)}_{\mathrm{IV}}(k) \equiv \frac{p_F}{M}\,g_{\ell,\mathrm{IV}}, \tag{19}$$

where the IV orbital g-factor is given by [3]

$$g_{\ell,\mathrm{IV}} = \frac{1}{2}\frac{M}{\epsilon_F}\left(1 - \frac{\epsilon_F v_F}{3p_F}(F_1 - F_1')\right) \tag{20}$$

in terms of two dimensionless Landau-Migdal parameters F_1 and F_1' and the Fermi energy ϵ_F (including the nucleon mass) [13]. Using the currents (18), (19) in eq. (17), we obtain $\Pi_{\mathrm{IV},A}(\boldsymbol{q} = 0, \omega = 0)|_{\omega\to 0rmfirst} = A/2M \cdot g_{\ell,\mathrm{IV}}$, and therefore the part $(1+\kappa_A)$ of the enhancement factor in eq. (15) becomes

$$1 + \kappa_A = 2\,g_{\ell,\mathrm{IV}} = 1 + 2\delta\, g_{\ell,\mathrm{IV}}, \tag{21}$$

that is, the part κ_A satisfies the FH relation $\kappa_A = 2\,\delta g_{\ell,\mathrm{IV}}$ with the *total* isovector correction $\delta g_{\ell,\mathrm{IV}}$ including all possible meson exchange current and configuration mixing effects. The part κ_B has no connection to $\delta g_{\ell,\mathrm{IV}}$.

4 Discussion

In the preceding section we have seen that the enhancement factor of the sum rule splits into two parts (see eq.(14)), where the term $1+\kappa_A$ originates from the correlator $\Pi_{\mathrm{IV},A}$ which involves intermediate 1p-1h states (fig. 1 and eq.(12)), and κ_B originates from $\Pi_{\mathrm{IV},B}$ which involves 2p-2h (or more complicated) states. The first part is related to the orbital g-factor via the FH relation (21). The problem of course is whether this part can be related to the experimentally measured sum rule or not. For this we note that *if* the part Π_A separately would satisfy a dispersion relation of the form (7) involving a well defined subset (n') of all possible intermediate states (n) of the current-current correlation function, the corresponding contribution to the sum rule (15) would be

$$(\mathrm{TRK})(1+\kappa_A) = 4\pi^2 \sum_{n'\epsilon n} |\langle 0|j_{\mathrm{IV}}(\boldsymbol{q}=0)|n'\rangle|^2 \frac{1}{\omega_{n'0}}. \tag{22}$$

The correlation function $\Pi_{\mathrm{IV},A}$ contains by definition *all* ph cuts, and if there were no other cuts in Π_A, eq.(22) would hold with n' referring to the particle-hole excited states. Since the particle-hole states give the main contribution to the strength in the region of the GDR, one would then conclude that this part of the sum rule could be identified with the integral over the GDR peak. However, the effective particle-hole vertex $\Gamma^{(\omega)}$ and the effective particle-hole interaction $(T^{(\omega)})$ contain also 2p-2h intermediate states (see figs. 3 and 4), and therefore due to eqs. (12),(13) $\Pi_{\mathrm{IV},A}$ has also cuts from 2p-2h states. That is, although the 1p-1h cuts are completely contained in Π_A, the 2p-2h (and higher) cuts contribute to both Π_A and Π_B, and therefore only the sum $\Pi_A + \Pi_B$ satisfies a dispersion relation. Therefore, due to the coupling between 1p-1h to 2p-2h states induced by the effective vertex $\Gamma^{(\omega)}$ and the effective interaction $T^{(\omega)}$, the relation (22) with n' referring to the 1p-1h excited states does not hold.

In the Landau-Migdal theory of finite nuclei the quantities $\Gamma^{(\omega)}$ and $T^{(\omega)}$ are treated as frequency (ω) independent quantities. In this case, the integral equation (13) for the vertex reduces to a familiar RPA equation with a frequency independent driving term and kernel, and as a result Γ has poles and cuts only due to the 1p-1h excited states. In this case Π_A satisfies a dispersion relation with the imaginary part exclusively due to 1p-1h states, and eq. (22) is satisfied. This approximation amounts to neglecting the coupling between the 1p-1h and 2p-2h states, and gives the same result for the sum rule as the 'quasiparticle approximation' used by Migdal et al [9].

The above discussion shows that the part κ_A satisfying the FH relation can be identified with the GDR contribution κ_{GDR}, if one assumes that the mixing of 2p-2h states into the GDR state on

[3] Eq. (20) holds also in relativistic field theory [13]. In a purely nonrelativistic approach, ϵ_F is replaced by M. We also note that, since F_1 is negative and typical values of F_1' are $F_1' \simeq -F_1/3$, the IV orbital g-factor (20) is enhanced relative to the free nucleon value $(\frac{1}{2})$, which is consistent with phenomenology [12]. In lowest order perturbation theory, this enhancement corresponds to the meson exchange current contributions discussed a long time ago by Miyazawa [14].

the sum rule is small. [4] To decide whether this assumption is reasonable or not requires model calculations. However, we would like to point out that the identification $\kappa_A = \kappa_{\rm GDR}$ seems to work phenomenologically: If the extension of the FH relation $\kappa_A = 2\,\delta g_{\ell,\rm IV}$ to the case N$\neq$Z is done as usually according to [8]

$$\kappa_A = \frac{A}{N}\delta g_{\ell,\pi} = -\frac{A}{Z}\delta g_{\ell,\nu}, \tag{23}$$

we can use the results of the calculations by Arima et al [12] for nuclei in the lead region ($\delta g_{\ell,\pi} \simeq 0.13$, $\delta g_{\ell,\nu} \simeq -0.08$) to obtain a theoretical value $\kappa_A \simeq 0.21$ for the lead region. On the other hand, the conclusion of Dale et al [7] emerging from a detailed analysis of photon scattering experiments on ^{209}Bi is that 'any reasonable prescription [to extract $\kappa_{\rm GDR}$] gives values between 0.2 and 0.3', which shows that the identification $\kappa_A = \kappa_{\rm GDR}$ at least does not lead to apparent contradictions between the measurements of $\kappa_{\rm GDR}$ and the calculations of the orbital g-factor.

In conclusion, we have derived generally the separation of the enhancement factor of the E1 sum rule into one part which is related to the orbital g-factor via the FH relation, and another part which has no connection to the orbital g-factor. We have also given arguments supporting the identification of the first part with the enhancement factor due to the integral over the strength in the region of the GDR, but further analysis is needed in order to arrive at firm conclusions.

ACKNOWLEDGEMENT

The author expresses his thanks to Prof. A. Arima (House of Councillors), Prof. K. Yazaki (Tokyo Woman's Christian Univ.), Prof. H. Sagawa (Univ. of Aizu), Prof. T. Suzuki (Fukui Univ.), Prof. T. Suzuki (Nihon Univ.), and Prof. K.W. Schmid (Univ. of Tübingen) for helpful discussions.

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