Description of Nucleons, Nuclear Matter and Quark Matter in an Effective Quark Theory

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Using an effective chiral quark theory for the single nucleon, we describe the equations of state (EOS) of nuclear matter (NM) and quark matter (QM), and discuss the phase transition between them. We pay special attention to the effects of the nucleon quark structure on the NM EOS, and of scalar diquark condensation (color superconductivity) on the QM EOS.

§1. Introduction

The behavior of matter at high baryon density, in particular the possibility of a phase transition from nuclear matter (NM) to quark matter (QM), is of great interest in connection with neutron stars and the possible existence of quark stars.[1)] In order to describe such a phase transition, one needs a consistent theoretical framework which can account for the properties of single nucleons, NM and QM at the same time. It has been shown recently[2)] that the Nambu-Jona-Lasinio (NJL) model[3)] is a strong candidate for this purpose: Besides a covariant description of the nucleon as a quark-diquark bound state,[4)] it also allows a description of a saturating NM equation of state (EOS)[5)] and of QM including the effects of color superconductivity.[6)] It is therefore of interest to study the conditions under which a transition between NM and QM becomes possible. It is the purpose of this paper to show our results on this phase transition. For details of the model and the formulation we refer to Ref. 2), and for other applications to Ref. 7).

§2. Description of nuclear matter and quark matter

The solution of the quark-diquark bound state equation in the NJL model leads to the nucleon mass $M_N(M)$ as a function of the constituent quark mass M. The effective (grand) potential for NM at zero temperature in the mean field approximation then has the form[5)]

$$V^{(\mathrm{NM})} = V_{\mathrm{vac}} + V_N + V_\omega \,, \tag{2·1}$$

where V_{vac} describes the polarization of the Dirac sea of quarks due to the presence of the valence nucleons, V_N arises from the Fermi motion of the valence nucleons and depends on the function $M_N(M)$, and V_ω is the contribution of the mean vector

field (ω_0) in NM. The conditions $\partial V^{(\mathrm{NM})}/\partial M = \partial V^{(\mathrm{NM})}/\partial \omega_0 = 0$ determine M and ω_0 for fixed chemical potential μ.

In effective field theories based on the linear realization of chiral symmetry, one often observes a collapse of the NM EOS because the σ mass decreases too rapidly as a function of the density.[8] It has been shown in Ref. 5), however, that the saturation properties of the NM EOS can be described if the quark structure of the nucleon is taken into account, provided that one eliminates the threshold for the unphysical decay of the nucleon into quarks. This can be done, for example, in the proper time regularization scheme by introducing an infrared cutoff (Λ_{IR}) in addition to the ultraviolet one.[9] The elimination of the unphysical decay threshold then leads to a positive scalar polarizability of the single nucleon, and this in turn gives rise to an effective $NN\sigma\sigma$ interaction which raises the σ meson mass and prevents the collapse.

The EOS of color superconducting QM in the mean field approximation has the form[2]

$$V^{(\mathrm{QM})} = V_{\mathrm{vac}} + V_Q + V_\Delta + V_\omega, \tag{2·2}$$

where V_{vac} describes the polarization of the Dirac sea of quarks due to the presence of the valence quarks, V_Q arises from the Fermi motion of the valence quarks without the effect of quark pairing, V_Δ arises from quark pairing in the scalar diquark channel and depends on the color superconducting gap (Δ), and V_ω is the contribution of the mean vector field (ω_0) in QM. The conditions $\partial V^{(\mathrm{QM})}/\partial M = \partial V^{(\mathrm{QM})}/\partial \Delta = \partial V^{(\mathrm{QM})}/\partial \omega_0 = 0$ determine M, Δ and ω_0 for fixed chemical potential μ.

After the calculation of the grand potential V, the EOS of NM and QM are obtained by applying the usual thermodynamic relationships. The chemical potential can be eliminated in favor of the baryon density ρ according to $\rho = -\partial V/\partial \mu$.

§3. Results

The elimination of unphysical quark decay thresholds by an infrared cutoff leads to a scalar polarizability of the single nucleon (curvature of the function $M_N(M)$), which stabilizes the NM EOS. This important effect is shown in Figs. 1 and 2.

Since we can describe stable NM as shown in Fig. 2, we can ask whether there is a phase transition to QM at high baryon densities. Figure 3 shows that there is no phase transition from NM (solid line) to normal, i.e., non-color superconducting, QM (dashed lines) in our model. For comparison, the EOS of a massless quark gas is also shown for two values of the bag constant B.

It is clear from Fig. 3 that the vector interaction makes the QM EOS stiff and hinders the phase transition very much. In Ref. 2) it has been shown that in QM one should actually use no vector interaction ($r_\omega = 0$). It has also been shown in that reference that one should not use an infrared cutoff in QM, since the role of this parameter is to mimic confinement. The results discussed below therefore refer to the choices $r_\omega = 0$ and $\Lambda_{\mathrm{IR}} = 0$ in QM.

We now investigate the effect of quark pairing in the scalar diquark channel (scalar diquark condensation) on the QM EOS and the phase transition. Figure 4

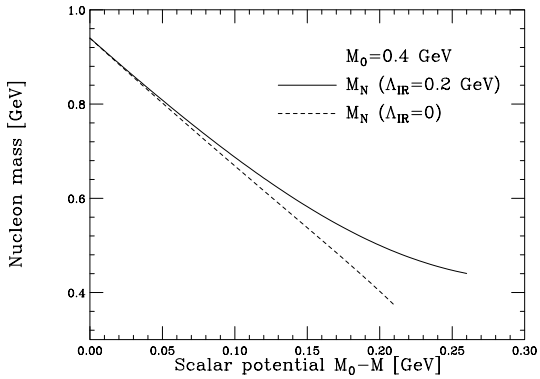

Fig. 1. Nucleon mass as a function of the scalar potential for $\Lambda_{\rm IR} = 0$ (dashed line) and $\Lambda_{\rm IR} = 0.2$ GeV (solid line).

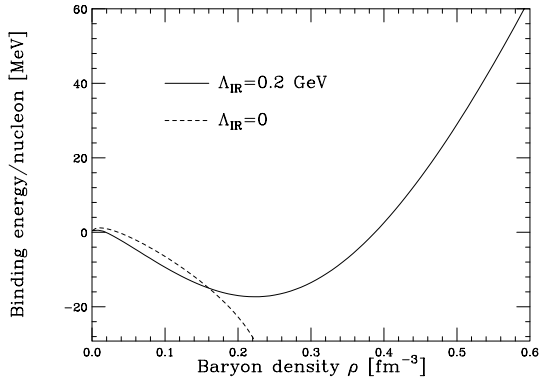

Fig. 2. The binding energy per nucleon in NM for $\Lambda_{\rm IR} = 0$ (dashed line) and $\Lambda_{\rm IR} = 0.2$ GeV (solid line).

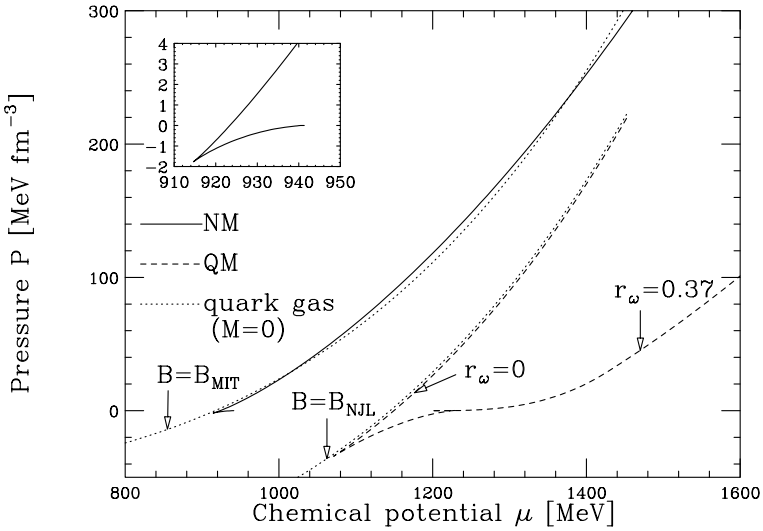

Fig. 3. The pressure as a function of the chemical potential for NM (solid line) and normal QM for two values of the strength of the vector interaction in QM (dashed lines). The value $r_\omega = 0.37$ follows from the saturation properties of NM, and also gives a reasonable value for the omega meson mass. The massless quark gas EOS is shown for comparison by the dotted lines, where $B_{\rm NJL}$ is the height of the Mexican hat shaped vacuum part $V_{\rm vac}$ in Eq. (2·2).

shows that the scalar diquark condensation gives rise to a substantial softening of the QM EOS, and to a phase transition from NM to QM at a transition density which decreases with increasing strength of the pairing interaction. The effective (constituent) quark mass and the color superconducting gap are shown for the same cases as functions of the baryon density in Figs. 5 and 6. Figure 5 demonstrates that the tendency toward chiral phase transition is much stronger in QM than in NM.

We now assume a particular value of the coupling constant in the scalar diquark channel, which leads to reasonable transition densities, and investigate the nature of the phase transition more closely. Figures 7 and 8 show the effective quark mass M^*

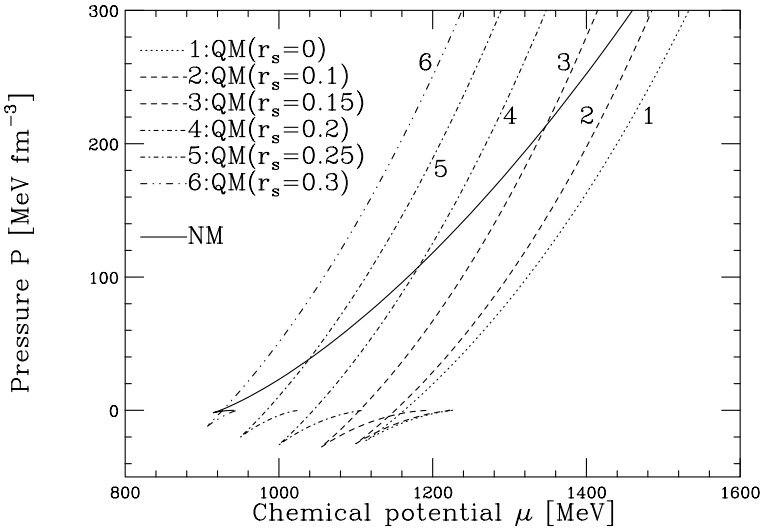

Fig. 4. Pressure as function of chemical potential in NM (solid line), and in QM for several values of r_s, which is the ratio of the coupling constant in the scalar diquark channel to the one in the pionic channel.

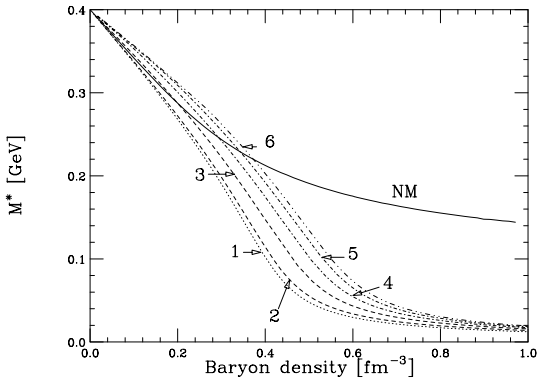

Fig. 5. Effective quark mass as function of baryon density for NM (solid line) and QM. Labels 1 to 6 correspond to the same values of the pairing strengths in QM as in Fig. 4.

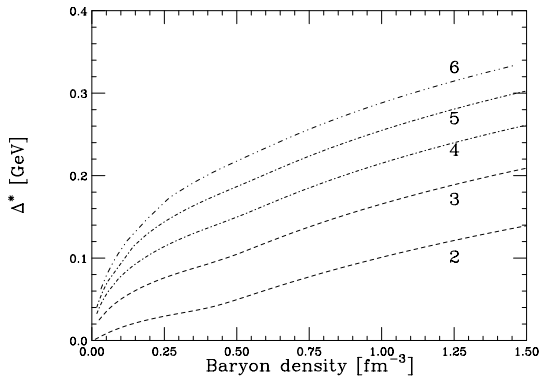

Fig. 6. Color superconducting gap as function of baryon density for QM. The labels 2 to 6 correspond to the same values of the pairing strengths in QM as in Fig. 4.

and the color superconducting gap Δ^* for the ground state of the system. The first order transitions from the vacuum (VAC) to NM, where in both phases chiral symmetry is broken and color symmetry is intact, and from NM to color superconducting QM, where chiral symmetry is largely restored and color symmetry is broken, as well as the mixed phases, can be clearly distinguished in these figures.

§4. Conclusions

We used the NJL model as an effective chiral quark theory to describe the single nucleon, the saturation properties of normal NM, and high density QM. We have

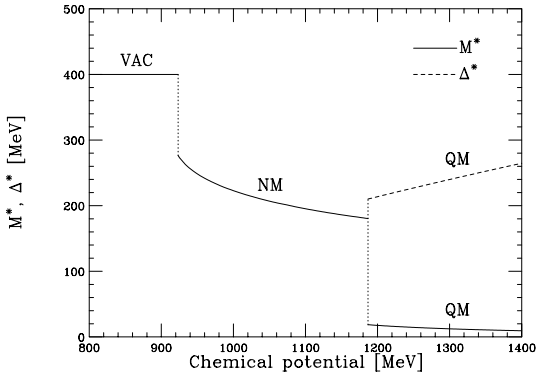

Fig. 7. Effective quark mass (solid lines) and gap (dashed line) of the ground state – vacuum (VAC), nuclear matter (NM) or superconducting quark matter (QM) – as functions of the chemical potential for $r_s = 0.2$.

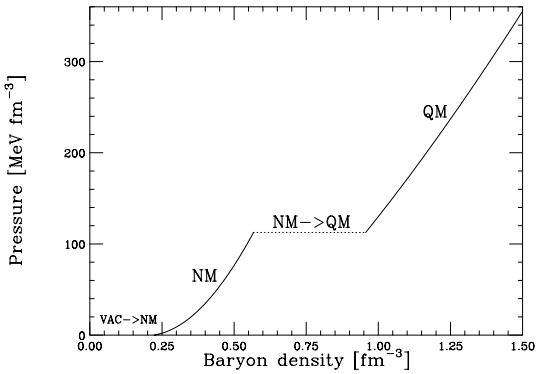

Fig. 8. Pressure of the ground state of the system – vacuum (VAC), nuclear matter (NM) or superconducting quark matter (QM) – as functions of the baryon density for $r_s = 0.2$. The mixed phases are also indicated.

shown that there is a phase transition from NM to QM, provided that the effects of scalar diquark condensation (color superconductivity) are taken into account. This phase transition is characterized by the restoration of chiral symmetry and the spontaneous breaking of color symmetry in the high density region.

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