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PROPERTIES OF NUCLEONS AND NUCLEAR MATTER IN THE QUARK–DIQUARK MODEL AND EXTENSION TO FINITE DENSITY

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We describe the nucleon as a quark-diquark bound state in a simple approximation to the full Faddeev method in the Nambu-Jona-Lasinio model. In the first part of this work, we concentrate on the role of axial vector diquark correlations for the static properties of the nucleon. In the second part, we construct an equation of state of nuclear matter based on the quark-diquark model for the single nucleon.

1. Introduction

The relativistic Faddeev approach [1,2] based on the Nambu-Jona-Lasinio (NJL) model [3] is a powerful method to study the structure of the nucleon in terms of quarks and composite diquarks. In particular, this method seems to work very well for the description of structure functions of the nucleon [4], if one assumes that the dominant configuration consists of a scalar diquark ($J^P = 0^+$) and a quark. The method seems also suitable for the investigation of modifications of nucleon properties inside the nuclear medium. For this purpose, however, one first has to construct a stable equation of state (EOS) for the nuclear matter ground state.

In the first part of this work, we will consider the role of correlations in the axial vector (a.v.) diquark channel ($J^P = 1^+$) for the static properties of the nucleon. In the second part, we will discuss a simple hybrid model of nuclear matter, which combines the quark-diquark description of the single nucleon with the mean field (Hartree) description of nuclear matter [6]. We will point out that a simple method based on the introduction of an infrared cut-off to mimic confinement effects leads to a saturating EOS.

2. Static properties of the nucleon

In our calculation we include the interacting scalar and a.v. diquark channels with the effective 4-fermi coupling constants g_s and g_a, respectively. The ratios $r_s = g_s/g_\pi$ and $r_a = g_a/g_\pi$ of these effective coupling constants to the one in the pionic $q\overline{q}$ channel (g_π) reflect different possible forms of the 4-fermi interaction lagrangian [1]. We will use r_a as a free parameter, and adjust r_s (for each r_a) so as to reproduce the nucleon mass as the pole of the quark-diquark t-matrix. For technical reasons, however, we will limit ourselves in this work to the "static approximation" to the full Faddeev equation [5] when describing the nucleon as a quark-diquark state. We use the constituent quark mass $M = 400$ MeV, and employ the three-momentum sharp cut-off scheme, which is equivalent to the Lepage-Brodsky invariant mass cut-off when used in terms of light cone variables [8].

In Table 1 we show three parameter sets for r_s and r_a together with the corresponding scalar diquark masses and the probabilities of the scalar diquark channel. (The "probability" is defined here as the contribution to the baryon number.) Set I (III) involves relatively strong (weak) scalar diquark correlations, while set II describes an intermediate situation. The value of r_a used in set III reproduces also a bound state for the delta isobar. For each parameter set we show the results for the magnetic moments, the isovector and isoscalar axial coupling constants, and the pion-nucleon coupling constant, including the effects of the (composite) pion cloud around the constituent quarks.

Table 1. The scalar diquark mass (M_s), the probability of the scalar diquark channel (W_s), and some static properties of the nucleon are shown for three different parameter sets.

case	I	II	III	exp.
r_a	0	0.25	0.66	
r_s	0.73	0.63	0.50	
M_s[MeV]	596	684	766	
W_s[%]	100	93	61	
μ_p	2.32	2.87	2.96	2.79
μ_n	-1.39	-2.08	-1.80	-1.91
$g_A^{(3)}$	0.66	0.76	0.81	1.26
$g_{\pi NN}$	7.5	12.82	15.34	13.2
$g_A^{(0)}$	0.60	0.41	0.30	$0.2 \sim 0.3$

In Fig. 1 we plot the proton and neutron magnetic moments as functions of r_a. These results indicate the necessity of some amount of a.v. diquark correlations, but not strong ones. In terms of probabilities, the admixture of the a.v. diquark channel should be less than 10%. Since the study of the nucleon structure functions leads to the same conclusion [7], our results support the view of a dominant scalar diquark configuration.

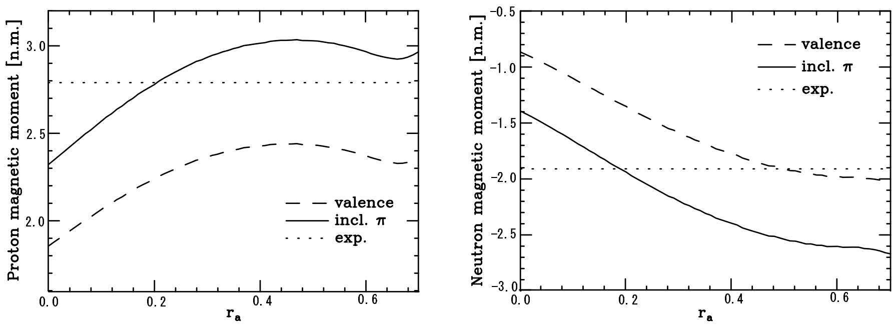

Fig. 1. The proton and neutron magnetic moments as functions of r_a. The dashed lines show the results without pion cloud contributions, and the solid lines include the effects of the pion cloud.

3. Nuclear matter EOS

In models based on the linear realization of chiral symmetry, the stability of the nuclear matter EOS is an old problem [9]. In these models, the vacuum effective potential (or energy density) $\mathcal{E}_V(\sigma)$ has a Mexican hat form as a function of the self consistent scalar field in the medium (σ). The curvature of this potential decreases as one moves away from the vacuum value σ_0 towards smaller values of σ, This implies strongly attractive contributions to the sigma meson mass $M_\sigma^2 \propto \partial^2\mathcal{E}/\partial\sigma^2$ and the effective NN interaction in the medium, and usually leads to a collapse of the system for increasing density.

On the other hand, if the scalar field is coupled to the quarks inside the nucleon instead of an elementary nucleon, the nucleon mass in the scalar field $M_N(\sigma)$ can have a finite curvature $\partial^2 M_N/\partial\sigma^2$. If this 'scalar polarizability' [10] of the nucleon is positive and large enough, it can lead to a repulsive contribution to $\partial^2\mathcal{E}/\partial\sigma^2$, i.e., the sigma meson mass, and can avoid the collapse discussed above. A positive curvature of M_N persisting also for large scalar potentials $\Phi \equiv \sigma_0 - \sigma \propto M - M_0$ is possible only if there are no unphysical thresholds which force the nucleon mass to vanish as $M \to 0$. This aspect of the confinement, namely the absence of unphysical quark- diquark or 3-quark thresholds, can be incorporated into the NJL model by introducing an infrared cut-off (μ) in addition to the standard ultraviolet one in the framework of the proper-time regularization scheme [11].

In Fig. 2 we plot the nucleon mass in the scalar field for the cases $\mu = 0$ and $\mu = 0.2$ GeV. We also show the binding energy per nucleon as a function of the density for these two cases. The details of this 'hybrid model' for the nuclear matter EOS, which is based on the quark-diquark description of the free nucleon discussed above and very similar in spirit to the successful model of Guichon and collaborators [12], are described in ref. [6]. The case $\mu = 0$ leads to the collapse of the system for the reasons explained above, while the case $\mu = 0.2$ GeV gives a positive curvature of $M_N(M)$ and a saturating EOS. Some interesting implications of this EOS, like the suppression of the famous "Z-graph contributions", are discussed in ref. [6].

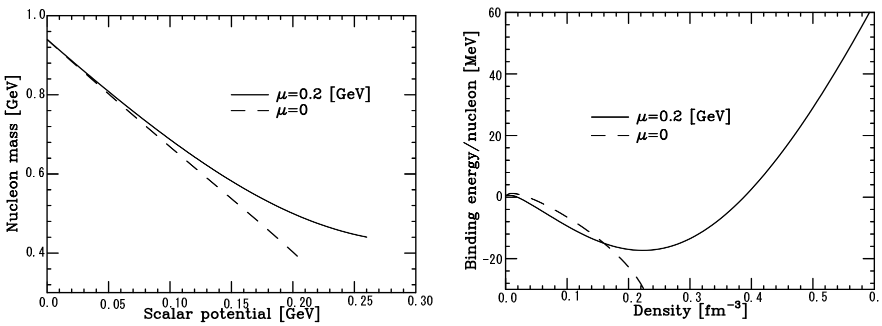

Fig. 2. The nucleon mass in the quark-diquark model as a function of the scalar potential (left figure) and the binding energy per nucleon as a function of the density (right figure) for the cases $\mu = 0$ (dashed lines) and $\mu = 0.2$ GeV (solid lines).

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