Gauge invariance and the E1 sum rule in nuclei *

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Abstract

The connection between the enhancement factor $(1+\kappa)$ of the photonuclear E1 sum rule and the orbital angular momentum g-factor (g_ℓ) of a bound nucleon is investigated in the framework of the Landau-Migdal theory for isospin asymmetric nuclear matter. Special emphasis is put on the role of gauge invariance to establish the $\kappa - g_\ell$ relation.

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1 Introduction

The enhancement factor $(1 + \kappa)$ of the photonuclear E1 sum rule and the orbital angular momentum g-factor (g_ℓ) of a bound nucleon have attracted the attention of nuclear theorists as well as experimentalists for a long time, since these quantities reflect the presence of exchange forces and mesonic degrees of freedom in nuclei [1]. More than 30 years ago, Fujita and Hirata [2] used the isospin symmetric Fermi gas as a model for an N=Z nucleus to derive the simple relation $1 + \kappa = 2\, g_{\ell,\mathrm{IV}}$ between κ and the isovector (IV) part of g_ℓ in first order perturbation theory. Later it has been shown [3] that, because of the presence of correlations between the nucleons, only a part of the total κ is related to $g_{\ell,\mathrm{IV}}$. It has been argued [4] that this part of κ is related to the sum of the E1 strength in the region of the isovector giant dipole resonance (GDR). In more recent years [5], this modified $\kappa - g_\ell$ relation has been used to analyse the results of photo-neutron experiments [6] and photon scattering experiments [7], in particular for nuclei in the lead region.

On the other hand, as early as 1965, Migdal and collaborators [8] used an approach based on a gas of quasiparticles to relate κ to the parameters characterizing the interaction between the quasiparticles (the Landau-Migdal parameters). Combining this relation with the more general one between $g_{\ell,\mathrm{IV}}$ and the Landau-Migdal parameters [9], their approach suggested that the relation $1+\kappa = 2\, g_{\ell,\mathrm{IV}}$ holds more generally without recourse to perturbation theory. The fact that their result involves the total κ instead of just a part of it reflects the quasiparticle gas approximation.

The main advantage of the Landau-Migdal theory [9], which is based on the Fermi liquid approach due to Landau [10], is that symmetries, like gauge invariance and Galilei invariance, are incorporated rigorously. The Fermi liquid approach to discuss sum rules in nuclear matter has therefore turned out to be very fruitful, and has been used in several papers on giant resonances [11]. However, to the best of our knowledge, a general discussion of the κ-g_ℓ relation is still lacking. In view of this fact, and also in view of the strong recent interest in nuclear giant resonances [12] and the recent attempts to extend the range of applicability of the Landau-Migdal theory [13], in this paper we will present a general discussion on the κ-g_ℓ relation in isospin asymmetric nuclear matter. The aims of our work are as follows: First, we will extend the relations obtained previously for the orbital g-factor [14] and the E1 enhancement factor [2] to the case of $N \neq Z$, putting special emphasis on the role of gauge invariance. Second, we will identify the physical processes which are taken into account in the κ-g_ℓ relation. For more detailed discussions and formal derivations of the results we refer to ref.[15].

2 The orbital g-factor

Consider a nucleon (quasiparticle) in nuclear matter at the Fermi surface. Its orbital angular momentum g-factor may be defined in terms of its electromagnetic current $\boldsymbol{j}$ by the relation[1]

$$\boldsymbol{j}(q=0) \equiv \frac{\boldsymbol{p}}{M}\, g_\ell\,, \tag{1}$$

where M is the free nucleon mass and $\boldsymbol{p}$ the momentum. In the Landau-Migdal theory, the current j is, up to a normalization factor, equivalent to

[1] This identification, which holds in nuclear matter, follows directly from the definition of the magnetic moment, see ref.[14].

the effective electromagnetic vertex $\Gamma_{\rm eff}$, which is defined as that part of the full vertex Γ which is irreducible in the particle-hole (ph) channel. That is, the integral equation for the full vertex Γ can be written in the form (see Fig.1)

$$\Gamma = \Gamma_{\rm eff} - i\,T_{\rm eff}\,A\,\Gamma \tag{2}$$

where A is the pole part of the ph propagator, and the vertex $\Gamma_{\rm eff}$ as well as the interaction $T_{\rm eff}$ are by definition irreducible in the ph channel[2] . The

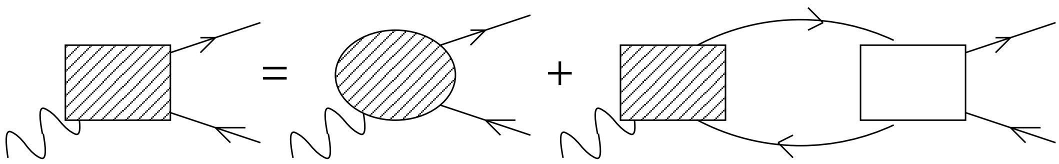

Figure 1: Integral equation for the vertex (shaded square) in terms of the ph-irreducible vertex (shaded circle) and the ph-irreducible interaction (white square).

effective vertex $\Gamma_{\rm eff}$ is renormalized by meson exchange currents, 2p-2h excitations, $N\overline{N}$ excitations, etc. Some examples are shown in Fig.2. Similarly, the effective interaction $T_{\rm eff}$ is also renormalized by processes like 2p-2h excitations, see Fig. 3 for examples.

The use of gauge invariance and Lorentz invariance (or Galilei invariance in the nonrelativistic case), combined with the integral equation (2), allows the determination of the quasiparticle current $\boldsymbol{j}(q = 0)$. The arguments,

[2] More precisely, they are irreducible with respect to states which have ph cuts, see ref.[15] for details. In the Landau-Migdal theory [9] these quantities are usually denoted as $\Gamma^{(\omega)}$ and $T^{(\omega)}$.

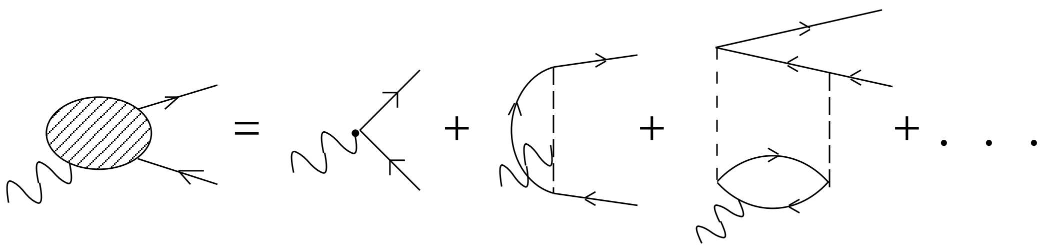

Figure 2: Examples for diagrams contained in the effective vertex Γ_{eff}. The first one represents the noninteracting part, the second a meson exchange current part, and the third one shows a 2p-2h excitation process.

which are very general and hold also in relativistic field theory [14], are explained in detail in ref.[15]. In terms of the orbital g-factor of eq.(1), the results for the proton (p) and neutron (n) can be expressed as follows:

$$g_\ell(p) = \frac{M}{\mu_p} - \frac{Mv_F}{3p_F}\, F_1(pn)\, \frac{\mu_n}{\mu_p}\, \frac{N}{A}\, \beta \tag{3}$$

$$g_\ell(n) = \frac{Mv_F}{3p_F}\, F_1(pn)\, \frac{Z}{A}\, \beta\,. \tag{4}$$

Here the quantity $\beta = \left[1 - \left(\frac{N-Z}{A}\right)^2\right]^{-\frac{1}{3}}$ expresses the neutron excess, μ_p and μ_n are the chemical potentials (i.e., Fermi energies including the rest masses) of protons and neutrons, p_F and v_F are the Fermi momentum and Fermi velocity for the case N=Z, and $F_1(pn)$ is the dimensionless $\ell = 1$ Landau-Migdal parameter for the pn-interaction at the Fermi surface [9][3] .

In spite of the simplicity of the results (3) and (4), they are very general and include all possible effects of meson exchange currents and configuration mixings. Fig.4 shows some examples for diagrams, drawn in the particle-

[3] The relation to the more familiar parameters F_1 and F_1' is $F_1(pn) = F_1 - F_1'$.

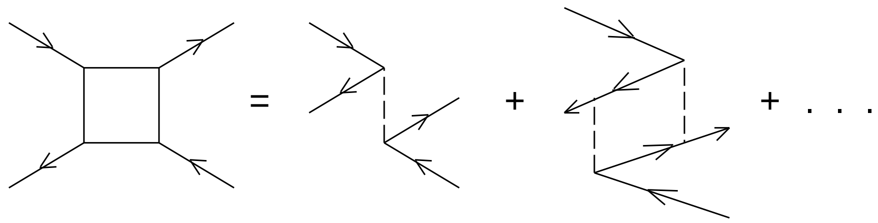

Figure 3: Examples for diagrams contained in the effective interaction T_{eff}. The first one represents the "bare" ph interaction, and the second one a 2p-2h excitation process. If the external particles are on the Fermi surface, T_{eff} becomes the Landau-Migdal interaction.

particle channel, which contribute to the renormalization of the orbital g-factors. We can make several important observations from eqs.(3) and (4):

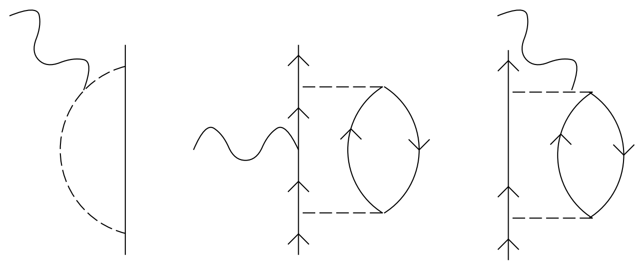

Figure 4: Examples for meson exchange current and configuration mixing processes which contribute to the renormalization of the orbital g-factors.

First, since $F_1(pn) < 0$, the orbital g-factor of the proton is enhanced, while that of the neutron is negative. Second, one can show that $F_1(pn)$ depends only weakly on the neutron excess, and up to $\mathcal{O}\left(\frac{N-Z}{A}\right)$, the dependence on the neutron excess is as shown by the factors N/A and Z/A in (3) and (4). In symmetric nuclear matter and in the nonrelativistic limit ($\mu_p = \mu_n = M$), the corrections to the g-factors are purely isovector, while in matter with neutron excess the proton g-factor is renormalized more strongly than the neutron one. Third, the g-factors satisfy the following relation:

$$\frac{Z}{A}\,\frac{\mu_p}{M}\,g_\ell(p) + \frac{N}{A}\,\frac{\mu_n}{M}\,g_\ell(n) = \frac{Z}{A}\,, \tag{5}$$

which is the extension to $N \neq Z$ of the well known fact [14, 16] that the isoscalar orbital g-factor is renormalized exclusively by relativistic effects.

3 The E1 sum rule and the $\boldsymbol{\kappa} - g_\ell$ relation

The strength function (cross section) for absorption of unpolarized photons by a nucleus in its ground state $|0>$ is given by

$$\begin{aligned} \sigma(\omega) &= 4\pi^2 \sum_n |\langle n|\hat{j}(\boldsymbol{q})\frac{1}{\sqrt{\omega}}|0\rangle|^2\,\delta(\omega-\omega_{n0}) && (6)\\ &\xrightarrow{\text{LWL}} 4\pi^2 \sum_n |\langle n|\hat{j}(\boldsymbol{q}=0)\frac{1}{\sqrt{\omega}}|0\rangle|^2\,\delta(\omega-\omega_{n0}), && (7) \end{aligned}$$

where $\hat{j}$ denotes the current operator[4] , $\omega_{n0} = E_n - E_0$ is the excitation energy of the state $|n>$, and the long wave length limit (LWL) indicated in (7) holds if $|\boldsymbol{q}|R << 1$, where R is the nuclear radius. The (energy non-weighted) sum rule then becomes:

$$\begin{aligned} S &\equiv \int_0^\infty \mathrm{d}\omega\,\sigma(\omega) = 4\pi^2 \sum_n \frac{1}{\omega_{n0}}\,|\langle n|\hat{j}(\boldsymbol{q}=0)|0\rangle|^2 && (8)\\ &= -2\pi^2\langle 0|\,[[H,D]\,,D]\,|0\rangle, && (9) \end{aligned}$$

[4] In order to eliminate the spurious effect of the center of mass motion, the current operator in this context is defined in terms of the effective charges $q_p = e_p\,N/A$ and $q_n = -e_p\,Z/A$, see ref. [15] for details.We also note that, for the case of unpolarized photons, the current operator $\hat{j}$ as well as the dipole operator D in eq.(9) refer to any space component, e.g., the 3-component. With this choice, the current-current correlation functions Π of this section refer to the 33-component.

where D is the dipole operator. This sum rule, however, is not directly observable, because the LWL contradicts the integration up to $\omega = \infty$. (Note that for a real photon we have $|\boldsymbol{q}| = \omega$.) It is therefore important to investigate whether it is possible to identify a part of the sum rule which is valid in the region of the GDR, where the LWL is justified. For this purpose, we again use the methods based on gauge invariance and the Landau-Migdal theory: The cross section in the LWL can be expressed in terms of the current-current correlation function $\Pi(\boldsymbol{q}, \omega)$ as

$$\sigma(\omega) = \frac{4\pi}{\omega}\, \mathrm{Im}\, \Pi(\boldsymbol{q} = 0, \omega) \qquad (\omega > 0). \tag{10}$$

Gauge invariance can then be used to derive a Ward-Takahashi identity for the correlation function [15], which gives in the low energy limit

$$S = 2\pi^2\, \Pi(\boldsymbol{q} = 0, \omega = 0). \tag{11}$$

In nuclear matter, one must take the limit $\omega \to 0$ first before setting $\boldsymbol{q} = 0$. Because the correlation function can be expressed in terms of the full vertex Γ, one can use the integral equation (2) to split Π into 2 pieces:

$$\Pi = i\Gamma_{\text{eff}}\, A\, \Gamma + i\Gamma_0\, B\, \Gamma_{\text{eff}} \equiv \Pi_A + \Pi_B\,, \tag{12}$$

where the quantity A is the same as in eq.(2) and represents the product of the pole parts of particle and hole propagators, and B is the rest (product of particle-particle propagators, etc.) in the decomposition of the product of two propagators ($SS = A + B$). The part Π_A is shown graphically in Fig. 5, and an example of a graph which contributes exclusively to Π_B is shown in Fig. 6.

The difference between Π_A and Π_B is that the former contains ph cuts while the latter has only cuts due to higher excited states like 2p-2h etc. If

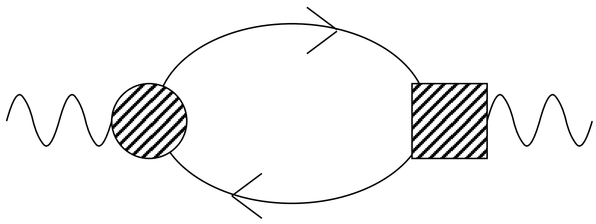

Figure 5: Graphical representation of the part Π_A in eq. (12). The symbols are the same as used in Fig.1.

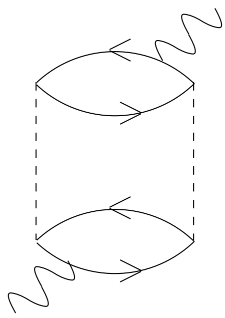

Figure 6: Example of a diagram which contributes exclusively to Π_B of eq.(12).

we use the RPA equation for the vertex (eq.(2)), we see that Π_A agrees with the familiar correlation function of the RPA theory, which is the starting point of almost all calculations of response functions, but processes like meson exchange currents, 2p-2h configuration mixings etc., are included in the definition of the effective vertex and the effective interaction.

In nuclear matter, one can now again use gauge invariance to completely specify the part Π_A and the associated part S_A of the sum rule (11). The result can be expressed by the following relation:

$$S_A = (\mathrm{TRK})\,(1+\kappa_A)\ , \tag{13}$$

where $(\mathrm{TRK}){=}2\pi^2 e_p^2 \dfrac{NZ}{AM}$ is the Thomas-Reiche-Kuhn sum rule value, and

$$1+\kappa_A = g_\ell(p) - g_\ell(n), \tag{14}$$

where the orbital g-factors are given in (3) and (4). An equivalent way to express the same result is as follows,

$$g_\ell(p) = 1 + \frac{N}{A}\kappa_A \tag{15}$$

$$g_\ell(n) = -\frac{Z}{A}\kappa_A \tag{16}$$

with

$$\kappa_A = -\frac{M v_F}{3\,p_F}\,F_1(pn)\,\left[1-\left(\frac{N-Z}{A}\right)^2\right]^{-\frac{1}{3}} . \tag{17}$$

The remaining part of the LWL sum rule, $S_B \equiv (\mathrm{TRK})\,\kappa_B$, which originates from Π_B, has no connection to the orbital g-factors and cannot be specified by using only symmetry principles.

One can now argue that the quantity κ_A can be approximately identified with the enhancement factor κ_{GDR}, which is extracted from the area under the Lorentzian of the GDR curve. The details are discussed in ref.[15], and the essential points of the argument are as follows:

First, only the part Π_A in eq.(12) involves the full vertex Γ, which is a solution to the RPA equation (2). Therefore, the associated strength function $\sigma_A(\omega)$ is the result of a renormalized RPA calculation, which produces a resonance structure due to the collective superposition of ph pairs. The part Π_B has no such resonance structure.

Second, in general the contribution $\Pi^{(i)}$ of a given time ordered diagram i to the correlation function can be split into its A and B-parts according to $\Pi^{(i)} = \Pi_A^{(i)} + \Pi_B^{(i)}$. Examples for diagrams which have both A and B-parts are shown in Fig. 7. The diagrams $\Pi^{(i)}$ in these examples involve

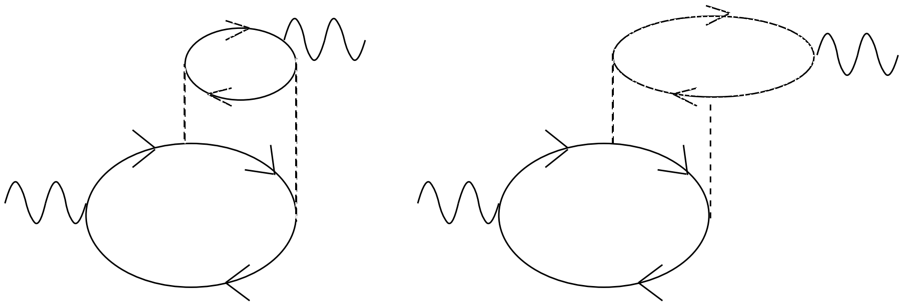

Figure 7: Example for diagrams with 2p-2h admixtures originating from the effective vertex (left) and from the effective interaction (right). These diagrams contribute to both Π_A and Π_B.

"high energy denominators" of the form $(\omega - \Delta E(2p-2h))^{-1}$. Their A-parts are obtained from $\Pi^{(i)}$ by replacing these high energy denominators by $(\Delta E_{\text{eff}}(1p-1h) - \Delta E(2p-2h))^{-1}$, where $\Delta E_{\text{eff}}(1p-1h)$ refers to the excitation energy of the ph state entering the vertex in the first diagram, and to the average of the ph states entering and leaving the interaction in the second diagram of Fig. 7. As long as, on the average, $\Delta E(2p-2h) >> \Delta E_{\text{eff}}(1p-1h)$, we have the approximate relation

$$\Pi^{(i)}(\boldsymbol{q}=0, \omega=0) \simeq \Pi_A^{(i)}(\boldsymbol{q}=0, \omega=0)\,, \tag{18}$$

which indicates that the A-part gives the dominant contribution of the diagram i to the sum rule: $S^{(i)} \simeq S_A^{(i)}$. Under this condition, the B-part, which gives rise to the spreading width of ph-states in the low energy region, gives only a minor contribution to the sum rule. This in turn implies that the main contribution from Π_B to the LWL sum rule comes from those diagrams which involve no ph cuts at all, like the example shown in Fig. 6. It is known that, because of the short range nature of the tensor force, these diagrams involve large excitation energies, on the average several hundred MeV [17]. Therefore, these diagrams, for which the LWL is unjustified anyhow, will contribute to the sum rule only in the high energy region well beyond the GDR.

In conclusion of the above discussion, we can say that the approximate relation $\kappa_A \simeq \kappa_{\rm GDR}$ holds, although it is difficult to specify it more quantitatively without using model calculations.

For the sake of illustration of the $\kappa - g_\ell$ relation (14), let us discuss the nuclei in the ^{208}Pb region. The empirical values of $g_\ell(p)$ and $g_\ell(n)$ for nuclei in the lead region have been derived by Yamazaki (see ref.[1]) as $g_\ell(p) \simeq 1 + 0.13$, $g_\ell(n) \simeq -0.08$. These values are very close to the ones calculated in ref. [16] from configuration mixing and meson exchange current effects, see table 7.12 of ref. [16]. From eq. (14) one then obtains the estimate $\kappa_A \simeq 0.21$. In the analysis of ref.[7], which uses the experimentally measured scattering cross section to extract the total photoabsorption cross section via dispersion relations, it was concluded that "any reasonable prescription gives (experimental) values of $\kappa_{\rm GDR}$ between 0.2 and 0.3", where $\kappa_{\rm GDR}$ was extracted from the area under a Lorentzian curve fitted to the GDR. This indicates at least a qualitative consistency between theory and experiment, since our κ_A can be identified with $\kappa_{\rm GDR}$ as discussed above.

4 Summary and conclusions

In this work we used the Landau-Migdal theory to discuss the orbital g-factor of a quasiparticle and the E1 sum rule for isospin asymmetric nuclear matter. The relations obtained for the orbital g-factors are in principle exact and hold also in relativistic field theory. For the E1 sum rule, we had to restrict ourselves to a nonrelativistic framework because of the problems arising from the center of mass motion.

We have split the strength function into two parts, where one comes from the p-h cuts including the effects of the higher excited states and meson exchange currents via their real parts, and the other comes from cuts at higher excitation energies. We have shown generally that the former part is related to the orbital g-factors, while the latter part has no relation to them. The former part has a close relation to the collective excitations of the system, i.e., the zero sound modes in infinite systems and the giant resonances in finite nuclei. We have discussed the importance of the $\kappa - g_\ell$ relation, which effectively separates the observable part of the LWL sum rule, which is related to the strength function in the low energy region, from the rest. Our discussions, which do not rely on perturbation theory, can serve to put many previous investigations on the $\kappa - g_\ell$ relation on a theoretically firm basis.

Concerning possible extensions, we would like to remark the following points: First, the methods used here to relate κ to g_ℓ refer to infinite nuclear matter, and it would be interesting to investigate to what extent they can be applied also to finite nuclei. Second, as we mentioned in the Introduction, very interesting attempts are now being made to extend the range of applicability of the Landau-Migdal theory to give a more general description of nuclear collective vibrations. The basic idea is to generalize the definition of the quantity A, which appears for example in the equation for the vertex (2), so as to include also more complicated configurations. It would be very interesting to see whether the results derived in this paper can be extended according to these lines.

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