Structure function in nuclear matter in the NJL model

H. Mineo*, W. Bentz†, N. Ishii**, A.W. Thomas‡ and K. Yazaki§

*Department of Physics, National Taiwan University,
1 Roosevelt, Section 4, Taipei, Taiwan
†Department of Physics, School of Science, Tokai University,
Hiratsuka-shi, Kanagawa 259-1292, Japan
**The Institute of Physical and Chemical Research (RIKEN),
Hirosawa, Wako-shi, Saitama 351-0198, Japan
‡Special Research Center for the Subatomic Structure of Matter, and
Department of Physics and Mathematical Physics, The University of Adelaide,
Adelaide, SA 5005, Australia
§Department of Physics, Tokyo Woman's Christian University,
Suginami-ku, Tokyo 167-8585, Japan

Abstract. We present a consistent description of the structure function of a free nucleon, the equation of state of nuclear matter, and the structure function of a nucleon bound in nuclear matter, in the framework of the Nambu-Jona-Lasinio (NJL) model. The important role of the mean vector field in the nuclear medium to explain the EMC effect is emphasized.

INTRODUCTION

In this work we will be concerned with the medium modifications of the spin independent nuclear structure functions measured in deep inelastic scattering of leptons, that is, the EMC effect [1, 2]. It is known [3, 4] that the Fermi motion and binding effects on the level of nucleons cannot explain the observed reduction of the nuclear structure function in the range of Bjorken x between 0.4 and 0.8. It is therefore a challenging task to describe the nuclear systems in terms of nucleons with internal quark structure, and to investigate whether the binding effects on the quark level can account for the observations. For this purpose, we will use the Nambu-Jona-Lasinio (NJL) model [5] as an effective chiral quark theory. It has been shown [6] that the quark-diquark description of the single nucleon, which is based on the relativistic Faddeev approach [7], can be combined successfully with the mean field description of the nuclear matter equation of state. Since the description of the nucleon is completely covariant, off-shell effects can be investigated unambiguously, and the Ward identities for baryon number and momentum conservation can be incorporated rigorously from the outset, which is particularly important for the description of structure functions. This framework is therefore a powerful tool to investigate the origin of the EMC effect in terms of binding on the level of quarks. This is the purpose of our present work, where we will consider the case of infinite nuclear matter as a first step. For simplicity, we will limit ourselves to a valence quark description, that is, the effects of sea quarks will not be considered in this paper.

QUARK LIGHT CONE MOMENTUM DISTRIBUTIONS IN NUCLEAR MATTER

The light cone momentum distribution of quarks per nucleon in a nucleus with mass number A is defined as

$$f_{q/A}(x_A) = \frac{P_-}{A^2} \int \frac{\mathrm{d}w^-}{2\pi} e^{iP_- x_A w^-/A} < A,P|\overline{\psi}(0)\gamma^+\psi(w^-)|A,P > \ . \tag{1}$$

Here P^μ is the total 4-momentum of the nucleus and P_- its "minus component" [1], and ψ is the quark field. Since we will refer to the case $N = Z$, we need only the isospin symmetric combination $f_{q/A} \equiv f_{u/A} + f_{d/A}$. The quantity x_A is the Bjorken variable of the nucleus, which is equivalent to the fraction of the total P_- carried by a quark, multiplied by A. In the rest system of the nucleus (nuclear matter) we have $P_-/A = \varepsilon_F/\sqrt{2}$, where ε_F is the Fermi energy (that is, the mass per nucleon or the chemical potential) of the nucleons.

To calculate the distribution (1), we make use of the familiar one-dimensional convolution formula

$$f_{q/A}(x_A) = \int \mathrm{d}y_A \int \mathrm{d}z\, \delta(x_A - y_A z)\, f_{q/N}(z)\, f_{N/A}(y_A)\,, \tag{2}$$

where $f_{q/N}(z)$ and $f_{N/A}(y_A)$ are the light cone momentum distributions of quarks in the nucleon, and of nucleons in the nucleus (per nucleon), respectively. (The definitions of these distributions can be found in Ref.[8].) The variable z is the fraction of the minus component of the nucleon momentum carried by a quark, and y_A is A times the fraction of the total P_- carried by a nucleon.

For the evaluation of the quark distribution in the nucleon ($f_{q/N}$), we describe the nucleon as a bound state of a quark and a scalar ($J^P = 0^+, T = 0$) diquark in the NJL model. The distribution $f_{q/N}$ can then be obtained by a straight forward Feynman diagram calculation [8, 9]. The presence of the nuclear medium is taken into account via scalar and vector mean fields which act on the quarks in the nucleon. In order to determine these mean fields for a given density, and also in order to evaluate the nucleon distribution function $f_{N/A}$, we make use of the successful mean field description of the nuclear matter ground state in the NJL model, which takes into account the quark substructure of the nucleons [6].

The most important relation of this approach, which shows the direct effect of the vector mean field on the quark distribution function, is as follows:

$$f_{q/A}(x_A) = \frac{\varepsilon_F}{E_F} f_{q/A0}(x'_A = \frac{\varepsilon_F}{E_F} x_A - \frac{V_0}{E_F}). \tag{3}$$

Here the distribution without the explicit effect of the mean vector field is denoted as $f_{q/A0}(x'_A)$, and the nucleon Fermi energy has the form $\varepsilon_F = E_F + 3V_0$, where $E_F =$

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[1] Our notations for light cone variables are $a^\pm = (a^0 \pm a^3)/\sqrt{2}$, $a_\pm = (a_0 \pm a_3)/\sqrt{2}$. The normalization of the nuclear state vector is $< A,P|\overline{\psi}\gamma^+\psi|A,P >= 3A$.

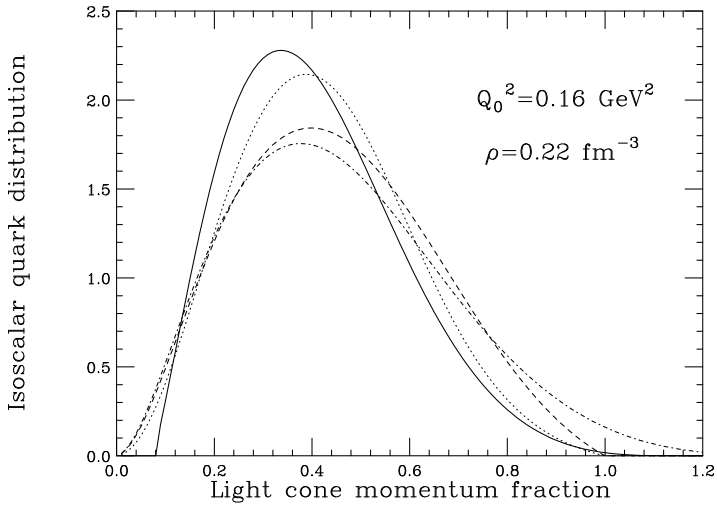

FIGURE 1. Sum of the valence up and down quark light cone momentum distributions at the low energy scale ($Q_0^2 = 0.16\,\mathrm{GeV}^2$). For explanation of the lines, see text.

$\sqrt{M_N^2 + p_F^2}$ (M_N is the effective nucleon mass), and V_0 is the vector field acting on the quarks in the nucleon. The actual calculation of $f_{q/A}$ therefore proceeds as follows: First we calculate the distribution for a free nucleon in the quark-diquark approach, then we replace the quark, diquark and nucleon masses by the effective ones according to the nuclear matter equation of state, then we include the effect of the Fermi motion of nucleons with effective mass M_N, and finally we perform the scale transformation (3) to include the direct effect of the vector mean field. The actual calculations are carried out in the proper time regularization scheme which avoids unphysical quark decay thresholds for the nucleon, see Ref.[8] for details.

RESULTS

The medium modifications of the isoscalar valence quark distribution at the saturation density of our nuclear matter equation of state are shown in Fig.1 for the low energy scale, for which we use the value $Q_0 = 0.4$ GeV. The dotted line shows the distribution in a free nucleon, which is consistent with the empirical parametrizations [10]. The dashed line shows the result when all masses are replaced by the effective ones, the dot-dashed line shows the result including the Fermi motion of nucleons without the direct effect of the mean vector field, and finally the solid line is obtained from the dot-dashed one by the scale transformation (3). It is clear from this figure, and can also be shown from Eq.(3), that the direct effect of the mean vector field is to squeeze the quark distribution from both the small and the large side of the Bjorken variable. The mean vector field is therefore essential to describe the depletion in the valence quark region, and also leads to an enhancement for smaller values of the Bjorken variable.

Fig. 2 shows our results for the structure function per nucleon in isospin symmetric nuclear matter (solid line), in comparison to the isoscalar structure function of a free

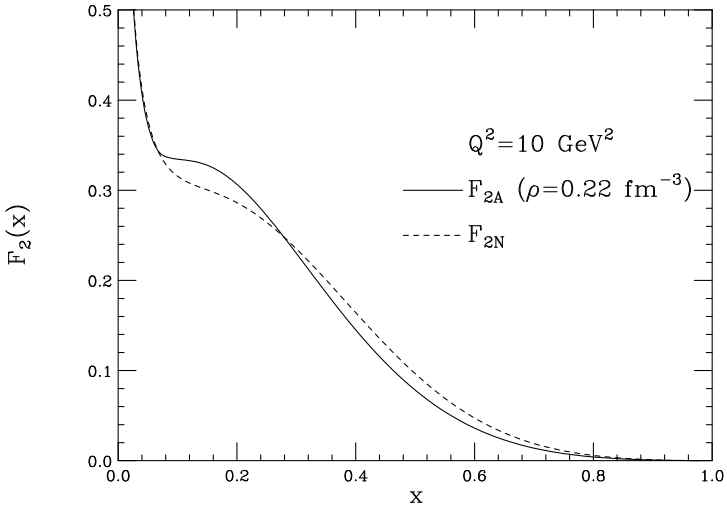

FIGURE 2. Structure function per nucleon in isospin symmetric nuclear matter (solid line), and isoscalar free nucleon structure function (dashed line) at $Q^2 = 10$ GeV^2.

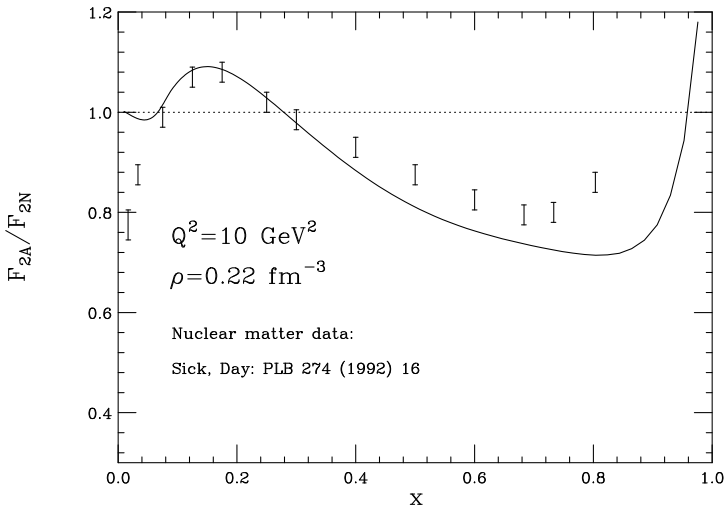

FIGURE 3. Ratio of the structure function per nucleon in isospin symmetric nuclear matter to the free isoscalar nucleon structure function at $Q^2 = 10$ GeV^2.

nucleon (dashed line). These curves were obtained by evolving the quark distributions shown by the solid and dotted lines in Fig. 1 up to $Q^2 = 10$ GeV^2 by using the Q^2 evolution code of Ref.[11]. Fig. 2 clearly shows the suppression of the structure function at large x and the enhancement at smaller x.

From the two curves of Fig. 2, we obtain the EMC ratio between the nuclear and the nucleon structure functions shown in Fig. 3. The data points in this figure are the extrapolations of nuclear EMC data to the nuclear matter case. We see that the calculation can reproduce the main features of the EMC data, namely the suppression at large x and the enhancement at smaller x.

SUMMARY

In this paper we discussed the modifications of the structure function of a nucleon bound in the nuclear medium. For this purpose, we used an effective chiral quark theory which can account for the quark substructure of a single nucleon, and for the saturation properties of the nuclear matter binding energy. Our description of nuclear matter was done in the mean field approximation, which is characterized by mean scalar and vector fields which couple to the quarks in the nucleons. Our aim was to assess the effects which arise from the quark structure of the bound nucleon.

It is well known that the effects of the scalar and vector mean fields on the level of nucleons, including the Fermi motion, cannot explain the EMC data. In our approach, we investigated these effects on the level of quarks, and our most important result can be summarized as follows: The mean vector field in the medium influences the form of the quark light cone momentum distributions directly, besides its indirect influence through the equation of state of the system. This direct modification of the quark distribution is expressed by Eq.(3), and is the principal agent to explain the EMC effect in the framework of a mean field description of nuclear matter. While the effect of the vector field on the nucleon momentum distribution alone tends to cancel the effect of the scalar field [4], there remains an appreciable net effect if the description is done on the quark level, and the result is consistent with the EMC data.

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