NUCLEAR MATTER AND THE TRANSITION TO QUARK MATTER IN AN EFFECTIVE QUARK THEORY

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We use an effective chiral quark theory to incorporate the nucleon structure into the description of nuclear matter, and use the resulting equation of state to discuss the EMC effect in nuclear matter. We also construct the equation of state for quark matter including the effect of color superconductivity, and describe the phase transition from nuclear matter to quark matter in the region of high baryon density.

1. Introduction

The question to what extent the properties of nuclear many body systems reflect the quark substructure of the single nucleon is an important research subject of intermediate energy hadronic physics. Directly related to this problem is the behavior of matter at high baryon densities, where one expects a phase transition to quark matter. In order to describe these phenomena reliably, one needs a theoretical framework which can account for not only for the quark substructure of the single nucleon, but also for (i) the properties of normal nuclear matter (NM), (ii) the medium modifications of nucleon properties, and (iii) the properties of high density quark matter (QM). It has been shown in recent works [1,2,3] that the Nambu-Jona-Lasinio (NJL) model [4] is a strong candidate for these purposes: Besides a covariant description of the nucleon as a quark-diquark bound state [5], it reproduces a saturating NM equation of state (EOS), the EMC effect in NM, and describes the transition to 2-flavor color superconducting QM. It is the purpose of this paper to discuss the most important results which we obtained in our studies of these three subjects.

The NJL model is characterized by a chiral symmetric contact interaction between quarks. Because of its simplicity, the relativistic Faddeev equation for the nucleon can be solved in the ladder approximation, taking into account the interactions in the scalar and axial vector diquark channels[5]. For our present investigations at finite density, however, we will restrict ourselves to the scalar diquark channel, and to a simple approximation to the Faddeev equation, where the momentum dependence of the quark exchange kernel is neglected[6]. The calculations presented in this paper are based on this "quark-diquark picture" of the single nucleon.

2. Description of nuclear matter

The effective (grand) potential for NM at zero temperature in the mean field approximation has the form [2]

$$V^{(NM)} = V_{\rm vac} + V_N + V_\omega \,, \tag{1}$$

where $V_{\rm vac}$ describes the polarization of the Dirac sea of quarks due to the presence of the valence nucleons, V_N arises from the Fermi motion of the valence nucleons depending on the nucleon mass $M_N(M)$, which is a solution of the quark-diquark bound state equation and depends on the constituent quark mass M, and V_ω is the contribution of the mean vector field (V_0) acting on the quarks in NM. The conditions $\partial V^{(NM)}/\partial M =$

$\partial V^{(NM)}/\partial V_0 = 0$ determine M and V_0 for fixed chemical potential μ.

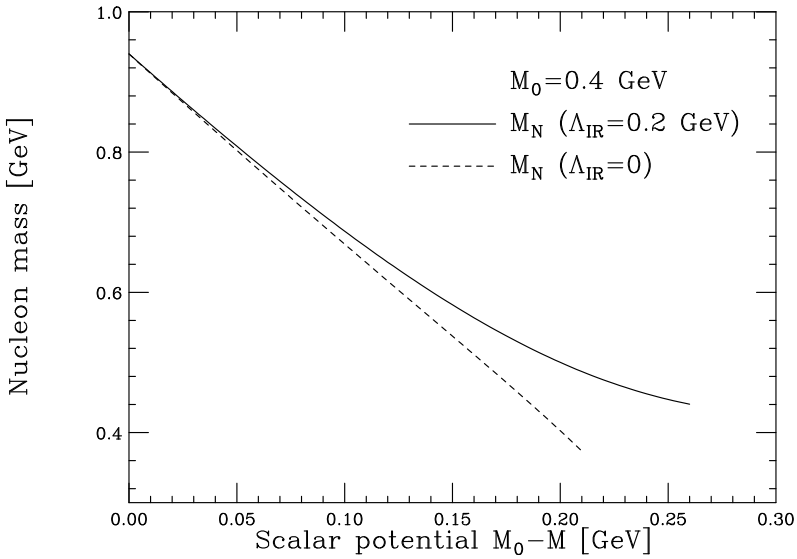

Figure 1. Nucleon mass as a function of the scalar potential for $\Lambda_{\rm IR} = 0$ (dashed line) and $\Lambda_{\rm IR} = 0.2$ GeV (solid line). M_0 is the constituent quark mass for zero density.

In effective meson-nucleon theories based on the linear realization of chiral symmetry, one often observes a collapse of the NM EOS because the σ mass decreases too rapidly as a function of the density [7]. It has been shown in Ref.[1], however, that the saturation properties of the NM EOS can be described if the quark structure of the nucleon is taken into account, provided that one eliminates the thresholds for the unphysical decay of the nucleon into quarks. This can be done, for example, in the proper time regularization scheme by introducing an infrared cut-off ($\Lambda_{\rm IR}$) in addition to the ultraviolet one [8]. The elimination of the unphysical decay threshold then leads to a positive scalar polarizability [9] of the single nucleon, and this in turn gives rise to an effective $NN\sigma\sigma$ interaction which raises the σ meson mass and prevents the collapse.

These points are shown in Figs. 1 and 2. Fig. 1 shows the function $M_N(M)$, which is the solution of the quark-diquark bound state equation for the nucleon. For the case $\Lambda_{\rm IR} = 0$, where the unphysical quark decay thresholds are present, one obtains an almost linear function. This is similar to the case of a point nucleon, and leads to the same kind of collapse as

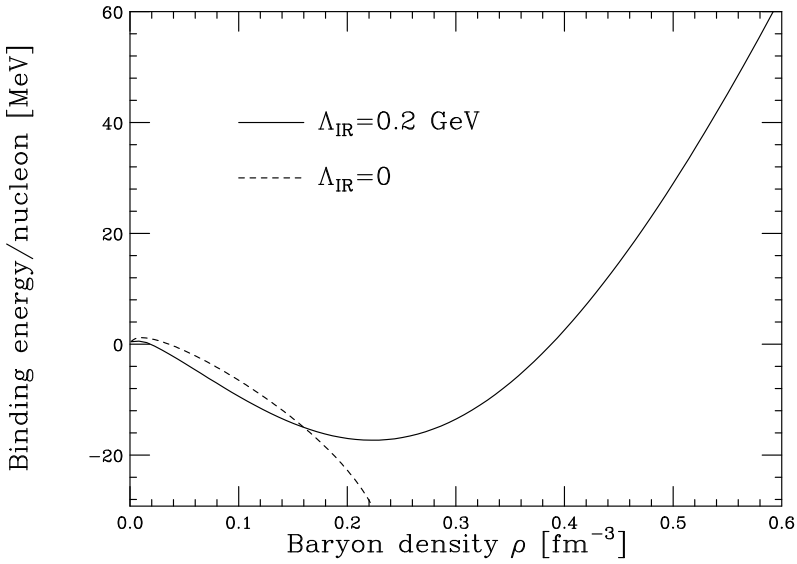

Figure 2. Binding energy per nucleon in NM as a function of the baryon density for $\Lambda_{\rm IR} = 0$ (dashed line) and $\Lambda_{\rm IR} = 0.2$ GeV (solid line).

observed in the linear sigma model, see the dashed line of Fig. 2. The elimination of unphysical quark decay thresholds by an infrared cut-off $\Lambda_{\rm IR}$, however, leads to a positive scalar polarizability (curvature of the function $M_N(M)$), which is clearly shown by the solid line in Fig. 1. This effect stabilizes the NM EOS, and leads to the saturation of the binding energy per nucleon, as shown by the solid line in Fig. 2. The saturation problem in effective chiral field theories can therefore be solved by taking into account the quark structure of the nucleon *and* a particular aspect of confinement physics, namely the absence of quark decay thresholds of the nucleon.

3. The EMC effect in nuclear matter

The isoscalar light cone momentum distribution ($f_{q/A} \equiv f_{u/A} + f_{d/A}$) of quarks per nucleon in a nucleus with mass number A is defined as[10]

$$f_{q/A}(x_A) = \frac{P_-}{A^2} \int \frac{{\rm d}w^-}{2\pi} e^{iP_- x_A w^-/A} < A, P|\overline{\psi}(0)\gamma^+\psi(w^-)|A, P > \ . \quad (2)$$

Here P^μ is the total 4-momentum of the nucleus and P_- its light cone "minus component", ψ is the quark field, and x_A is the Bjorken variable of the nucleus, which is equivalent to the fraction of the total P_- carried

by a quark times A. In the rest system of the nucleus (NM) we have $P_-/A = \epsilon_F/\sqrt{2}$, where ϵ_F is the Fermi energy (that is, the mass per nucleon or the chemical potential) of the nucleons.

The distribution (2) can be expressed as a convolution of the light cone momentum distributions of quarks in the nucleon and of nucleons in the nucleus[3]. For the evaluation of the quark distribution in the nucleon, we describe the nucleon as a bound state of a quark and a scalar diquark in the NJL model as in the previous section. The presence of the nuclear medium is taken into account via the same scalar and vector mean fields which were used to describe the EOS of NM. The most important relation of this approach, which shows the direct effect of the vector mean field on the quark distribution function, is as follows[3]:

$$f_{q/A}(x_A) = \frac{\epsilon_F}{E_F} f_{q/A0}(x'_A = \frac{\epsilon_F}{E_F} x_A - \frac{V_0}{E_F}). \tag{3}$$

Here the distribution without the explicit effect of the mean vector field is denoted as $f_{q/A0}(x'_A)$, and the nucleon Fermi energy has the form $\epsilon_F = E_F + 3V_0$, where $E_F = \sqrt{M_N^2 + p_F^2}$ and V_0 is the mean vector field as in sect.1.

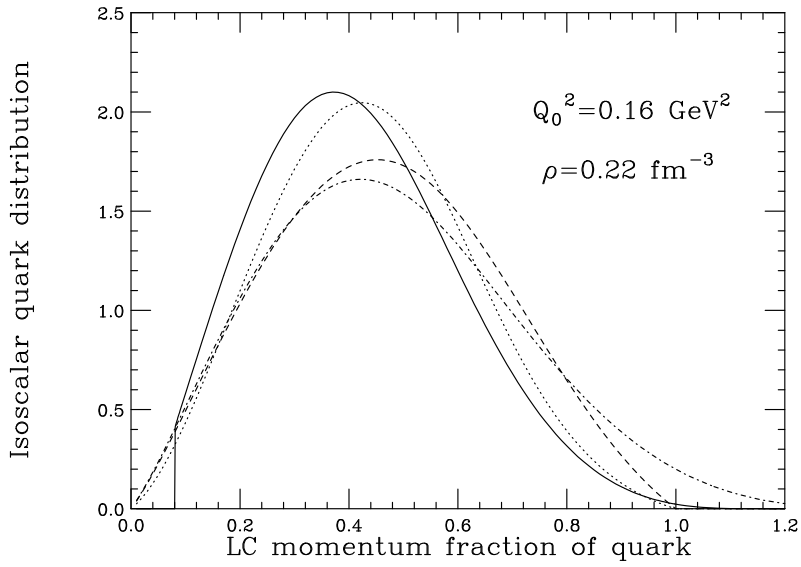

Figure 3. Sum of the up and down quark light cone momentum distributions at the low energy scale $Q_0^2 = 0.16$ GeV^2. For explanation of the lines, see text.

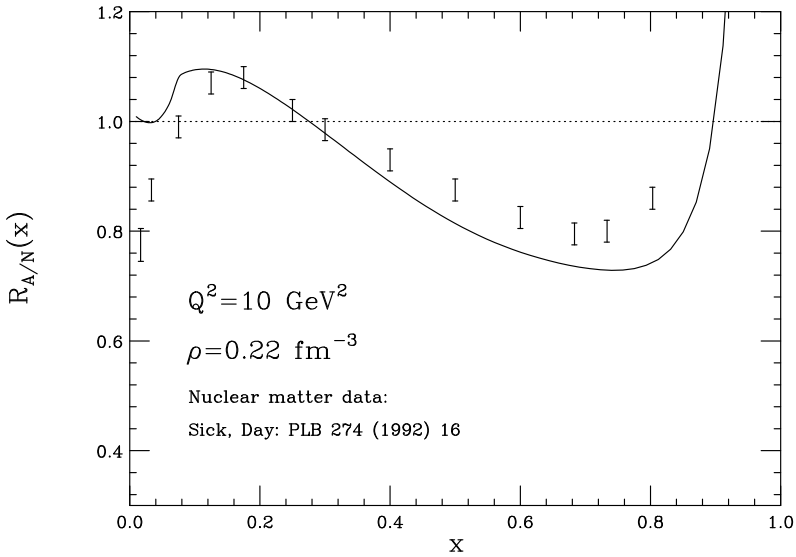

Figure 4. Ratio of the structure function per nucleon in isospin symmetric nuclear matter to the free isoscalar nucleon structure function at $Q^2 = 10$ GeV^2.

The actual calculation of $f_{q/A}$ therefore proceeds as follows: First we calculate the distribution for a free nucleon in the quark-diquark approach, then we replace the quark, diquark and nucleon masses by the effective ones according to the EOS of NM determined in sect.1, then we include the effect of the Fermi motion of nucleons with effective mass M_N, and finally we perform the scale transformation (3) to include the direct effect of the vector mean field. The results for these 4 steps are shown in Fig.3 at the saturation density of our NM EOS for the low energy scale $Q_0^2 = 0.16$ GeV^2. The dotted line shows the distribution in a free nucleon, which is consistent with the empirical parametrizations[11]. The dashed line shows the result when all masses are replaced by the effective ones, the dot-dashed line shows the result including the Fermi motion of nucleons without the direct effect of the mean vector field, and finally the solid line is obtained from the dot-dashed one by the scale transformation (3). We see that the direct effect of the mean vector field is to squeeze the quark distribution from both the small and the large side of the Bjorken variable. The mean vector field is therefore essential to describe the depletion in the valence quark region, and also leads to an enhancement for smaller values of the Bjorken variable. This is shown by the EMC ratio[12] in Fig. 4, which has

been obtained by performing the Q^2 evolution up to $Q^2 = 10$ GeV^2[13]. We see that the calculation can reproduce the main features of the EMC effect, namely the suppression at large x and the enhancement at smaller x.

4. Phase transition to color superconducting quark matter

The EOS of color superconducting QM in the mean field approximation has the form[2]

$$V^{(QM)} = V_{\rm vac} + V_Q + V_\Delta + V_\omega \,, \tag{4}$$

where $V_{\rm vac}$ describes the polarization of the Dirac sea of quarks due to the presence of the valence quarks, V_Q arises from the Fermi motion of the valence quarks without the effect of quark pairing, V_Δ arises from quark pairing in the scalar diquark channel and depends on the color superconducting gap (Δ)[14], and V_ω is the contribution of the mean vector field (V_0) in QM. The conditions $\partial V^{(QM)}/\partial M = \partial V^{(QM)}/\partial \Delta = \partial V^{(QM)}/\partial V_0 = 0$ determine M, Δ and V_0 for fixed chemical potential μ.

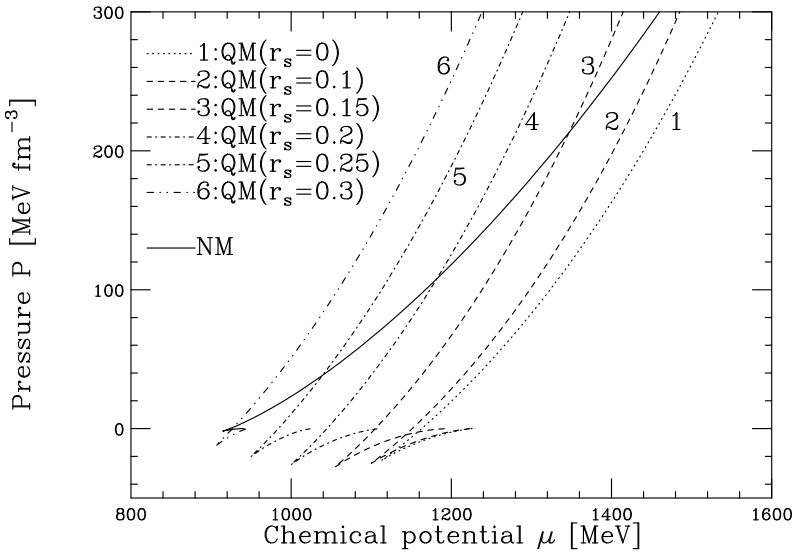

Figure 5. Pressure as function of chemical potential in NM (solid line), and in QM for several values of r_s, which is the ratio of the coupling constant in the scalar diquark channel to the one in the pionic channel.

Fig. 5 shows the EOS for QM with several values for the strength of the pairing interaction in the scalar diquark channel (r_s). Curve 1 corresponds

to normal, i.e., non-color superconducting, QM, and the other curves show the results for increasing strength of the pairing interaction. We also show the EOS for NM by the solid line.

It is clear from Fig. 5 that there is no phase transition from NM to normal QM in our model. The scalar diquark condensation, however, gives rise to a substantial softening of the QM EOS, and to a phase transition from NM to QM at a transition density which decreases with increasing strength of the pairing interaction.

We now assume a particular value of the coupling constant in the scalar diquark channel, which leads to reasonable transition densities, and investigate the nature of the phase transition more closely. Fig. 6 shows the resulting pressure of the ground state as a function of the baryon density for the case corresponding to $r_s = 0.2$ of Fig. 5. We obtain first order transitions from the vacuum (VAC) to NM, where in both phases chiral symmetry is broken and color symmetry is intact, and from NM to color superconducting QM, where in the latter phase chiral symmetry is largely restored and color symmetry is broken. The present calculation gives large color superconducting gaps in the QM phase ($\Delta > 200$ MeV), see ref.[2] for details.

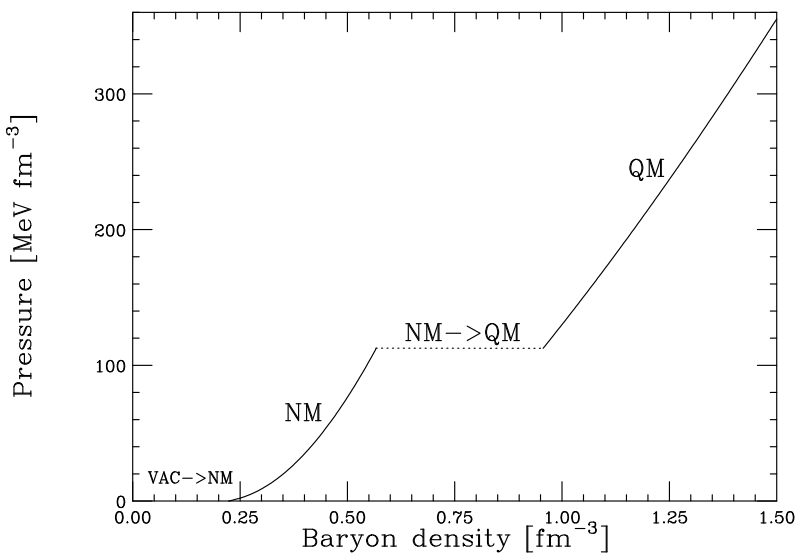

Figure 6. Pressure of the ground state of the system - vacuum (VAC), nuclear matter (NM) or superconducting quark matter (QM) - as function of the baryon density for $r_s = 0.2$. The mixed phases are also indicated.

5. Summary

Let us summarize the three most important points which we found in our investigations on the role of quark degrees of freedom in strongly interacting baryonic matter: First, the effects of quark substructure of the nucleons can solve the saturation problem in chiral many-body theories. The main difference to the point nucleon case is that the extended nucleon has a positive scalar polarizability, which leads to a curvature of the nucleon mass function $M_N(M)$, an in-medium sigma meson mass which depends only weakly on the density, and a saturation of the binding energy per nucleon. Second, the coupling of the mean vector field in nuclear matter to the quarks inside the nucleons influences the form of the quark light cone momentum distributions directly, besides its indirect influence through the equation of state of the system. This direct modification of the quark distribution is expressed by Eq.(3), and is the principal agent to explain the EMC effect in the framework of a mean field description of nuclear matter. Third, there is a phase transition from nuclear matter to quark matter, provided that the effects of scalar diquark condensation (color superconductivity) are taken into account. This phase transition is characterized by the restoration of chiral symmetry and the spontaneous breaking of color symmetry in the high density region.

Acknowledgments

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