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On the DLY Relation for Fragmentation Functions†

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[FRAGMENTATION FUNCTIONS, Drell-Levy-Yan Relation, Effective Quark Theories]

The Drell-Levy-Yan (DLY) relation[1)] expresses the quark fragmentation function $D_q^h(z)$ by the quark distribution function $f_q^h(x)$ in the unphysical region of the Bjorken variable ($x > 1$). Here we outline two independent derivations of this relation, referring to the case of the nucleon ($h = N$), and give numerical examples, which indicate that the predictions for $D_q^h(z)$ based on this relation and a simple effective quark theory are smaller than the empirical values by factors of ten to hundred.

We consider the Green function $< p_n|T\left(\mathcal{O}(0)\Psi_N(p)\right)|0>$, where $\Psi_N(p)$ is the Fourier transform of the nucleon field operator, $\mathcal{O}$ is a local field operator, and $|p_n>$ is a hadronic state. If $\Gamma(p, p_n)$ denotes the amputated Green function, the spectral representation (or reduction formula) leads to the following relations:

$$
\begin{aligned}
< p_n|\mathcal{O}|p > &= \overline{\Gamma}(p, p_n)\, u_N(p) \\
< \overline{p}, p_n|\mathcal{O}|0 > &= \pm\overline{\Gamma}(-p, p_n)\, v_N(p)\,.
\end{aligned}
\tag{1}
$$

Here $|p>$ ($|\overline{p}>$) denotes a nucleon (antinucleon) state with 4-momentum p^μ, and u_N, v_N are the nucleon spinors. The plus (minus) sign holds if $\mathcal{O}$ is a fermion (boson) type operator.

If we take $\mathcal{O}$ to be the current operator in the definition of the hadronic tensors $W^{\mu\nu}$ and $\overline{W}^{\mu\nu}$ for the processes $ep \rightarrow e'X$ and $e^+e^- \rightarrow pX$, respectively, Eq. (1) gives $\overline{W}^{\mu\nu}(p, q) = -W^{\mu\nu}(-p, q)$, which leads to the DLY relation

$$
D_q^h(z) = \pm\frac{z}{6}\, f_q^h(\frac{1}{z})\,. \tag{2}
$$

For $h = N$ ($h = \pi$) the minus (plus) sign holds. We can also take $\mathcal{O}$ to be the quark field in the operator definitions of $f_q^h(x)$ and $D_q^h(z)$, and Eq. (1) again leads to (2). One important observation from Eq.(2) is the following: For the case of the nucleon, the generalized distribution *must* cross zero at $x = 1$, but for the pion it *must not* cross zero because both the distribution and the fragmentation functions should be positive.

As an illustration, we show the generalized u distribution for the π^+ and proton in Figs. 1 and 2. Here we use a simple NJL model description of the pion as a $q\overline{q}$ state, and the nucleon as a quark-scalar diquark state. It is particularly evident from Fig.2 that the "tail" of the distribution ($x > 1$) is extremely small. Indeed, the fragmentation function predicted by Eq.(2) turns out to be smaller than the empirical one by factors of ten to hundred, and a similar - though less severe - problem occurs for the pion, too. Because the distributions for $x < 1$ describe the empirical ones very well[2)], this is a puzzling problem. We found that sea quark effects, induced by the pion cloud, do not improve the situation.

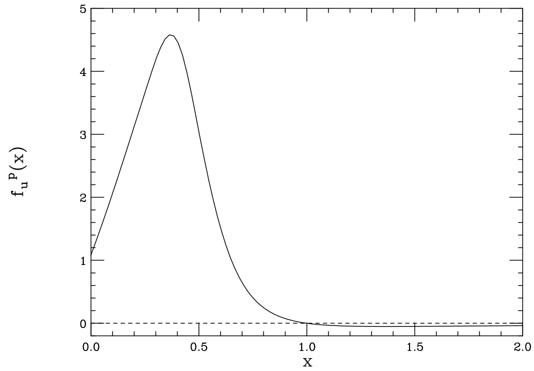

Fig. 1. Generalized u distribution in proton.

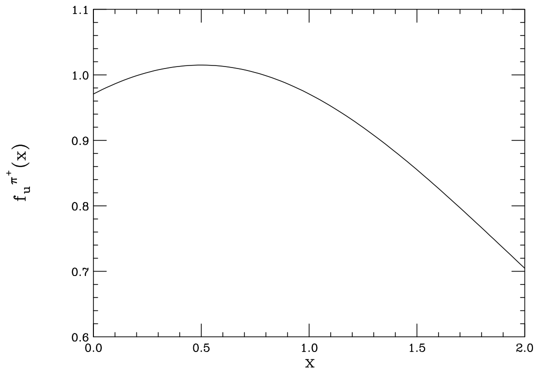

Fig. 2. Generalized u distribution in π^+.

References

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