

Reassessment of the NuTeV determination of the weak mixing angle

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In light of the recent discovery of the importance of the isovector EMC effect for the interpretation of the NuTeV determination of $\sin^2 \theta_W$, it seems timely to reassess the central value and the errors on this fundamental Standard Model parameter derived from the NuTeV data. We also include earlier work on charge symmetry violation and the recent limits on a possible asymmetry between s and \bar{s} quarks. With these corrections we find a revised NuTeV result of $\sin^2 \theta_W = 0.2221 \pm 0.0013(\text{stat}) \pm 0.0020(\text{syst})$, which is in excellent agreement with the running of $\sin^2 \theta_W$ predicted by the Standard Model. As a further check, we find that the separate ratios of neutral current to charge current cross sections for neutrinos and for antineutrinos are both in agreement with the Standard Model, at just over one standard deviation, once the corrections described here are applied.

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Using a very careful comparison of the charged and neutral current total cross sections for ν and $\bar{\nu}$ on an iron target, the NuTeV collaboration reported a three standard deviation discrepancy with the Standard Model value of $\sin^2 \theta_W$ [1]. This was initially taken as an indication of possible new physics, however attempts to understand this anomaly in terms of popular extensions of the Standard Model have proven unsuccessful [2, 3]. At the same time a number of possible corrections within the Standard Model have been suggested [4–10], most of which have a sign likely to reduce this discrepancy.

The correction associated with charge symmetry violation (CSV), arising from the u - and d -quark mass differences [11, 12], has been shown to be largely model independent and to reduce the discrepancy by about 1σ [4]. If the momentum fraction carried by s -quarks in the proton exceeds that carried by \bar{s} -quarks, as suggested by chiral physics [13, 14] and recent experimental analysis [8], there could be a further reduction, albeit with large uncertainties at present [9]. Finally, in Ref. [10] it was recently pointed out that the excess neutrons in iron lead to an isovector EMC effect that modifies the parton distribution functions (PDFs) of *all* the nucleons in the nucleus. Qualitatively this has the same sign as the CSV correction and a quantitative estimate suggests that it reduces the NuTeV discrepancy with the Standard Model by about 1.5σ [10].

These effects are essentially independent and can therefore be combined in a straightforward manner. It is then immediately clear that the corrected NuTeV data will be more consistent with the Standard Model. Rather than continuing to report that the data is 3σ above expectations, we suggest that it is timely to update the derived value of $\sin^2 \theta_W$.

In this Letter we will examine the corrections in turn, assign to each a central value and a conservative error,

then combine them to produce a revised value for $\sin^2 \theta_W$. Our final value is

$$\sin^2 \theta_W = 0.2221 \pm 0.0013(\text{stat}) \pm 0.0020(\text{syst}), \quad (1)$$

which is in excellent agreement with the corresponding Standard Model result, namely 0.2227 ± 0.0004 [1, 15] in the on-shell renormalization scheme. The original NuTeV result was $\sin^2 \theta_W = 0.2277 \pm 0.0013(\text{stat.}) \pm 0.0009(\text{syst.})$ [1].

The NuTeV experiment involved a precise measurement on a steel target of the ratios R^ν and $R^{\bar{\nu}}$, which are the ratios of the neutral current (NC) to charged current (CC) total cross sections for ν and $\bar{\nu}$, respectively. Integral to the NuTeV extraction of $\sin^2 \theta_W$ was a detailed Monte Carlo simulation of the experiment. However, NuTeV have provided functionals which allow one to accurately estimate the effect of any proposed correction [16].

The NuTeV study was motivated by the observation of Paschos and Wolfenstein [17] that a ratio of cross sections for neutrinos and antineutrinos on an isoscalar target allowed an independent extraction of the weak mixing angle. The so-called Paschos-Wolfenstein (PW) ratio is given by [17, 18]

$$R_{\text{PW}} = \frac{\sigma_{\text{NC}}^{\nu A} - \sigma_{\text{NC}}^{\bar{\nu} A}}{\sigma_{\text{CC}}^{\nu A} - \sigma_{\text{CC}}^{\bar{\nu} A}} \equiv \frac{R^\nu - rR^{\bar{\nu}}}{1 - r}. \quad (2)$$

In Eq. (2), R_{PW} is the PW ratio, A represents the nuclear target, and $r = \sigma_{\text{CC}}^{\bar{\nu} A} / \sigma_{\text{CC}}^{\nu A}$. Expressing the total cross-sections in terms of quark distributions, ignoring the heavy quark flavours and $\mathcal{O}(\alpha_s)$ corrections, the PW ratio becomes

$$R_{\text{PW}} = \frac{\left(\frac{1}{6} - \frac{4}{9}s_W^2\right)\langle x_A u_A^- \rangle + \left(\frac{1}{6} - \frac{2}{9}s_W^2\right)\langle x_A d_A^- + x_A s_A^- \rangle}{\langle x_A d_A^- + x_A s_A^- \rangle - \frac{1}{3}\langle x_A u_A^- \rangle}, \quad (3)$$

where $s_W^2 \equiv \sin^2 \theta_W$, x_A is the Bjorken scaling variable for the nucleus multiplied by A , $\langle \dots \rangle$ implies integration over x_A , and $q_A^- \equiv q_A - \bar{q}_A$ are the non-singlet quark distributions of the target.

Ignoring quark mass differences, strange quark effects and electroweak corrections, the u - and d -quark distributions of an isoscalar target will be identical, and in this limit Eq. (3) becomes $R_{\text{PW}} \xrightarrow{N=Z} \frac{1}{2} - \sin^2 \theta_W$. If corrections to this result are small the PW ratio provides an independent determination of the weak mixing angle. Expanding Eq. (3) about the $u_A^- = d_A^-$ and $s_A^- \ll u_A^- + d_A^-$ limits, we obtain the leading PW correction term, namely

$$\Delta R_{\text{PW}} \simeq \left(1 - \frac{7}{3} s_W^2\right) \frac{\langle x_A u_A^- - x_A d_A^- - x_A s_A^- \rangle}{\langle x_A u_A^- + x_A d_A^- \rangle}. \quad (4)$$

Extensive studies of neutrino-nucleus reactions have concluded that the most important contributions to Eq. (4) arise from nuclear effects, CSV and strange quarks. These corrections will be discussed in turn below.

In discussing the extraction of the weak mixing angle from neutrino reactions, it is customary and pedagogically useful to refer to corrections to the PW ratio, and we follow this practice. However, it is important to remember that in the NuTeV analysis the measured quantities were the NC to CC ratios for neutrinos and antineutrinos, and that the weak mixing angle was extracted through a Monte Carlo analysis. For a given effect, the PW ratio will give only a qualitative estimate of the correction to the weak mixing angle. Quantitative corrections are obtained by using the functionals provided by NuTeV [16]. Throughout this work we denote a contribution to Eq. (4) by ΔR_{PW}^i , while the best estimate of the correction to the NuTeV determination of $\sin^2 \theta_W$, calculated using a NuTeV functional, is denoted by $\Delta R^i \equiv \Delta^i \sin^2 \theta_W$, where, in each case, i labels the type of correction. For completeness, at the end of our discussion we also report the effect of the corrections which we have considered on the separate ratios R^ν and $R^{\bar{\nu}}$.

Nuclear corrections — For sufficiently large Q^2 , nuclear corrections to the PW ratio for an isoscalar nucleus are thought to be negligible. However, the NuTeV experiment was performed on a steel target and it is essential to correct for the neutron excess before extracting $\sin^2 \theta_W$. NuTeV removed the contribution of the excess neutrons to the cross-section by assuming that the target was composed of free nucleons.

However, the recent results of Cloët *et al.* [10] have shown that the excess neutrons in iron have an effect on *all* the nucleons in the nucleus, which is not accounted for by a subtraction of their naive contribution. In particular, the isovector-vector mean-field generated by the difference in proton and neutron densities, $\rho_p(r) - \rho_n(r)$, acts on every u - and d -quark in the nucleus and results in the break down of the usual assumption that $u_p(x) = d_n(x)$ and $d_p(x) = u_n(x)$ for the bound nucle-

ons. An explicit calculation of this correction was made in Ref. [10], using the approach of Bentz *et al.* [19–21].

The correction associated with the neutron excess can be evaluated in terms of the consequent contribution to $\langle x_A u_A^- - x_A d_A^- \rangle$, using Eq. (4). For nuclei with $N > Z$ the u -quarks feel less vector repulsion than the d -quarks, and in Ref. [10] it was shown that a model independent consequence of this is that there is a small shift in quark momentum from the u - to the d -quarks. Therefore, the momentum fraction $\langle x_A u_A^- - x_A d_A^- \rangle$ in Eq. (4) will be negative [10], even after standard isoscalar corrections are applied. Correcting for the isovector-vector field therefore has the *model independent effect of reducing the NuTeV result for $\sin^2 \theta_W$* .

To estimate the effect on the NuTeV experiment, Cloët *et al.* used a nuclear matter approximation, chose the Z/N ratio to correspond to the NuTeV experimental neutron excess and calculated the quark distributions at an effective density appropriate for Fe, namely 0.89 times nuclear matter density [22]. Using Eq. (4), this gave an estimate of the isovector correction of $\Delta R_{\text{PW}}^{\rho^0} = -0.0025$. Finally, the NuTeV CSV functional [16] was used to obtain an accurate determination of this effect on the NuTeV result. This gave $\Delta R^{\rho^0} = -0.0019$, which accounts for between 1.0 and 1.5σ of the NuTeV discrepancy with the Standard Model.

The sign of this effect is model independent and because it depends only on the difference in the neutron and proton densities in iron and the symmetry energy of nuclear matter, which are both well known, the magnitude is expected to be well constrained. As a conservative estimate of the uncertainty we assign an error twice that of the difference between the PW correction obtained at nuclear matter density and at 0.89 times that, this gives

$$\Delta R^{\rho^0} = -0.0019 \pm 0.0006. \quad (5)$$

Other studies of nuclear corrections to the PW ratio have largely been focused on Fermi motion [10, 23] and nuclear shadowing [23–25] effects. Fermi motion corrections were found to be small [10, 23] and the NuTeV collaboration argue that, given their Q^2 -cuts, sizeable corrections from shadowing would be inconsistent with data [26]. Therefore we have not included a correction from shadowing here.

Charge Symmetry Violation — Before the NuTeV result, two independent studies of the effect of quark mass differences on proton and neutron PDFs, by Sather [11] and by Rodionov *et al.* [12], reached very similar conclusions. These mass differences violate charge symmetry, the invariance of the QCD Hamiltonian under a rotation by 180 degrees about the 2-axis in isospin space. They lead to the CSV differences

$$\delta d^-(x) = d_p^-(x) - u_n^-(x), \quad (6)$$

$$\delta u^-(x) = u_p^-(x) - d_n^-(x), \quad (7)$$

where the subscripts p and n label the proton and neutron, respectively. The contribution of CSV in the nucleon can be found through Eq. (4) and has the form:

$$\Delta R_{\text{PW}}^{\text{CSV}} = \frac{1}{2} \left(1 - \frac{7}{3} \sin^2 \theta_W \right) \frac{\langle x \delta u^- - x \delta d^- \rangle}{\langle x u_p^- + x d_p^- \rangle}. \quad (8)$$

Londergan and Thomas [4] explained the similarity of the results obtained by Sather and Rodionov *et al.* by demonstrating that the leading contribution to the moment $\langle x \delta u^- - x \delta d^- \rangle$ is largely model independent and simply involves the ratio of the up-down mass difference to the nucleon mass. The contribution arising from the quark mass differences to Eq. (8) was found to be $\Delta R_{\text{PW}}^{\delta m} \simeq -0.0020$ and the corresponding NuTeV CSV functional result was $\Delta R^{\delta m} \simeq -0.0015$ [4]. We assign an error of 20% to this term, which is conservative in view of its demonstrated model independence.

An additional CSV effect arises from QED splitting [6, 7], associated with the Q^2 evolution of photon emission from the quarks. Because $|e_u| > |e_d|$ the u -quarks lose momentum to the photon field at a greater rate than the d -quarks. Therefore a model independent consequence of QED splitting is that it will reduce the NuTeV result for $\sin^2 \theta_W$. Glück *et al.* [6] calculated this effect on the NuTeV result and obtained $\Delta R_{\text{PW}}^{\text{QED}} = -0.002$, corresponding to $\Delta R^{\text{QED}} = -0.0011$ using the NuTeV CSV functional. A similar study was undertaken by the MRST group [7] who explicitly included QED splitting effects in the PDF evolution and found $\Delta R_{\text{PW}}^{\text{QED}} = -0.0021$ at $Q^2 = 20 \text{ GeV}^2$. This correction has the same sign as the CSV term arising from quark mass differences and the two contributions are almost independent so we simply add them. The sum of the two terms explains roughly half of the NuTeV discrepancy with the Standard Model. Assigning a conservative 100% error to the QED splitting result and combining the errors in quadrature gives a total CSV correction of

$$\Delta R^{\text{CSV}} = -0.0026 \pm 0.0011. \quad (9)$$

The only experimental information regarding CSV effects on the PDFs is obtained from an MRST study [5]. In this case a global analysis was performed on a set of high energy data allowing for explicit CSV in the PDFs. From their global analysis [5] the MRST group found $\Delta R_{\text{PW}}^{\text{CSV}} = -0.002$, with a 90% confidence interval of $-0.007 < \Delta R_{\text{PW}}^{\text{CSV}} < 0.007$. The MRST study implicitly include both sources of CSV considered here, quark-mass and QED effects. The 90% confidence interval obtained by MRST allows a rather large range of valence quark CSV.

We have not adopted the MRST value and error for partonic CSV, for the following reasons. First, the MRST results are based on the assumption of a specific functional form for the CSV parton distributions. The assumed function had an overall strength parameter that

was varied to obtain the best fit to the global analysis. MRST also imposed relations between the valence quark CSV PDFs. For convenience in their global analysis they neglected the Q^2 dependence of the CSV distributions. Finally, the experiments in the global set of high energy data have different treatments of radiative corrections. It is not clear that these different radiative corrections have been treated consistently in an analysis of CSV effects. If the CSV effects are as large as are allowed within the MRST 90% confidence limit, it should be possible to observe such effects [27]. However it will be some time before experiments can further constrain this result.

Strange Quark Asymmetry — A difference in shape between $s(x)$ and $\bar{s}(x)$ in the nucleon was first proposed on the basis of chiral symmetry in Ref. [13]. However, the size of $s^-(x)$ is not constrained by symmetries and badly needs further input from experiment or lattice QCD [28]. The strange quark correction arises from the term $\langle x s_A^- \rangle$ on the right hand side of Eq. (4). The best direct experimental information on $\langle x s_A^- \rangle$ comes from opposite sign dimuon production in reactions induced by neutrinos or antineutrinos. Such experiments have been carried out by the CCFR [29] and NuTeV [30] groups. A precise extraction of $\langle x s_A^- \rangle$ is the NuTeV analysis by Mason *et al.* [8], which found $\langle x s_A^- \rangle = 0.00196 \pm 0.00143$ at $Q^2 = 16 \text{ GeV}^2$, where we have added the various errors in quadrature.

Global PDF analyses have also provided estimates of $s^-(x)$. The recent examination by the NNPDF collaboration found $\langle x s^- \rangle = 0.0005 \pm 0.0086$ [9] at $Q^2 = 20 \text{ GeV}^2$, which has an error more than six times larger than that cited by NuTeV. However, unlike the NuTeV dimuon experiment, this analysis is not directly sensitive to the s -quark distributions, evidenced by the fact that their upper limit on $s^-(x)$ is an order of magnitude larger than any other sea quark distribution at $x \sim 0.5$. This large uncertainty is a consequence of their neural network approach, which was primarily aimed at accurately determining V_{cd} and V_{cs} , not $s^-(x)$ [9]. Alekhin *et al.* [31] obtained $\langle x s^- \rangle = 0.0013 \pm 0.0009(\text{exp}) \pm 0.0002(\text{QCD})$ when they imposed a constraint on the semileptonic branching ratio B_μ from production rates of charmed hadrons in other experiments. The MSTW collaboration find a momentum fraction very similar to that of NuTeV, namely $\langle x s^- \rangle = 0.0016_{-0.0009}^{+0.0011}$ [32] at $Q^2 = 10 \text{ GeV}^2$, while CTEQ report $\langle x s^- \rangle = 0.0018$ [33] at $Q^2 = 1.69 \text{ GeV}^2$, with a 90% confidence interval of $-0.001 < \langle x s^- \rangle < 0.005$. These results are summarized in the second column of Table I.

The quantity $s_A^-(x)$ must have at least one zero-crossing since its first moment vanishes. For each of the above analyses the central best fit curve crosses zero at values of x less than 0.03 for $Q^2 > 2 \text{ GeV}^2$, with the exception of the NNPDF result which has a zero-crossing at $x = 0.13$ for $Q^2 = 2 \text{ GeV}^2$. For example the NuTeV result has the zero-crossing at $x = 0.004$, which is a very small

| | $\langle x s^- \rangle$ | ΔR^s | ΔR^{total} | $\sin^2 \theta_W \pm \text{syst.}$ |
|----------------------------|--------------------------------|---------------------------------|-------------------------------|------------------------------------|
| Mason <i>et al.</i> [8] | 0.00196 ± 0.00143 | -0.0018 ± 0.0013 | -0.0063 ± 0.0018 | 0.2214 ± 0.0020 |
| NNPDF [9] | 0.0005 ± 0.0086 | -0.0005 ± 0.0078 | -0.0050 ± 0.0079 | $0.2227 \pm \text{large}$ |
| Alekhin <i>et al.</i> [31] | $0.0013 \pm 0.0009 \pm 0.0002$ | $-0.0012 \pm 0.0008 \pm 0.0002$ | -0.0057 ± 0.0015 | 0.2220 ± 0.0017 |
| MSTW [32] | $0.0016^{+0.0011}_{-0.0009}$ | $-0.0014^{+0.0010}_{-0.0008}$ | -0.0059 ± 0.0015 | 0.2218 ± 0.0018 |
| CTEQ [33] | $0.0018^{+0.0016}_{-0.0004}$ | $-0.0016^{+0.0014}_{-0.0004}$ | $-0.0061^{+0.0019}_{-0.0013}$ | $0.2216^{+0.0021}_{-0.0016}$ |
| This work (Eq. (10)) | 0.0 ± 0.0020 | 0.0 ± 0.0018 | -0.0045 ± 0.0022 | 0.2232 ± 0.0024 |

TABLE I: A summary of the recent estimates of the strangeness asymmetry, $\langle x s^- \rangle$, the correction to the Paschos-Wolfenstein ratio after applying the NuTeV functional, ΔR^s , and the total correction, ΔR^{total} , obtained by combining ΔR^{ρ^0} , ΔR^{CSV} and ΔR^s , with the errors added in quadrature. The final column shows the value of $\sin^2 \theta_W$ deduced in each case by applying the total correction to the published NuTeV result. Note that we show only the systematic error, which is obtained by treating the error on ΔR^{total} as a systematic error and combining it in quadrature with the NuTeV systematic error.

x value (it is smaller than the lowest x point measured in the CCFR and NuTeV experiments), and moreover is extremely unlikely on theoretical grounds [13, 34, 35]. In any quark model calculation the zero-crossing will occur near $x \simeq 0.15$ (a value similar to that found by NNPDF). In this case the NuTeV strange quark momentum fraction becomes $\langle x s_A^- \rangle = 0.00007$ [8], with a moderate increase in the χ^2 compared to the best value of Mason *et al.*. Since relatively little is known about the s -quark distributions we ignore nuclear effects and therefore assume $\langle x s_A^- \rangle \equiv \langle x s^- \rangle$ throughout this discussion.

Clearly the correction to the PW ratio from the strange quark asymmetry has a significant uncertainty. On the theoretical grounds explained earlier, we prefer the NuTeV analysis based on a zero-crossing at $x \approx 0.15$, which means that $\langle x s^- \rangle$ is essentially zero. For the uncertainty we choose the difference between this and the NuTeV determination noted above with the zero-crossing at $x = 0.004$, this gives $\langle x s^- \rangle = 0.0 \pm 0.0020$ at 16 GeV². This is a conservative choice for the error since it is substantially larger than the original uncertainty quoted by NuTeV and covers all of the central values of the analyses mentioned above. Including the effect of the NuTeV functional leads to our preferred value for the strange quark correction to the NuTeV $\sin^2 \theta_W$ result, namely

$$\Delta R^s = 0.0 \pm 0.0018. \quad (10)$$

The s -quark corrections to the NuTeV result obtained from the other analyses discussion here are summarized in column three of Table I.

Conclusion — The errors associated with the three corrections given in Eqs. (5), (9) and (10) are systematic and to a very good approximation independent errors. We therefore combine them in quadrature with the original systematic error quoted by NuTeV. The statistical error is, of course, unchanged from the NuTeV analysis.

Because of the uncertainty over the strangeness asymmetry, in the last column of Table I we show the effect

on $\sin^2 \theta_W$ for each of the recent analyses [8, 9, 31–33] as well as our own preferred value given in Eq. (10). Every one of the six results lies within one standard deviation of the Standard Model value for $\sin^2 \theta_W$. As a best estimate of the corrected value we take the average of these six values. For the systematic error we note that (apart from NNPDF which is unrealistically large) they are all very similar. Because of the correlations between them, the final quoted systematic error is a simple average of all the entries in the last column of Table I except NNPDF. This yields the revised value for $\sin^2 \theta_W$, including all of the corrections discussed here, namely:

$$\sin^2 \theta_W = 0.2221 \pm 0.0013(\text{stat}) \pm 0.0020(\text{syst}), \quad (11)$$

which is in excellent agreement with the Standard Model expectation of $\sin^2 \theta_W = 0.2227 \pm 0.0004$ [1, 15]. Correction terms of higher order than Eq. (4) and also $\mathcal{O}(\alpha_s)$ corrections were also investigated and found to be negligible.

This updated value for the NuTeV determination of $\sin^2 \theta_W$ is also shown in Fig. 1, now in the $\overline{\text{MS}}$ -scheme and labelled as ν -DIS, along with the results of a number of other completed experiments and the anticipated errors of several future experiments, which are shown at the appropriate momentum scale Q .

In this paper we have summarized various estimates of the size of both partonic CSV effects and a possible strange quark momentum asymmetry. For valence quark CSV we have relied on well founded theoretical arguments to constrain the magnitude of CSV effects arising from quark mass differences. We have also used theoretical guidance on the zero crossing in $s^-(x)$, as well as the most recent analyses of the experimental data to constrain the strange quark momentum asymmetry. When re-evaluated, the NuTeV point is within one standard deviation of the Standard Model prediction for all analyses of this asymmetry. As the experimental information on the strange quark asymmetry or charge symmetry viola-

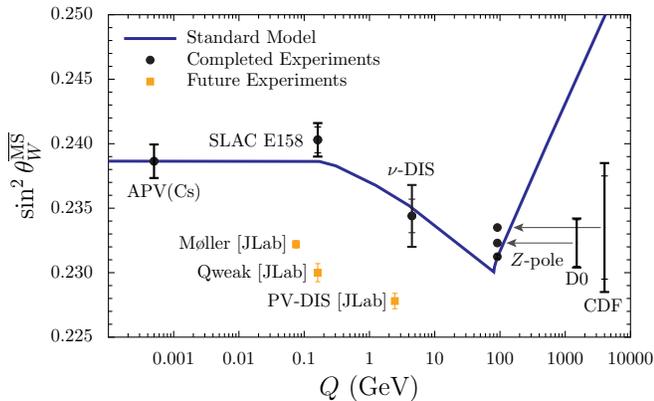


FIG. 1: The curve represents the running of $\sin^2 \theta_W$ in the $\overline{\text{MS}}$ renormalization scheme [36]. The Z -pole point represents the combined results of six LEP and SLC experiments [37]. The CDF [38] and D0 [39] collaboration results (at the Z -pole) and the SLAC E158 [40] result, are labelled accordingly. The atomic parity violating (APV) result [41] has been shifted from $Q^2 \rightarrow 0$ for clarity. The inner error bars represent the statistical uncertainty and the outer error bars the total uncertainty. At the Z -pole, conversion to the $\overline{\text{MS}}$ scheme was achieved via $\sin^2 \theta_W^{\text{eff}} = 0.00029 + \sin^2 \theta_W^{\overline{\text{MS}}}$ [37]. For the results away from the Z -pole, the discrepancy with the Standard Model curve reflects the disagreement with the Standard Model in the renormalization scheme used in the experimental analysis.

tion improves it is a simple matter to update the current analysis.

As a final point, we return to the fact that the NuTeV experiment actually measured R^ν and $R^{\bar{\nu}}$, not R_{PW} . We might ask how the corrections that we have applied affect the individual values for these two ratios. For the quantity R^ν NuTeV measured 0.3916 ± 0.0013 , compared with 0.3950 in the Standard Model, while for $R^{\bar{\nu}}$ they obtained 0.4050 ± 0.0027 , compared with 0.4066. The corrections to the Standard Model ratios arising from the isovector EMC effect and CSV are both included through the non-zero value of $\langle x_A u_A^- - x_A d_A^- \rangle$:

$$\delta R^\nu = \frac{2(g_{Lu}^2 + g_{Ru}^2) \langle x_A u_A^- - x_A d_A^- \rangle}{\langle 3x_A u_A + 3x_A d_A + x_A \bar{u}_A + x_A \bar{d}_A + 6x_A s_A \rangle}, \quad (12)$$

$$\delta R^{\bar{\nu}} = \frac{-2(g_{Ld}^2 + 3g_{Rd}^2) \langle x_A u_A^- - x_A d_A^- \rangle}{\langle x_A u_A + x_A d_A + 3x_A \bar{u}_A + 3x_A \bar{d}_A + 6x_A \bar{s}_A \rangle}, \quad (13)$$

where

$$g_{Lu} = \frac{1}{2} - \frac{2}{3} \sin^2 \theta_W \quad g_{Ru} = -\frac{2}{3} \sin^2 \theta_W, \quad (14)$$

$$g_{Ld} = -\frac{1}{2} + \frac{1}{3} \sin^2 \theta_W \quad g_{Rd} = \frac{1}{3} \sin^2 \theta_W. \quad (15)$$

It is clear from our earlier discussion that $\langle x_A u_A^- - x_A d_A^- \rangle$ is negative and allowing for the NuTeV functional we find

$\delta R^\nu = -0.0017 \pm 0.0008$ and $\delta R^{\bar{\nu}} = +0.0016 \pm 0.0008$. (Note that the errors quoted also include our estimated error on $\langle x s^- \rangle$). Subtracting δR^ν from the NuTeV result yields a value 0.3933 ± 0.0015 , which is in good agreement with the Standard Model value, namely 0.3950. The corresponding $\bar{\nu}$ correction yields $R^{\bar{\nu}} = 0.4034 \pm 0.0028$, which is just over one standard deviation from the Standard Model value, namely 0.4066 - again, in quite good agreement. After including our corrections from nuclear effects, partonic CSV and strange quarks, the total χ^2 for R^ν and $R^{\bar{\nu}}$ compared with the Standard Model values moves from 7.19 to 2.58. This represents a very significant improvement.

In conclusion, it should be clear that there is no longer any significant discrepancy between the predictions of the Standard Model evolution and the existing data. However, we look forward to the much higher accuracy in the weak mixing angle which is anticipated in future experiments [42–44]. With regard to NuTeV itself, the greatest single improvement in the accuracy with which one could extract $\sin^2 \theta_W$ would come from a more precise determination of $\langle x s^- \rangle$. A decrease in the error associated with $R^{\bar{\nu}}$ would also set tight constraints on QCD corrections to the NuTeV result.

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