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The orbital g-factor and related sum rules

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The renormalization of the orbital g-factor in nuclei is discussed on the basis of gauge invariance. The relation of the orbital g-factor to the integrated E1 photoabsorption cross section is reviewed, and its relation to the M1 sum rule for the scissors mode of deformed nuclei is examined.

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1 Introduction

The orbital angular momentum g-factor (g_ℓ) of a bound nucleon has attracted the attention of nuclear physicists or a long time, since this quantity reflects the presence of exchange forces and meson exchange currents in nuclei[1, 2]. In infinite nuclear matter, this renormalization can be determined from gauge invariance by using the concepts of the Landau-Migdal theory[3, 4]. Very general considerations, which can be formulated also in relativistic field theory[4, 5], show that the isoscalar part $g_\ell^{(0)}$ is almost unrenormalized, but the isovector part $g_\ell^{(1)}$ is enhanced mainly by pion and sigma meson exchange currents.

A very interesting relation between g_ℓ and the enhancement factor $(1 + \kappa)$ of the photonuclear E1 sum rule has been derived almost 40 years ago in the framework of the Fermi gas model[2], and has been discussed more rigorously in Ref.[5]. The data are largely consistent with this relation, although it has been derived only for nuclear matter so far. On the other hand, the relation between g_ℓ and the M1 sum rule for the "scissors mode" in deformed nuclei has been investigated recently[6], and the results seem to contradict the information which was obtained previously from nuclear magnetic moments and the E1 sum rule.

In this paper we will first review the results for the renormalization of g_ℓ in Sect.2. The relation to the E1 sum rule will be discussed in Sect.3, and in Sect. 4 we will review the analysis of the M1 sum rule for the scissors mode. A summary will be presented in Sect. 5.

2 The orbital g-factor

We first show some numerical results for δg_ℓ obtained in Ref.[7] for nuclei in the lead region in Table 1. These results are obtained in the framework of the nonrelativistic shell model, including higher order configuration mixing and meson exchange currents.

From this Table we see that δg_ℓ is positive for protons and negative for neutrons, and the absolute value of the correction is larger for protons than for neutrons. This result is consistent with observations[8].

For a nucleon in an infinite nuclear medium, g_ℓ can be defined in terms of the current[4] by $\vec{j}(q=0) = \frac{\vec{p}}{M} \cdot g_\ell$, where M is the free nucleon mass. Because the current for $q = 0$ is determined from the Ward identity (gauge invariance), the

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Table 1: Calculated results of Ref.[7] for δg_ℓ in the lead region. "1-pion" refers to the one-pion exchange currents, and "total" includes also other meson exchange currents and configuration mixing.

	Nucleus	1-pion	total
$\delta g_\ell(p)$	^{209}Bi$(h_{9/2})$	0.09	0.13
	^{209}Bi$^*(i_{13/2})$	0.08	0.13
	^{207}Tl$^*(d^{-1}_{3/2})$	0.11	0.15
$\delta g_\ell(n)$	^{209}Pb$(g_{9/2})$	-0.04	-0.07
	^{209}Pb$^*(i_{11/2})$	- 0.05	-0.07
	^{207}Pb$(p^{-1}_{1/2})$	-0.06	-0.09

following general relations can be derived[5]:

$$g_\ell(p) = 1 - \frac{M}{M^*}\frac{F_1(pn)}{3}\frac{N}{A}\,, \tag{1}$$

$$g_\ell(n) = \frac{M}{M^*}\frac{F_1(pn)}{3}\frac{Z}{A}\,. \tag{2}$$

Here $F_1(pn) = F_1 - F_1' < 0$ is the spin-independent $\ell = 1$ Landau-Migdal parameter for the pn interaction. The isoscalar F_1 is related to the effective nucleon mass by the Landau relation $1 - \frac{M}{M^*}\frac{F_1}{3} = \frac{M}{M^*}$. The relations (1) and (2) show the main features discussed above, i.e., $g_\ell(p) > 0$, $g_\ell(n) < 0$, and $|g_\ell(p)| > |g_\ell(n)|$.

From the relations (1) and (2), we can form the "isoscalar" and "isovector" combinations

$$\frac{Z}{A}g_\ell(p) + \frac{N}{A}g_\ell(n) = \frac{Z}{A}\,, \tag{3}$$

$$g_\ell(p) - g_\ell(n) = 1 - \frac{M}{M^*}\frac{F_1(pn)}{3} = \frac{M}{M^*}\left(1 + \frac{F_1'}{3}\right). \tag{4}$$

The isoscalar combination is unrenormalized, and the isovector one is enhanced. (Note that $F_1' > 0$ and $M^* < M$.) It is known that pion exchange gives an important contribution to F_1', and sigma exchange is important for M^*. Because the nonrelativistic analysis of Ref.[7] has shown that the enhancement of $g_\ell^{(1)}$ from pion exchange alone is in agreement with observations, Eq. (4) clearly demonstrates that there is an "overenhancement problem" in relativistic theories[9], where sigma exchange usually leads to a small effective nucleon mass.

It is important to keep in mind that the formulae (1) - (4) incorporate all meson exchange currents and configuration mixing processes which result from the interaction of the valence nucleon with the core nucleons. They have been generalized to relativistic field theory in Refs.[4, 5], where it was shown that relativistic effects sometimes lead to interesting modifications, for example a small enhancement of the isoscalar orbital g-factor.

3 Photoabsorption on nuclei: The Giant Dipole Resonance (GDR)

The GDR around 15 MeV excitation energy is the most prominent feature of photoabsorption on nuclei. The peak in the cross section is usually parametrized by a Lorentzian curve [10]. We define the enhancement factor $\kappa_{\rm GDR}$ by the area under the Lorentzian as follows:

$$S_{\rm GDR} = \int_{\rm GDR} {\rm d}\omega\, \sigma(\omega) = \left(2\pi^2 e^2 \frac{NZ}{MA}\right)(1 + \kappa_{\rm GDR})\,. \tag{5}$$

The first factor on the r.h.s. of (5) is the well known Thomas-Reiche-Kuhn (TRK) result for the energy weighted sum rule of the dipole operator (which is equivalent, by current conservation, to the energy inverse weighed sum rule of the current operator[5]). Typical experimental values are $\kappa_{\rm GDR} \simeq 0.2$, see Fig.1b below.

As will be explained below, in the nuclear matter picture one can use the Ward identity (gauge invariance) to derive the relation[5] $1 + \kappa_{\rm GDR} = g_\ell(p) - g_\ell(n)$, or:

$$g_\ell(p) = 1 + \frac{N}{A}\kappa_{\rm GDR}\,, \qquad g_\ell(n) = -\frac{Z}{A}\kappa_{\rm GDR}\,, \tag{6}$$

where

$$\kappa_{\rm GDR} = -\frac{M}{M^*}\frac{F_1(pn)}{3}\,. \tag{7}$$

For the case $N = Z$, these are the relations derived by Fujita and Hirata[2] in the Fermi gas model.

In Fig.1a we show experimental data for $S_{\rm GDR}$ in comparison to the TRK values, and in Fig.1b we plot the corresponding enhancement factors $\kappa_{\rm GDR}$. In Figs.2a and 2b we show the values of $g_\ell(p)$ and $g_\ell(n)$ as obtained from Eq.(6) by using the $\kappa_{\rm GDR}$ of Fig.2b. It is interesting to note that the 3 spikes in these figures correspond to nuclei with magic proton or neutron numbers. If we discard these magic nuclei, we see that the $\kappa - g_\ell$ relation is qualitatively consistent with observations: The typically observed values of $\kappa_{\rm GDR} \simeq 0.2$ lead to the orbital g-factors $g_\ell(p) \simeq 1.1$ and $g_\ell(n) \simeq -0.1$, which are consistent with the observed nuclear magnetic moments[8].

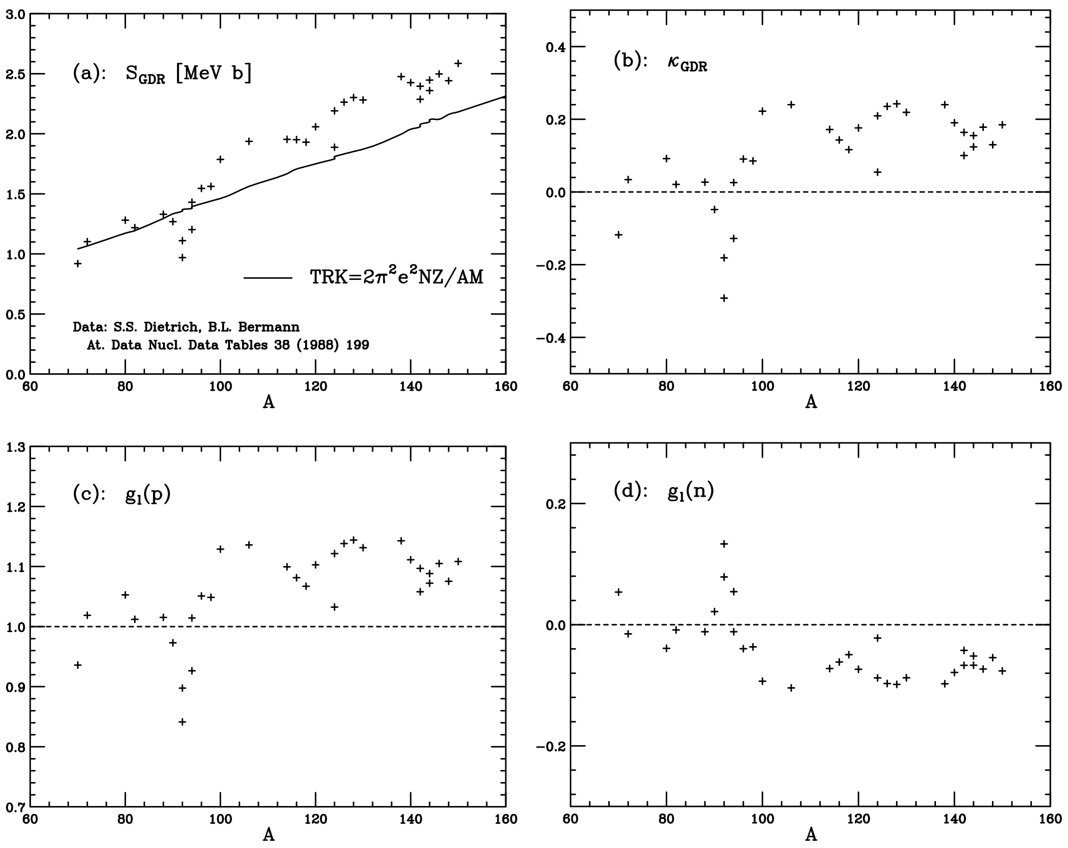

Figure 1: (a): Experimental data for $S_{\rm GDR}$ (crosses) in comparison to the TRK values (sold line). (b): Values for $\kappa_{\rm GDR}$ obtained from the data shown in (a). (c) and (d): Values of the orbital g-factors obtained from Eq.(6) and the observed $\kappa_{\rm GDR}$ shown in (b). The three spikes in these figures correspond to the magic numbers $N = 50$, $Z = 50$ and $N = 82$.

The derivation of the $\kappa - g_\ell$ relation given in Ref.[5] is based on the Fourier transform of the current-current correlation function $\Pi = i\langle 0|T\left(\hat{j}\hat{j}\right)|0\rangle$, where $\hat{j}$ stands for the 3-component (or any other space component) of the current operator. In the long wave length limit ($|\vec{q}| << 1/R$) one can express the cross section and its integral in terms of Π as follows:

$$\sigma(\omega) = \frac{4\pi}{\omega}\mathrm{Im}\Pi(\omega)\,, \tag{8}$$

$$S \equiv \int_0^\infty \mathrm{d}\omega\sigma(\omega) = 2\pi^2\Pi(\omega = 0)\,. \tag{9}$$

The polarization Π can be split into 2 parts: $\Pi = \Pi_A + \Pi_B$, where Π_A arises from particle-hole cuts (see Fig.2), while Π_B has no particle-hole cuts. (Examples of time-ordered diagrams contributing to Π_A and Π_B are discussed in Ref.[5].) The vertex $\Gamma_{\rm eff}$ in Fig.2 is irreducible in the particle-hole channel, i.e., it is the effective particle-hole vertex which includes renormalizations from meson exchange currents and higher order configuration mixing. It is precisely this vertex which is also responsible for the renormalization of g_ℓ. The vertex Γ, on the other hand, is the total vertex including effects of particle-hole rescattering, i.e., it satisfies an RPA-type equation with the driving term $\Gamma_{\rm eff}$ and the Landau-Migdal interaction as the particle-hole interaction kernel. In the limit $\omega \to 0$ first followed by $\vec{q} \to 0$ ("static limit"), the vertex Γ is determined by the Ward identity, and by using the RPA equation one also obtains the vertex $\Gamma_{\rm eff}$[4, 5]. It is then not difficult to derive the relations (6) and (7), where $\kappa_{\rm GDR}$ is identified with the contribution from Π_A to the sum rule (9). This identification is valid as long as one can consider the GDR as a superposition of collective particle-hole states, like in the RPA. The contributions of 2-particle 2-hole (order higher) cuts are contained in Π_B, and are important for energies above the GDR.

These arguments show that a part of the enhancement factor κ, namely $\kappa_{\rm GDR}$, is related to the orbital g-factors by (6), while the remaining part of κ has no relation to g_ℓ. Using a somewhat different approach, the same conclusion was obtained in Ref.[11].

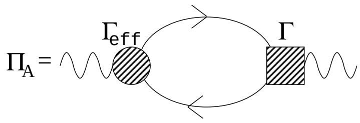

Figure 2: Graphical representation of Π_A. The effective vertex $\Gamma_{\rm eff}$ is irreducible in the particle-hole channel, and Γ is the total vertex including the rescattering of the particle and the hole.

4 M1 sum rule for the scissors mode

The scissors mode[12] is often discussed in terms of the 2-rotor model[13], which is based on the assumption of deformed proton and neutron bodies characterized by the moments of inertia I_p and I_n. In this picture, the rotational energy can be expressed as

$$H_{\rm rot} = \frac{\vec{L}_p^2}{2I_p} + \frac{\vec{L}_n^2}{2I_n} = \frac{\vec{L}^2}{2I} + \frac{\vec{L}^{(1)2}}{2I^{(1)}}\,, \qquad (10)$$

where the total (isoscalar) and relative (isovector) orbital angular momenta and moments of inertia are defined as

$$\vec{L} = \vec{L}_p + \vec{L}_n\,, \qquad \vec{L}^{(1)} = \frac{2I_n}{I}\vec{L}_p - \frac{2I_p}{I}\vec{L}_n\,, \qquad (11)$$

$$I = I_p + I_n\,, \qquad I^{(1)} = \frac{4I_pI_n}{I}\,. \qquad (12)$$

The corresponding separation of the orbital M1 operator is:

$$\vec{\mu} = g_\ell(p)\,\vec{L}_p + g_\ell(n)\,\vec{L}_n = g_\ell^{(0)}\,\vec{L} + g_\ell^{(1)}\,\vec{L}^{(1)}\,, \qquad (13)$$

where

$$g_\ell^{(0)} = \frac{I_p}{I}g_\ell(p) + \frac{I_n}{I}g_\ell(n)\,, \qquad g_\ell^{(1)} = \frac{1}{2}\left(g_\ell(p) - g_\ell(n)\right)\,. \qquad (14)$$

Let us first consider the isoscalar part of Eq.(13). The g-factor of the 2^+ state of the g.s. rotational band is

$$\begin{aligned} g(2^+) = g_\ell^{(0)} &= \frac{I_p}{I}g_\ell(p) + \frac{I_n}{I}g_\ell(n) \\ &\simeq \frac{Z}{A}g_\ell(p) + \frac{N}{A}g_\ell(n) = \frac{Z}{A}\,, \end{aligned} \qquad (15)$$

which is often called the "rotor value". In the last equality, we used Eq.(3). The experimental data for $g(2^+)$ are qualitatively consistent with Eq.(15)[6].

The isovector part of Eq.(13) contributes to the M1 excitation of the scissors mode [14]. Enders et al[6] used their data to determine the following energy-inverse weighted M1 sum rule:

$$S_{-1} \equiv 2\sum_n \frac{1}{\omega_n}|\langle n|\mu_x^{(1)}|0\rangle|^2 \simeq \frac{B(M1,{\rm sciss})}{\omega_{\rm sciss}}\,, \qquad (16)$$

where the symmetry axis is taken along the z direction, and the two independent rotations are around the x and y axes. The "naive" value of S_{-1} is given by

$$\begin{aligned} S_{-1} &= \left(g_\ell^{(1)}\right)^2 \sum_n \frac{2}{\omega_n}\left[|\langle n|L_{px}|0\rangle|^2\left(\frac{2I_n}{I}\right)^2 + |\langle n|L_{nx}|0\rangle|^2\left(\frac{2I_p}{I}\right)^2\right] \\ &= \left(g_\ell^{(1)}\right)^2\left[I_p\left(\frac{2I_n}{I}\right)^2 + I_n\left(\frac{2I_p}{I}\right)^2\right] = \left(g_\ell^{(1)}\right)^2\,\frac{4I_pI_n}{I} \\ &= \left(g_\ell^{(1)}\right)^2\,I^{(1)}\,, \end{aligned} \qquad (17)$$

where in the second equality we used the Inglis expressions for the proton and neutron moments of inertia. The first equality corresponds to the assumption that crossing terms between L_p and L_n can be neglected, i.e., that the effects of the proton-neutron (exchange) interaction, which can change a proton particle-hole pair into a neutron one, can be neglected.

The analysis of Ref.[6] shows that for all nuclei under consideration the observed value of S_{-1} (which is identified with the r.h.s. of Eq.(16)) is well be reproduced by $\left(g_\ell^{(0)}\right)^2 I^{(1)}$, i.e., by replacing the isovector orbital g-factor in Eq.(17) by the isoscalar one, which is roughly equal to Z/A and therefore *smaller* than the free nucleon value of $1/2$. On the other hand, Eq.(17) tells that S_{-1} is given by $\left(g_\ell^{(1)}\right)^2 I^{(1)}$, where the isovector $g_\ell^{(1)}$ is *larger* than the free nucleon value of $1/2$. This is a puzzling situation, and should be investigated more closely by using theoretical models to describe the scissors mode, for example the RPA.

5 Summary

The renormalization of the orbital g-factor in nuclei is a fascinating subject where nuclear structure physics must work together with hadronic and quark physics. It has an impact not only on nuclear magnetic moments, but also on electric and magnetic sum rules for collective nuclear excitations. In this paper we discussed the relation between g_ℓ and the E1 sum rule in the region of the GDR. This relation, which is much more general than the original derivation in the Fermi gas model, is largely consistent with experimental data. The relation between g_ℓ and the recently determined M1 sum rule for the scissors mode in deformed nuclei, however, remains a puzzle which has to be closely examined in future works.

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