Skewness of nuclear matter and three-particle correlations

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We present a study of the skewness of nuclear matter, which is proportional to the third derivative of the energy per nucleon with respect to the baryon density at the saturation point, in the framework of the Landau-Migdal theory. We derive an exact relation between the skewness, the nucleon effective mass, and two-particle and three-particle interaction parameters. We also present qualitative estimates, which indicate that three-particle correlations play an important role for the skewness.

PhySH: Nuclear matter; Nuclear forces; Nuclear many-body theory.

I. INTRODUCTION

The properties of infinite nuclear matter have been the subject of intensive investigations for many decades, mainly because of the wide range of applications, including the properties of heavy nuclei and their excitation modes [1, 2], heavy ion collisions [3], the physics of supernova explosions [4, 5], and neutron stars [6]. Besides the binding energy per nucleon and the saturation density, other basic properties of nuclear matter like the effective mass of nucleons and its momentum dependence [7], the incompressibility [8], the symmetry energy and its density dependence [9], and recently the skewness [10], provide basic information on the effective interaction between nucleons in the medium. Theoretical descriptions of those and other important properties of nuclear matter have been based on Brückner's G-matrix approach [11–13], relativistic theories based on meson exchange [14–16], effective interactions of the Skyrme [17] and Landau-Migdal type [18], and effective field theories [19–23].

In this article we focus our attention on the skewness of nuclear matter and its implication for three-particle correlations. Skewness is defined as $J = 27\rho^3 \left(\mathrm{d}^3 E_A/\mathrm{d}\rho^3\right)$ at $\rho = \rho_0$, where E_A is the energy per nucleon in isospin symmetric nuclear matter, ρ is the baryon density, and ρ_0 is the saturation density ($\rho_0 = 0.15$ fm^{-3}). This physical quantity has been receiving interest in connection to phenomenological applications of the liquid drop model to heavy nuclei [24], and relativistic approaches to nuclear matter [25, 26]. Recently, empirical values have been extracted in Refs. [10, 27] from flow data in heavy ion collisions and neutron star observations. Based on the analogy to the incompressibility, which is defined as $K = 9\rho^2 \left(\mathrm{d}^2 E_A/\mathrm{d}\rho^2\right)$ at $\rho = \rho_0$, and which is known to be related to the spin-isospin averaged two-particle interaction in the medium [28–30], one may expect that J is related to the spin-isospin averaged three-particle interaction. The aim of this paper is to work out this relation in detail and to discuss the result in connection to empirical values. To achieve this we will use the framework of the Landau-Migdal theory of nuclear matter [28, 29], which is based on Landau's Fermi liquid theory [31–34]. The main advantage of this theory is that symmetries, like gauge invariance and Galilei invariance

(or Lorentz invariance in its relativistic extension [35]) are incorporated rigorously, and that the description of collective excitations of the system is physically very appealing [36, 37]. However, to the best of our knowledge, the theory has not yet been applied to directly relate the skewness of nuclear matter to three-particle interactions. It is well known that the effects of three-particle interactions can be incorporated by using effective density-dependent two-particle interactions [38], and this approach has been used in many recent studies with Skyrme-type interactions [39]. However, in view of the long history of studies on three-particle correlations in nuclear matter [40–42], it is desirable to know a more direct relation of a physical quantity, which is connected to observables, to three-particle interaction parameters.

In Sec. II we derive the relation between the skewness of nuclear matter and the three-particle Landau-Migdal parameters, which are defined in a similar way as the familiar two-particle parameters. In Sec. III we discuss this relation in connection to empirical values, and in Sec. IV we present further qualitative discussions, which indicate that three-particle correlations play an important role for the skewness. A summary is presented in Sec. V, and additional details are collected in the Appendices.

II. THEORETICAL FRAMEWORK

Here we discuss the form of the first three derivatives of the energy density, $E(\rho)$, of spin saturated isospin symmetric nuclear matter w.r.t. the baryon density $\rho = 2\,p_F^3/(3\pi^2)$, where p_F is the Fermi momentum. Extending Landau's formula [31–34] for the variation of E w.r.t. spin-isospin independent deviations of the quasiparticle occupation numbers from a reference Fermi distribution (corresponding to a density $\rho_0 = 2\,p_{F0}^3/(3\pi^2) > 0$) to include the third order term, we can write

$$
\begin{aligned}
\delta E(\rho) = {} & 4 \int \frac{\mathrm{d}^3 k}{(2\pi)^3}\, \varepsilon(\boldsymbol{k};\rho_0)\, \delta n_{\boldsymbol{k}} \\
& + \frac{1}{2} \left(\prod_{i=1}^{2} 4 \int \frac{\mathrm{d}^3 k_i}{(2\pi)^3}\, \delta n_{\boldsymbol{k}_i} \right) f(\boldsymbol{k}_1, \boldsymbol{k}_2; \rho_0) \\
& + \frac{1}{6} \left(\prod_{i=1}^{3} 4 \int \frac{\mathrm{d}^3 k_i}{(2\pi)^3}\, \delta n_{\boldsymbol{k}_i} \right) h(\boldsymbol{k}_1, \boldsymbol{k}_2; \boldsymbol{k}_3; \rho_0).
\end{aligned} \tag{1}
$$

Here $\varepsilon(\boldsymbol{k};\rho)$ is the energy of a quasiparticle with momentum $\boldsymbol{k}$; $f(\boldsymbol{k}_1, \boldsymbol{k}_2; \rho)$ is the spin-isospin averaged forward scattering

amplitude of two quasiparticles with momenta $\boldsymbol{k}_1, \boldsymbol{k}_2$; and $h(\boldsymbol{k}_1, \boldsymbol{k}_2; \boldsymbol{k}_3; \rho)$ is the spin-isospin averaged three-particle forward scattering amplitude. The amplitudes f and h are totally symmetric with respect to interchanges of the momentum variables, and can be represented by a set of connected diagrams with four and six external nucleon lines, respectively. Besides the momentum variables, we also indicate the dependence on the background density explicitly.

The form of $\delta n_{\boldsymbol{k}}$ corresponding to a change of the Fermi momentum ($p_{F0} \rightarrow p_F$) is independent of the direction of $\boldsymbol{k}$ and given, to first order in $(p_F - p_{F0})$, by

$$\delta n_k = (p_F - p_{F0}) \, \delta(p_{F0} - k) \,. \tag{2}$$

The corresponding first order variation of E is given by

$$\delta E = 4 \int \frac{\mathrm{d}^3 k}{(2\pi)^3} \varepsilon(\boldsymbol{k}; \rho_0) \, \delta n_k = \varepsilon(p_{F0}; \rho_0) \cdot \delta\rho, \tag{3}$$

where $\delta\rho = \frac{2p_{F0}^2}{\pi^2} \cdot (p_F - p_{F0})$. Because the reference density $\rho_0 > 0$ is arbitrary,[1] we get the well known result [43]

$$\frac{\mathrm{d}E(\rho)}{\mathrm{d}\rho} = \varepsilon(p_F; \rho) \,. \tag{4}$$

Next we consider the first order variation of the quasiparticle energy:

$$\delta\varepsilon(k_1; \rho) = 4 \int \frac{\mathrm{d}^3 k_2}{(2\pi)^3} f(\boldsymbol{k}_1, \boldsymbol{k}_2; \rho_0) \, \delta n_{k_2}$$
$$= f_0(k_1, p_{F0}; \rho_0) \cdot \delta\rho, \tag{5}$$

where the $\ell = 0$ two-particle forward scattering amplitude, which is proportional to the s-wave scattering length in the medium, is the angular average of $f(\boldsymbol{k}_1, \boldsymbol{k}_2; \rho)$. The definition, together with the $\ell = 1$ amplitude which will be used below, is

$$\frac{1}{2\ell + 1} f_\ell(k_1, k_2; \rho) = \int \frac{\mathrm{d}\Omega_2}{4\pi} \left(\hat{\boldsymbol{k}}_1 \cdot \hat{\boldsymbol{k}}_2 \right)^\ell f(\boldsymbol{k}_1, \boldsymbol{k}_2; \rho), \tag{6}$$

where $\ell = 0, 1$, and we use the notation $\mathrm{d}\Omega_i \equiv \mathrm{d}\Omega_{\boldsymbol{k}_i}$. From Eq. (5) we obtain the partial derivative of the quasiparticle energy w.r.t. the density as

$$\frac{\partial\varepsilon(k_1; \rho)}{\partial\rho} = f_0(k_1, k_2 = p_F; \rho) \,. \tag{7}$$

For the total derivative of the Fermi energy (chemical potential) w.r.t. the density we then obtain the well known result [37][2]

$$\frac{\mathrm{d}\varepsilon(p_F; \rho)}{\mathrm{d}\rho} = \frac{\mathrm{d}^2 E}{\mathrm{d}\rho^2} = v(p_F; \rho) \frac{\pi^2}{2p_F^2} + f_0(p_F, p_F; \rho)$$
$$\equiv \frac{\pi^2}{2p_F M^*(p_F; \rho)} + f_0(p_F, p_F; \rho) \equiv \frac{\pi^2}{2p_F M^*} (1 + F_0) \,. \tag{8}$$

[1] Expansions around $\rho_0 = 0$ are analytic in p_F but not in ρ, and are not considered in this paper.
[2] To simplify the notation, quantities like f_ℓ, F_ℓ and M^* (without arguments) are defined on the Fermi surface. A partial derivative $\partial/\partial p_F$ acts only on the momenta, $\partial/\partial\rho$ acts only on the background density, and $(\mathrm{d}/\mathrm{d}\rho) = \pi^2/(2p_F^2)(\partial/\partial p_F) + (\partial/\partial\rho)$ is the total derivative w.r.t. the density.

Here $v(k; \rho) = \partial\varepsilon(k; \rho)/\partial k$ is the magnitude of the velocity of the quasiparticle, and we introduced the momentum dependent effective nucleon mass (Landau effective mass) $M^*(k; \rho)$ by

$$v(k; \rho) \equiv \frac{k}{M^*(k; \rho)} \,. \tag{9}$$

In the last relation of Eq. (8) we defined the dimensionless Landau-Migdal parameters as

$$F_\ell = \frac{2p_F M^*}{\pi^2} f_\ell \,. \tag{10}$$

Before proceeding to the third order derivative, we note that the effective mass is related to the $\ell = 1$ two-particle forward scattering amplitude by[3]

$$\frac{k}{M^*(k; \rho)} + \frac{2p_F^2}{3\pi^2} f_1(k, p_F; \rho) = \frac{k}{M} \,, \tag{11}$$

which for $k = p_F$ becomes the familiar Landau effective mass relation [31–33, 36].

Next we consider the derivative of Eq. (8) w.r.t. the density, which gives

$$\frac{\mathrm{d}^3 E}{\mathrm{d}\rho^3} = -\frac{\pi^4}{4p_F^4 M^*} - \frac{\pi^2}{2p_F M^{*2}} \frac{\mathrm{d}M^*}{\mathrm{d}\rho} + \frac{\mathrm{d}f_0}{\mathrm{d}\rho} \,. \tag{12}$$

By using the derivative of Eq. (7) w.r.t. k_1, we can write

$$\frac{\partial M^*}{\partial\rho} = -\frac{M^{*2}}{2p_F} \frac{\partial f_0}{\partial p_F} \,. \tag{13}$$

Recall (see footnote 1) that the quantity $\partial f_\ell/\partial p_F$ actually means

$$\frac{\partial f_\ell}{\partial p_F} \equiv \left[\left(\frac{\partial}{\partial k_1} + \frac{\partial}{\partial k_2} \right) f_\ell(k_1, k_2; \rho) \right]_{k_1 = k_2 = p_F} \,,$$

and because of the symmetry of the 2-particle amplitude this is the same as the derivative w.r.t. only one momentum variable, multiplied by 2. Further we note that, from Eq. (1), a change of the Fermi momentum leads to the following first order variation of the two-particle scattering amplitude:

$$\delta f(\boldsymbol{k}_1, \boldsymbol{k}_2; \rho) = 4 \int \frac{\mathrm{d}^3 k_3}{(2\pi)^3} h(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3; \rho_0) \, \delta n_{k_3}$$
$$= \delta\rho \cdot \int \frac{\mathrm{d}\Omega_3}{4\pi} h(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3; \rho_0)|_{k_3 = p_{F0}} \,. \tag{14}$$

By taking the $\ell = 0, 1$ moments of this relation, we obtain

$$\frac{\partial}{\partial\rho} f_\ell(k_1, k_2; \rho) = h_\ell(k_1, k_2, k_3 = p_F; \rho), \tag{15}$$

where the $\ell = 0, 1$ three-particle amplitudes are defined by

$$\frac{1}{2\ell + 1} h_\ell(k_1, k_2, k_3; \rho)$$
$$= \int \frac{\mathrm{d}\Omega_2}{4\pi} \int \frac{\mathrm{d}\Omega_3}{4\pi} \left(\hat{\boldsymbol{k}}_1 \cdot \hat{\boldsymbol{k}}_2 \right)^\ell h(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3; \rho) \,. \tag{16}$$

[3] For convenience, we collect the relations which arise from Galilei invariance in App. A.

Substituting Eqs. (13) and (15) into Eq. (12) gives

$$\frac{d^3E}{d\rho^3} = -\frac{\pi^4}{4p_F^4 M^*} + \frac{3\pi^2}{4p_F^2}\frac{\partial f_0}{\partial p_F} - \frac{\pi^4}{4p_F^3 M^{*2}}\frac{\partial M^*}{\partial p_F} + h_0 \,. \quad (17)$$

In order to eliminate the derivative of f_0 from this equation, we take the partial derivative w.r.t. the density on both sides of Eq. (11), and then set $k = p_F$. Using again Eq. (13) we obtain

$$p_F \frac{\partial}{\partial p_F}\left(f_0 + \frac{1}{3}f_1\right) + \frac{4}{3}f_1 + \frac{4p_F^3}{3\pi^2} h_1 = 0 \,. \quad (18)$$

As explained in App. A, this relation can also be derived directly from the Galilei invariance of the two-particle scattering amplitude. The f_1 terms in Eq. (18) can be expressed by the effective mass, by using Eq. (11) and its derivative w.r.t. k at the Fermi surface. In this way we can rewrite Eq. (18) as

$$p_F \frac{\partial f_0}{\partial p_F} - \frac{3\pi^2}{p_F}\frac{M - M^*}{MM^*} + \frac{\pi^2}{M^{*2}}\frac{\partial M^*}{\partial p_F} + \frac{4p_F^3}{3\pi^2} h_1 = 0 \,. \quad (19)$$

Using this relation to eliminate the derivative of f_0 from Eq. (17) we finally obtain

$$\frac{d^3E}{d\rho^3} = \frac{3\pi^4}{4p_F^4 M^*}$$
$$\times \left(-\frac{1}{3} + 3\frac{M - M^*}{M} - \frac{4}{3}\frac{p_F}{M^*}\frac{\partial M^*}{\partial p_F} + (H_0 - H_1)\right). \quad (20)$$

Here we have defined the dimensionless three-particle interaction parameters by[4]

$$H_\ell = \left(\frac{2p_F M^*}{\pi^2}\rho\right) h_\ell = \frac{4p_F^4 M^*}{3\pi^4} h_\ell \,. \quad (21)$$

One should note that Eq. (20) is an exact relation.

III. DISCUSSION ON EMPIRICAL VALUES

In order to connect the relations of Sec. II to empirical values, we note that the energy per nucleon (E_A) is related to the energy density (E) by

$$E_A(\rho) = E(\rho)/\rho \,. \quad (22)$$

All the relations which follow will be given for $\rho = \rho_0$, where ρ_0 is the saturation density defined by $dE_A/d\rho|_{\rho=\rho_0} = 0$.

Taking the first derivative of Eq. (22) and using Eq. (4) we obtain the well known relation $E_A(\rho) = \varepsilon(p_F; \rho)$ [43]. The second derivative of Eq. (22) and its relation to the incompressibility (K) of nuclear matter is given by [7, 10]

$$\frac{d^2 E_A}{d\rho^2} = \frac{1}{\rho}\frac{d^2 E}{d\rho^2} \equiv \frac{K}{9\rho^2} \,. \quad (23)$$

The third derivative of Eq. (22) and its relation to the skewness (J) of nuclear matter is given by [7, 10]

$$\frac{d^3 E_A}{d\rho^3} = -\frac{3}{\rho^2}\frac{d^2 E}{d\rho^2} + \frac{1}{\rho}\frac{d^3 E}{d\rho^3} \equiv \frac{J}{27\rho^3} \,. \quad (24)$$

By using Eqs. (8) and (20), the incompressibility and the skewness can be expressed as

$$K = \frac{3p_F^2}{M^*}(1 + F_0) \,, \quad (25)$$

$$J = -9K + \frac{9p_F^2}{M^*}$$
$$\times \left(-\frac{1}{3} + 3\frac{M - M^*}{M} - \frac{4}{3}\frac{p_F}{M^*}\frac{\partial M^*}{\partial p_F} + (H_0 - H_1)\right). \quad (26)$$

Recent analyses [7, 10] which take into account constraints from heavy ion collisions and astrophysical observations, suggest the following ranges of K and J at the saturation density $\rho_0 = 0.15$ fm^{-3}:

$$0.20\,[\text{GeV}] < K < 0.30\,[\text{GeV}], \quad (27)$$
$$-0.50\,[\text{GeV}] < J < -0.01\,[\text{GeV}]. \quad (28)$$

From these values and Eqs. (25) and (26) we can make a few observations: First, the Fermi gas value of the incompressibility $K_{\text{free}} = 3\frac{p_F^2}{M} \simeq 0.22$ GeV is inside the range of Eq. (27), i.e., the detailed values of M^* and F_0 result in fine-tuning of K but do not lead to order of magnitude changes. On the other hand, the Fermi gas value for the skewness $J_{\text{free}} = -10K_{\text{free}} \simeq -2.2$ GeV is very large in magnitude compared to the empirical values of Eq. (28), indicating that some of the correction terms in Eq. (26) must be large and positive. In fact, by using the lower limits of Eqs. (27) and (28), we see that Eq. (26) implies

$$\frac{8}{3}\frac{M - M^*}{M^*} - \frac{4}{3}\frac{p_F M}{M^{*2}}\frac{\partial M^*}{\partial p_F} + \frac{M}{M^*}(H_0 - H_1) > 2.38. \quad (29)$$

The value of the effective mass at the saturation density of symmetric nuclear matter has been the subject of intensive investigations during the last decades [2, 7], and most of them are consistent with the range

$$0.7 < \frac{M^*}{M} < 1.0. \quad (30)$$

The momentum dependence of the effective mass has been intensively studied by Mahaux *et al.* [2]. Their studies, as well as subsequent work of Blaizot *et al.* [44], indicate that $M^*(k; \rho)$ has a pronounced peak very close to the Fermi surface (i.e., $\partial M^*/\partial p_F \simeq 0$), and it was shown that this behavior comes mainly from the same polarization effects which cause an imaginary part of the single particle energy away from the Fermi surface. More recent studies [45] indicate that this peak of the effective mass may be slightly above the Fermi surface, i.e., $M^*(k; \rho)$ is still increasing at $k = p_F$ before it reaches a maximum.[5] In our subsequent estimates we will therefore assume that $\partial M^*/\partial p_F > 0$. We then obtain an upper limit for the first two terms in Eq. (29):

$$\frac{8}{3}\frac{M - M^*}{M^*} - \frac{4}{3}\frac{p_F M}{M^{*2}}\frac{\partial M^*}{\partial p_F} < 1.14. \quad (31)$$

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[4] As noted in App. B, some care has to be taken when comparing the magnitudes of the dimensionless two-particle and three-particle parameters.

[5] Fig. 4 of Ref. [45] indicates that $\frac{\partial M^*}{\partial p_F} \simeq 1.8$ at the saturation density.
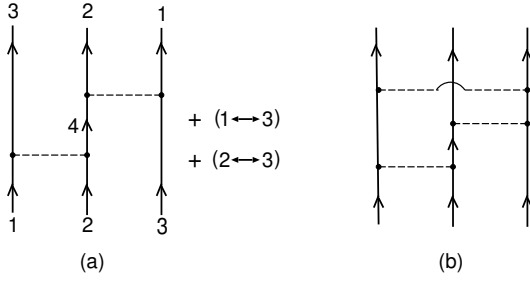
Figure 1. (a) Lowest order diagram contributing to the two-particle correlation piece $h^{(2pc)}$. (b) An example of a contribution to the three-particle correlation piece $h^{(3pc)}$. In each case the solid lines are nucleons, and the dashed lines denote the nuclear potential, which is assumed to be antisymmetrized according to Eq. (35).

By comparing Eqs. (29) and (31) we clearly see that one needs a large positive contribution from the three-body terms in order to explain the empirical values:

$$\frac{M}{M^*}(H_0 - H_1) > 1.24. \tag{32}$$

IV. QUALITATIVE DISCUSSIONS ON THE THREE-PARTICLE AMPLITUDE

The three-particle amplitude $h(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3; \rho)$ contains 1-particle reducible pieces, i.e., diagrams which can be made disconnected by cutting a single intermediate nucleon line, and 1-particle irreducible pieces. In perturbation theory, the former ones start with terms of second order in the nuclear potential, see Fig. 1(a). We will refer to them as the "two-particle correlation" (2pc) contributions to the three-particle amplitude. On the other hand, the 1-particle irreducible pieces start with third order perturbation theory, and involve interactions between all three pairs of the particles. (See Fig. 1(b) for an example.) We will call them the "three-particle correlations" (3pc). We therefore split the three-particle amplitude as follows:

$$h = h^{(2pc)} + h^{(3pc)}. \tag{33}$$

The expression for the lowest order process shown in Fig. 1(a) can be derived from the definition given in Eq. (1) by taking the second functional derivative of the energy density in second order perturbation theory. Those familiar relations [13, 37] are summarized in App. B, and the second (2nd) order result for h is

$$h^{(2nd)}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3; \rho) = \sum_4 \delta_{\mathbf{k}_1 + \mathbf{k}_2, \mathbf{k}_3 + \mathbf{k}_4}$$
$$\times P \frac{|\langle 12| \hat{v} |34\rangle_a|^2}{\epsilon_3 + \epsilon_4 - \epsilon_1 - \epsilon_2} + (1 \leftrightarrow 3) + (2 \leftrightarrow 3). \tag{34}$$

In this schematic notation, $1 \sim 4$ represent the momenta $\mathbf{k}_1 \sim \mathbf{k}_4$ as well as the associated spin and isospin components, though an average over the spin and isospin components of 1, 2, 3 is assumed implicitly. The sum represents momentum integration and summation over spin and isospin components, the δ symbol represents a momentum conserving δ-function, P denotes the principal value, and the ϵ's represent the free
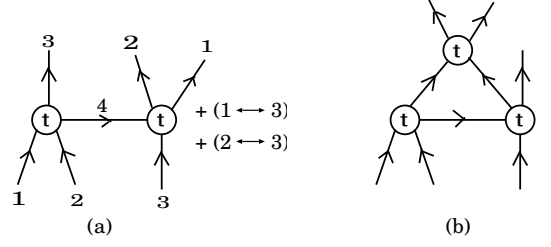

Figure 2. (a) The two-particle correlation piece $h^{(2pc)}$ of Eq. (36). (b) An example of the three-particle correlation contribution $h^{(3pc)}$. In each case the solid lines are nucleons, and t represents a 2-body off-forward scattering matrix. The two diagrams actually represent the first two terms in the Faddeev series.

nucleon kinetic energies $\epsilon_i = k_i^2/(2M)$. The antisymmetrized matrix elements of the nuclear potential ($\hat{v}$) are defined by

$$\langle 12| \hat{v} |34\rangle_a = \langle 12| \hat{v} |34\rangle - \langle 12| \hat{v} |43\rangle. \tag{35}$$

The fact that there is no Fermi step-function n_4 associated with the intermediate nucleon 4 in Eq. (34) is consistent with the 1-particle reducible character: If there were a dependence on n_4, one further variation could be carried out to derive a connected four-particle amplitude. This, however, is not possible for the process shown in Fig. 1(a), because cutting the intermediate nucleon line gives a disconnected diagram.

A few points of this simple perturbative example can be generalized (see also App. B for examples): According to the definition given in Eq. (1), the function h can be obtained from a given diagram for the energy density (effective potential) by successively cutting three intermediate nucleon lines which we denote by 1, 2, and 3.[6] If the cuts are taken in such a way that, among the remaining lines, there is one (we denote it by 4) whose momentum is fixed by momentum conservation, then the four lines 1, 2, 3, 4 constitute the external legs of a two-particle scattering amplitude (matrix elements of an operator $\hat{t}$), which is the off-forward generalization of the function f defined by Eq. (1). In this case, the resulting diagram can be made disconnected by cutting the line 4, i.e., this line must connect two 2-particle irreducible t-blocks, as shown in Fig. 2(a). The expression for this process is then obtained by a generalization of Eq. (34), i.e., replacing $\hat{v} \to \hat{t}$ and $\epsilon_i \to \varepsilon_i$, where the ε_i's are the quasiparticle energies:

$$h^{(2pc)}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3; \rho) = \sum_4 \delta_{\mathbf{k}_1 + \mathbf{k}_2, \mathbf{k}_3 + \mathbf{k}_4}$$
$$\times P \frac{|\langle 12| \hat{t} |34\rangle_a|^2}{\varepsilon_3 + \varepsilon_4 - \varepsilon_1 - \varepsilon_2} + (1 \leftrightarrow 3) + (2 \leftrightarrow 3). \tag{36}$$

The energy denominator in this expression comes from the intermediate state where only the particles 1, 2, 3, 4 are "in the air", and is the same irrespective of whether particle 4 propagates forward or backward in time. Hence, there is no dependence on the Fermi step function n_4 of particle 4, which is consistent with the 1-particle reducible character of the process,

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[6] We can draw the resulting diagram for h in the "particle channel", i.e., if a cut line (p) was *beginning* at an interaction vertex A and *terminating* at a vertex B, we draw a particle line p *outgoing* from A and *incoming* into B.

as explained above for the lowest order case. On the other hand, if the cuts in the diagram for the energy density are taken in any other way, the resulting diagram for h is 1-particle irreducible, and the example shown in Fig. 2(b) is a generalization of the lowest order process of Fig. 1(b). We see that actually the process of Fig. 2(a) is the driving term of the Faddeev equation (one particle is exchanged between an interacting pair and the third particle), and Fig. 2(b) appears in the first iteration of the Faddeev equation [40–42].

In order to get a rough estimate of the two-particle correlation piece $h^{(2\mathrm{pc})}$ and its contribution to the inequality given in Eq. (32), we assume that the 2-particle t-matrix in Eq. (36) can be represented by an effective contact interaction, i.e., by the in-medium scattering length [46]. In this case, the angular averages of Eq. (16) concern only the energy denominator of Eq. (36), and with the further assumption that the latter can be obtained from the free expression in Eq. (34) by the replacement $M \to M^*$, assuming an average value of the effective mass in the range given in Eq. (30), the angular integrals can be done analytically (see App. C):

$$\int \frac{\mathrm{d}\Omega_2}{4\pi} \int \frac{\mathrm{d}\Omega_3}{4\pi} \left(\frac{P}{\varepsilon_3 + \varepsilon_4 - \varepsilon_1 - \varepsilon_2} + (1 \leftrightarrow 3) + (2 \leftrightarrow 3) \right)$$
$$= \frac{M^*}{p_F^2} \, (3 \ln 2) , \qquad (37)$$

$$3 \int \frac{\mathrm{d}\Omega_2}{4\pi} \int \frac{\mathrm{d}\Omega_3}{4\pi} \left(\hat{\boldsymbol{k}}_1 \cdot \hat{\boldsymbol{k}}_2 \right)$$
$$\times \left(\frac{P}{\varepsilon_3 + \varepsilon_4 - \varepsilon_1 - \varepsilon_2} + (1 \leftrightarrow 3) + (2 \leftrightarrow 3) \right)$$
$$= -\frac{M^*}{p_F^2} \, (1 - \ln 2) . \qquad (38)$$

Because these simple expressions indicate that $\ell = 1$ contributions are suppressed by large factors compared to the $\ell = 0$ contributions, we neglect the $\ell = 1$ terms in the following. To be specific, we assume that the matrix elements $_a\langle 34| \hat{t} |12\rangle$ can be replaced by the $\ell = 0$ part of an effective interaction of the Landau-Migdal type [28–30]:

$$_a\langle 34| \hat{t} |12\rangle = f_0 \, (\delta_{31} \cdot \delta_{42}) + f_0' \, (\boldsymbol{\tau}_{31} \cdot \boldsymbol{\tau}_{42})$$
$$+ g_0 \, (\boldsymbol{\sigma}_{31} \cdot \boldsymbol{\sigma}_{42}) + g_0' \, (\boldsymbol{\sigma}_{31} \cdot \boldsymbol{\sigma}_{42}) \, (\boldsymbol{\tau}_{31} \cdot \boldsymbol{\tau}_{42}), \quad (39)$$

where the notation indicates that the spin and isospin operators are defined to act in the particle-hole channel. As usual, the effect of exchange terms is assumed to be included in the interaction parameters. Performing then the spin-isospin sum over 4 as well as the spin-isospin averages over 1, 2, 3 in Eq. (36), we obtain

$$h_0^{(2\mathrm{pc})} = \frac{M^*}{p_F^2} \cdot (3 \ln 2) \cdot \frac{1}{4} \left(f_0^2 + 3 f_0'^2 + 3 g_0^2 + 9 g_0'^2 \right) . \quad (40)$$

In terms of the dimensionless interaction parameters of Eqs. (10) and (21), this can be expressed as

$$H_0^{(2\mathrm{pc})} = \ln 2 \cdot \frac{1}{4} \left(F_0^2 + 3 F_0'^2 + 3 G_0^2 + 9 G_0'^2 \right) . \quad (41)$$

In order to estimate the 2pc contributions to the inequality given in Eq. (32), we refer to recent calculations done by using several parameter sets of the extended Skyrme interaction [17] and chiral effective field theory [23]. (Both calculations are consistent with the ranges given in Eq. (27) for the incompressibility and Eq. (30) for the effective mass.) Using the values for the Landau-Migdal parameters given in Table IV of Ref. [17], and in Figs. 9 and 10 of Ref. [23] at the saturation density, we see that the values for the quantity $\frac{M}{M^*} H_0^{(2\mathrm{pc})}$ are between 0.75 and 1.23. Comparison with Eq. (32) then indicates that the 2pc piece alone is too small to reproduce the empirical results. This gives us a hint that the three-particle correlation term in Eq. (33) plays an important role for the skewness of nuclear matter.

V. SUMMARY

In this paper we used the framework of the Landau-Migdal theory to express the skewness of nuclear matter in terms of the effective nucleon mass, its slope as a function of momentum, and two-body as well as three-body interaction parameters. Our main result is summarized by the formula given in Eq. (26), where H_0 and H_1 are the dimensionless three-body Landau-Migdal parameters, which we defined in Eq. (21) via the $\ell = 0$ and $\ell = 1$ moments of the three-body forward scattering amplitude h. We pointed out that, in order to explain the range of empirical values for J given in Eq. (28), one needs a large positive contribution from the three-body interaction terms, as expressed by Eq. (32). We attempted to make a rough estimate of the contributions of two-nucleon correlations [see Figs. 1(a) and 2(a)] to those three-body parameters, and found that they are of the right sign but very likely too small to explain the empirical values of J. We find this to be an important hint that three-particle correlation processes, like those shown in Figs. 1(b) and 2(b), play an important role for the skewness of nuclear matter.

Usually the effect of three-particle interactions are incorporated by using effective density-dependent two-particle interactions. In view of the long and important history of studies on three-particle correlations in nuclear matter, however, it would be interesting to assess our results more quantitatively, by using for example the Faddeev method in the framework of effective field theories.

A. GALILEI INVARIANCE

For completeness we review here the derivation of the Landau relation between M^* and f_1: One considers the variation of the quasiparticle energy which arises from the change of the distribution function due to a Galilei transformation from the

rest system of nuclear matter to a system which moves with velocity $\boldsymbol{u} \equiv \boldsymbol{p}/M$, where M is the free nucleon mass. To first order in p these variations are given by

$$\delta n_{\boldsymbol{k}} = -(\hat{\boldsymbol{k}} \cdot \boldsymbol{p}) \, \delta(p_F - k), \tag{A1}$$

$$\delta\varepsilon(\boldsymbol{k};\rho) = 4 \int \frac{\mathrm{d}^3 k_2}{(2\pi)^3} \, f(\boldsymbol{k}, \boldsymbol{k}_2; \rho) \, \delta n_{\boldsymbol{k}_2}$$
$$= -\frac{2p_F^2}{3\pi^2} \left(\hat{\boldsymbol{k}} \cdot \boldsymbol{p} \right) f_1(k, p_F; \rho). \tag{A2}$$

On the other hand, the quasiparticle energy should transform in the same way as a Hamiltonian in classical mechanics, i.e; $\varepsilon'(\boldsymbol{k}';\rho) = \varepsilon(\boldsymbol{k};\rho) - \frac{\boldsymbol{k}\cdot\boldsymbol{p}}{M} + \frac{p^2}{2M}$, where $\boldsymbol{k}' = \boldsymbol{k} - \boldsymbol{p}$. From this it follows that $\varepsilon'(\boldsymbol{k};\rho) = \varepsilon(\boldsymbol{k} + \boldsymbol{p};\rho) - \frac{\boldsymbol{k}\cdot\boldsymbol{p}}{M} - \frac{p^2}{2M}$, and to first order in p,

$$\delta\varepsilon(\boldsymbol{k};\rho) = (\boldsymbol{k} \cdot \boldsymbol{p}) \left(\frac{1}{M^*(k,\rho)} - \frac{1}{M} \right), \tag{A3}$$

where we used the definition given in Eq. (9). The requirement that Eqs. (A2) and (A3) are identical leads to the desired relation of Eq. (11) in the main text:

$$\frac{k}{M^*(k;\rho)} + \frac{2p_F^2}{3\pi^2} f_1(k, p_F; \rho) = \frac{k}{M}, \tag{A4}$$

which holds for any values of k and ρ. For the case $k = p_F$, this becomes the familiar Landau effective mass relation.

As explained in the main text, by taking derivatives of Eqs. (7) and (11) one can derive Eq. (18). Here we show that this relation can also be derived by the requirement of Galilei invariance of the 2-particle forward scattering amplitude: Under the change given by Eq. (A1) of the quasiparticle distribution function, the scattering amplitude changes according to

$$\delta f(\boldsymbol{k}_1, \boldsymbol{k}_2; \rho) = -4 \int \frac{\mathrm{d}^3 k_3}{(2\pi)^3}$$
$$h(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3; \rho) \left(\boldsymbol{p} \cdot \hat{\boldsymbol{k}}_3 \right) \delta(p_F - k_3). \tag{A5}$$

On the other hand, the Galilei invariance of the scattering amplitude is expressed as $f'(\boldsymbol{k}_1', \boldsymbol{k}_2'; \rho) = f(\boldsymbol{k}_1, \boldsymbol{k}_2; \rho)$, where $\boldsymbol{k}_i' = \boldsymbol{k}_i - \boldsymbol{p}$. From this it follows, to first order in p, that $\delta f(\boldsymbol{k}_1, \boldsymbol{k}_2; \rho) = \boldsymbol{p} \cdot \left(\boldsymbol{\nabla}_{k_1} + \boldsymbol{\nabla}_{k_2} \right) f(\boldsymbol{k}_1, \boldsymbol{k}_2; \rho)$. The requirement that this change is the same as Eq. (A5) gives

$$\left(\boldsymbol{\nabla}_{k_1} + \boldsymbol{\nabla}_{k_2} \right) f(\boldsymbol{k}_1, \boldsymbol{k}_2; \rho)$$
$$= -4 \int \frac{\mathrm{d}^3 k_3}{(2\pi)^3} h(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3; \rho) \, \hat{\boldsymbol{k}}_3 \, \delta(p_F - k_3). \tag{A6}$$

Multiplying this relation by $\hat{\boldsymbol{k}}_1$ and integrating over the directions of $\boldsymbol{k}_2$ this becomes

$$\int \frac{\mathrm{d}\Omega_2}{4\pi} \left(\hat{\boldsymbol{k}}_1 \cdot \boldsymbol{\nabla}_{k_1} + \hat{\boldsymbol{k}}_1 \cdot \boldsymbol{\nabla}_{k_2} \right) f(\boldsymbol{k}_1, \boldsymbol{k}_2; \rho)$$
$$= -\frac{2p_F^2}{\pi^2} \int \frac{\mathrm{d}\Omega_2}{4\pi} \int \frac{\mathrm{d}\Omega_3}{4\pi} \left(\hat{\boldsymbol{k}}_1 \cdot \hat{\boldsymbol{k}}_3 \right) h(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3; \rho)|_{k_3 = p_F}. \tag{A7}$$
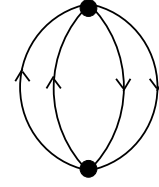

Figure 3. Hugenholtz diagram for the second order contribution to the energy density. The solid lines represent nucleons, and the black circles represent the antisymmetrized nuclear potential.

We consider this relation for the case $k_1 = k_2 = p_F$. In the first term on the l.h.s. we simply have $\hat{\boldsymbol{k}}_1 \cdot \boldsymbol{\nabla}_{k_1} = \partial/\partial k_1$. For the second term, we can use a partial integration in $\boldsymbol{k}_2$ to show that

$$\int \frac{\mathrm{d}\Omega_2}{4\pi} \left(\hat{\boldsymbol{k}}_1 \cdot \boldsymbol{\nabla}_{k_2} \right) f(\boldsymbol{k}_1, \boldsymbol{k}_2; \rho)$$
$$= \int \frac{\mathrm{d}\Omega_2}{4\pi} \left(\hat{\boldsymbol{k}}_1 \cdot \hat{\boldsymbol{k}}_2 \right) \left(\frac{2}{p_F} + \frac{\partial}{\partial k_2} \right) f(\boldsymbol{k}_1, \boldsymbol{k}_2; \rho)$$
$$= \frac{1}{3} \left(\frac{2}{p_F} + \frac{\partial}{\partial k_2} \right) f_1(k_1, k_2; \rho).$$

Substituting this for the second term on the l.h.s. of Eq. (A7), and using the definition given in Eq. (16) as well as the symmetry of h in the momentum variables, leads to Eq. (18).

B.   PERTURBATION THEORY

For illustration of the discussions in Sec. IV, we summarize the familiar expressions for the energy density and the two- and three-particle forward scattering amplitudes in second order[7] perturbation theory [13, 37]. The expression for the energy density is

$$E^{(2)} = -\frac{1}{4} \sum_{1234} \delta_{\boldsymbol{k}_1 + \boldsymbol{k}_2, \boldsymbol{k}_3 + \boldsymbol{k}_4} \frac{|\langle 12|\hat{v}|34\rangle_a|^2}{\epsilon_3 + \epsilon_4 - \epsilon_1 - \epsilon_2}$$
$$\times n_1 n_2 (1 - n_3)(1 - n_4). \tag{B1}$$

Here we use the schematic notations explained in Sec. IV, and $n_i \equiv n_{k_i} = \theta(p_F - k_i)$ is the Fermi step function. The corresponding Hugenholtz diagram [37] is shown in Fig. 3. The second order contribution to the two-particle forward scattering amplitude obtained from Eq. (B1) is given by

$$f^{(2)}(\boldsymbol{k}_1, \boldsymbol{k}_2; \rho) = \frac{\delta^2 E^{(2)}}{\delta n_1 \, \delta n_2}$$
$$= \frac{1}{2} \sum_{34} \delta_{\boldsymbol{k}_1 + \boldsymbol{k}_2, \boldsymbol{k}_3 + \boldsymbol{k}_4} \frac{|\langle 12|\hat{v}|34\rangle_a|^2}{\epsilon_3 + \epsilon_4 - \epsilon_1 - \epsilon_2} (n_3 + n_4 - 1) \tag{B2}$$
$$+ \sum_{34} \delta_{\boldsymbol{k}_1 + \boldsymbol{k}_3, \boldsymbol{k}_2 + \boldsymbol{k}_4} \frac{|\langle 13|\hat{v}|24\rangle_a|^2}{\epsilon_2 + \epsilon_4 - \epsilon_1 - \epsilon_3} (n_3 - n_4). \tag{B3}$$

The corresponding Hugenholtz diagrams are shown in Fig. 4. The term Eq. (B2) is represented by the left diagram, where the bubble graph has two possible time orderings corresponding to

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[7] In this Appendix we use a superscript (2) to denote expressions derived in second order perturbation theory.
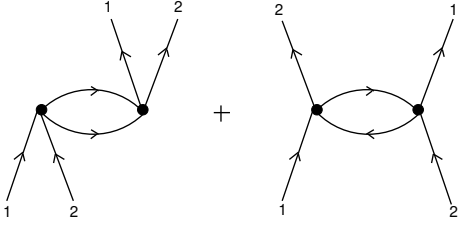
Figure 4. Hugenholtz diagrams for the two-particle forward scattering amplitude. The left diagram represents Eq. (B2), and the right diagram represents Eq. (B3). The solid lines represent nucleons, and the black circles represent the nuclear potential.

two particles or two holes in the intermediate state. The term Eq. (B3) is represented by the right diagram, where again the bubble graph has two possible time orderings, corresponding to forward and backward propagating particle-hole intermediate states. An average over the spin and isospin components of particles 1, 2 are implicitly assumed in the above schematic notations.

By relabeling indices, it is easy to verify that

$$f^{(2)}(\boldsymbol{k}_1, \boldsymbol{k}_2; \rho) = f^{(2)}(\boldsymbol{k}_1, \boldsymbol{k}_2; \rho = 0) + \sum_3 n_3$$

$$\times \left[\sum_4 \delta_{\boldsymbol{k}_1 + \boldsymbol{k}_2, \boldsymbol{k}_3 + \boldsymbol{k}_4} \frac{|\langle 12| \hat{v} |34\rangle_a|^2}{\epsilon_3 + \epsilon_4 - \epsilon_1 - \epsilon_2} + (2 \leftrightarrow 3) + (1 \leftrightarrow 3) \right]. \tag{B4}$$

From this expression it is clear that the second order contribution to the three-particle forward scattering amplitude:

$$h^{(2)}(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3; \rho) = \frac{\delta f^{(2)}(\boldsymbol{k}_1, \boldsymbol{k}_2; \rho)}{\delta n_3}, $$

is given by Eq. (34). The only non-trivial point to note is how the principal value emerges. To see this, one has to go back to Eq. (B1) and add the infinitesimal $-i\delta$ in the denominator, although this is irrelevant for the energy density and the two-particle scattering amplitude, which are automatically real. Going through the derivation, one finds Eq. (34), but with an additional contribution

$$\sum_4 \delta_{\boldsymbol{k}_1 + \boldsymbol{k}_2, \boldsymbol{k}_3 + \boldsymbol{k}_4} |\langle 12|\hat{v}|34\rangle_a|^2 \, i\pi\delta \left(\epsilon_3 + \epsilon_4 - \epsilon_1 - \epsilon_2\right) (1 - 2n_4)$$

$$+ (2 \leftrightarrow 3) + (1 \leftrightarrow 3) . \tag{B5}$$

Because $k_i = p_F$ for $i = 1, 2, 3$, the energy conserving delta function in Eq. (B5) requires that also $k_4 = p_F$. In order to avoid such an unphysical imaginary part, one therefore has to define the step function $\theta(x)$ so that $\theta(x = 0) = \frac{1}{2}$, which is also suggested by the finite temperature form of n_4, taking the limit $\epsilon_4 \to \epsilon_F$ before the limit $T \to 0$.

We note that Eq. (B4), which reads symbolically as $f(1, 2) = f_{\text{vac}}(1, 2) + \sum_3 h(1, 2, 3) n_3$, does not hold in the general case, only its differential form is general. This indicates that some care should be taken in a direct comparison of magnitudes of the dimensionless two-body [Eq. (10)] and three-body [Eq. (21)] parameters.
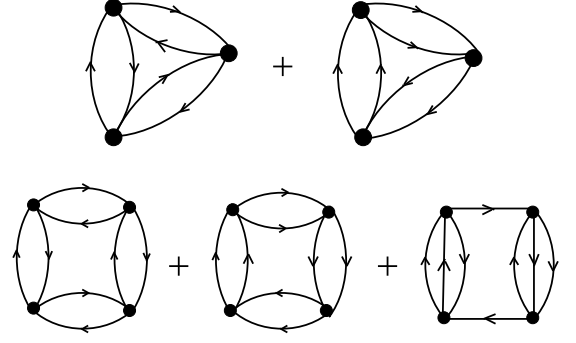

Figure 5. *Top panel:* Hugenholtz diagram for the third order contribution to the energy density. *Bottom panel:* Hugenholtz diagram for the fourth order contribution to the energy density. In both cases the solid lines represent nucleons, and the black circles represent the antisymmetrized nuclear potential.

The discussions given above may be extended to higher orders in perturbation theory, and we just give the relevant Hugenholtz diagrams for the energy density in third and fourth order in Fig. 5 [20–22]. One may use those figures to follow the line of arguments in Sec. IV, leading from the second order result, Eq. (34), to its extension given by Eq. (36).

C. ANGULAR INTEGRALS

If we chose a coordinate system where $\boldsymbol{k}_1$ points along the z axis, the energy denominator of the first term in Eq. (37) can be expressed as

$$\frac{1}{\epsilon_3 + \epsilon_4 - \epsilon_1 - \epsilon_2} \equiv \frac{1}{\epsilon_{\boldsymbol{k}_3} + \epsilon_{\boldsymbol{k}_1 + \boldsymbol{k}_2 - \boldsymbol{k}_3} - \epsilon_{\boldsymbol{k}_1} - \epsilon_{\boldsymbol{k}_2}}$$

$$= \frac{M}{p_F^2} \frac{1}{(1 + z_2)(1 - z_3) - \sqrt{(1 - z_2^2)(1 - z_3^2)} \cos \phi} . \tag{C1}$$

Here z_2, z_3 are the polar angles of $\boldsymbol{k}_2$, $\boldsymbol{k}_3$, and ϕ is the relative azimuthal angle between $\boldsymbol{k}_2$ and $\boldsymbol{k}_3$. The principal value integration over ϕ can be performed by using the formula

$$P \int_0^{2\pi} \frac{\mathrm{d}\phi}{a + b \cos \phi} = \frac{2\pi}{\sqrt{a^2 - b^2}} \theta(a^2 - b^2), \tag{C2}$$

where $a > 0$. This gives

$$\int \frac{\mathrm{d}\Omega_2}{4\pi} \int \frac{\mathrm{d}\Omega_3}{4\pi} P \frac{1}{\epsilon_3 + \epsilon_4 - \epsilon_1 - \epsilon_2} = \frac{M}{p_F^2} \frac{\sqrt{2}}{8} I_0, \tag{C3}$$

$$\int \frac{\mathrm{d}\Omega_2}{4\pi} \int \frac{\mathrm{d}\Omega_3}{4\pi} P \frac{\hat{\boldsymbol{k}}_1 \cdot \hat{\boldsymbol{k}}_2}{\epsilon_3 + \epsilon_4 - \epsilon_1 - \epsilon_2} = \frac{M}{p_F^2} \frac{\sqrt{2}}{8} I_1, \tag{C4}$$

where the integrals I_0 and I_1 over the polar angles are evaluated as

$$I_0 = \int_{-1}^1 \mathrm{d}z_2 \int_{-1}^{z_2} \mathrm{d}z_3 \frac{1}{\sqrt{(1 + z_2)(1 - z_3)(z_2 - z_3)}}$$

$$= 4\sqrt{2} \ln 2, \tag{C5}$$

$$I_1 = \int_{-1}^{1} z_2 \, \mathrm{d}z_2 \int_{-1}^{z_2} \mathrm{d}z_3 \, \frac{1}{\sqrt{(1+z_2)(1-z_3)(z_2-z_3)}}$$

$$= \frac{4}{3}\sqrt{2}\,(1 - \ln 2). \tag{C6}$$

the results are

$$\int \frac{\mathrm{d}\Omega_2}{4\pi} \int \frac{\mathrm{d}\Omega_3}{4\pi} \left(\frac{P}{\epsilon_3 + \epsilon_4 - \epsilon_1 - \epsilon_2} + (1 \leftrightarrow 3) + (2 \leftrightarrow 3) \right)$$

$$= \frac{M}{p_F^2} \frac{3\sqrt{2}}{8} I_0, \tag{C7}$$

$$3 \int \frac{\mathrm{d}\Omega_2}{4\pi} \int \frac{\mathrm{d}\Omega_3}{4\pi} \left(\hat{\boldsymbol{k}}_1 \cdot \hat{\boldsymbol{k}}_2 \right)$$

$$\times \left(\frac{P}{\epsilon_3 + \epsilon_4 - \epsilon_1 - \epsilon_2} + (1 \leftrightarrow 3) + (2 \leftrightarrow 3) \right)$$

$$= -\frac{M}{p_F^2} \frac{3\sqrt{2}}{8} I_1, \tag{C8}$$

The remaining two terms in Eq. (37) can be obtained from the above integrals by a redefinition of integration variables, and

which gives Eqs. (37) and (38) of the main text.

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