Transverse momentum dependent jet model for quark fragmentation functions†

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Fragmentation functions (FFs) describe the semi-inclusive production of hadrons in deep inelastic scattering (DIS) of leptons on nuclear targets[1)]. The leading order process for the case of pion production is represented by Fig. 1, which shows a high energy virtual quark with momentum k and polarization s after the interaction with the lepton, fragmenting into the observed pion with momentum p and a spectator state, which includes a quark and, in general, also unobserved hadrons. The FF describing this process has the form

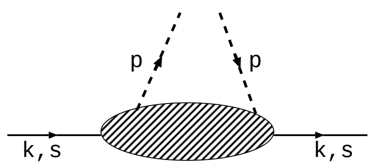

Fig. 1. Cut diagram representing the process where a quark with momentum k and polarization s fragments into a pion with momentum p. The shaded oval represents the spectator states, and the cut goes through the shaded oval.

$$F^{(q\to\pi)}(z,\mathbf{p}_\perp;\mathbf{s}) = D^{(q\to\pi)}(z,\mathbf{p}_\perp^2) + \frac{1}{m_\pi z}\left(\mathbf{p}_\perp \times \mathbf{s}\right)^3 H^{\perp(q\to\pi)}(z,\mathbf{p}_\perp^2)\,, \qquad (1)$$

where the direction of the 3-momentum of the fragmenting quark is assumed along the z axis. The produced pion has a fraction z of the initial quark's longitudinal momentum and transverse momentum $\mathbf{p}_\perp$. The two functions in (1) are the unpolarized FF $D^{(q\to\pi)}$ and the so-called Collins function $H^{\perp(q\to\pi)}$. The observed single-spin asymmetries in semi-inclusive pion production in DIS of polarized electrons on unpolarized protons[2)] have shown that the Collins function is non-zero, while more precise information has not yet been obtained.

The two FFs of (1) are subject to important sum rules. If we consider only the case of inclusive pion production and quark flavor $SU(2)$ for simplicity, the sum rules for the longitudinal and transverse momentum, and the isospin sum rule are expressed as follows:

$$\sum_{\tau_\pi}\int_0^1 \mathrm{d}z\, z \int \mathrm{d}^2 p_\perp D^{(q\to\pi)}(z,\mathbf{p}_\perp^2) = 1\,, \qquad (2)$$

$$\sum_{\tau_\pi}\int_0^1 \frac{\mathrm{d}z}{2zm_\pi}\int \mathrm{d}^2 p_\perp \mathbf{p}_\perp^2 H^{\perp(q\to\pi)}(z,\mathbf{p}_\perp^2) = 0\,, \qquad (3)$$

$$\sum_{\tau_\pi}\tau_\pi\int_0^1 \mathrm{d}z \int \mathrm{d}^2 p_\perp D^{(q\to\pi)}(z,\mathbf{p}_\perp^2) = \frac{\tau_q}{2}\,. \qquad (4)$$

Here the isospin labels for the pions and quarks are $\tau_\pi = (1,0,-1)$ for (π^+,π^0,π^-), and $\tau_q = (1,-1)$ for (u,d).

In this work we extend our previous quark-jet model description[3)] of the unpolarized FF to the polarized case. In order to account for multi-fragmentation processes, we make a product ansatz for $F^{(q\to\pi)}$ in terms of the functions describing the elementary fragmentation processes. The proper treatment of the spin of the quark in the intermediate states requires several more elementary FFs, in addition to the elementary counterparts of the two functions in (1). We have worked out the coupled integral equations for the two FFs in Eq.(1), which can readily be used for numerical calculations. An important result of our investigation is that the sum rules (2) - (4) are satisfied automatically in this transverse momentum dependent jet-model.

Important tasks for future investigations are to obtain numerical solutions of the integral equations derived in this work, as well as the extension to include additional hadron production channels.

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References

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