On the role of three-particle interactions in nuclear matter†

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In a previous publication[1)] we discussed an interesting relation between the skewness J of nuclear matter ($J = 27\rho^3 \left(\mathrm{d}^3 E_A/\mathrm{d}\rho^3\right)$, where ρ is the baryon density and E_A the energy per nucleon in isospin symmetric nuclear matter) and the isoscalar three-particle interaction parameters. In this paper, we wish to discuss an equally interesting relation between the slope parameter L of the symmetry energy ($L = 3\rho\frac{\mathrm{d}a_s}{\mathrm{d}\rho}$, where $a_s \simeq 32$ MeV is the symmetry energy) and the isovector three-particle interaction parameters.

We extend Landau's basic formula[2)] for the variation of the energy density of nuclear matter to include the third order term, which involves the spin-averaged three-particle forward scattering amplitude $h^{(\tau_1\tau_2\tau_3)}(\vec{k}_1, \vec{k}_2, \vec{k}_3)$. Here $\tau_i = (p, n)$, and h is symmetric under simultaneous interchanges of the momentum variables $\vec{k}_i$ and the isospin variables τ_i. Taking finally the isospin symmetric limit, we can derive the following exact relations for J and L in terms of the incompressibility K and the symmetry energy a_s:

$$J = -9K + \frac{9p_F^2}{M} \times \left[\left(-3 + \frac{8}{3}\frac{M}{M^*}\right) - \frac{4}{3}\frac{Mp_F}{M^{*2}}\frac{\partial M^*}{\partial p_F} + \frac{M}{M^*}\left(H_0 - H_1\right)\right],$$

$$L = 3a_s - \frac{p_F^2}{2M} \times \left[\left(1 - \frac{2}{3}\frac{M}{M^*}\right) + \mu\left(\frac{M}{M^*}\right)^2 - \frac{M}{M^*}\left(H_0' - \frac{1}{3}H_1\right)\right].$$

Here p_F is the Fermi momentum, M the free nucleon mass, M^* the Landau effective mass, $\frac{\partial M^*}{\partial p_F}$ refers to the momentum dependence of M^* at the Fermi surface, and $\mu = \rho\frac{\partial}{\partial \rho^{(3)}}\left(\frac{\Delta M^*}{M}\right)$ expresses the dependence of $\Delta M^* = M^{*(p)} - M^{*(n)}$ on the isovector density $\rho^{(3)} = \rho^{(p)} - \rho^{(n)}$. The dimensionless isoscalar and isovector three-particle interaction parameters

$$H_\ell = \left(\frac{2p_F M^*}{\pi^2}\rho\right) h_\ell\,, \quad H_\ell' = \left(\frac{2p_F M^*}{\pi^2}\rho\right) h_\ell'$$

are the $\ell = 0, 1$ moments of the isoscalar ($h_\ell = \frac{1}{4}\left(h_\ell^{(ppp)} + 3h_\ell^{(ppn)}\right)$) and isovector ($h_\ell' = \frac{1}{4}\left(h_\ell^{(ppp)} - h_\ell^{(ppn)}\right)$) combinations of the 3-particle forward scattering amplitude at the Fermi surface.

By using empirical information, it was shown in Ref.[1)] that the above expression for J requires a large positive 3-particle term $\frac{M}{M^*}\left(H_0 - H_1\right) > 1.24$. On the other hand, if we use the canonical value $a_s = 32$ MeV together with $\mu \simeq 0.27$, which is the central value of the empirical range $\mu = 0.27 \pm 0.25$ reported in Ref.[3)], the sum of the first two terms in $[\ldots]$ in the expression for L is ~ 0.6, almost independent of M^* within the empirical range $0.7 < M^*/M < 1$. The empirical range of the slope parameter[3)] $L = 59 \pm 16$ MeV then implies that the 3-particle term $\frac{M}{M^*}\left(H_0' - \frac{1}{3}H_1\right)$ is negative, with a magnitude smaller than unity.

Theoretically the three-particle amplitudes should be calculated from the Faddeev equation, which is illustrated by Fig. 1. The driving term, which we call

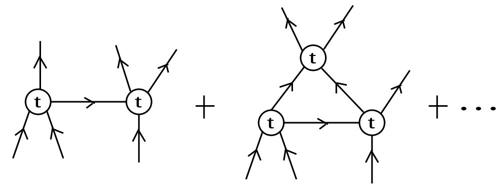

Fig. 1. First two terms in the Faddeev series. Circles represent two-body t-matrices.

the "2-particle correlation (2pc) term", can be easily estimated by using effective contact interactions of the Landau-Migdal type. This gives the analytic results

$$H_0^{(2\mathrm{pc})} = \frac{\ln 2}{4}\left(F_0^2 + 3F_0'^2 + 3G_0^2 + 9G_0'^2\right),$$

$$H_0'^{(2\mathrm{pc})} = \frac{\ln 2}{4}\left(\frac{1}{3}F_0^2 + \frac{4}{3}F_0F_0' - \frac{1}{3}F_0'^2 + G_0^2 + 4G_0G_0' - G_0'^2\right).$$

While the isoscalar $H_0^{(2\mathrm{pc})}$ is positive definite and of the order of unity or even larger, depending mainly on the magnitude of G_0', the isovector $H_0'^{(2\mathrm{pc})}$ is negative and small compared to unity for most of the published sets of Landau-Migdal parameters. Because the p-wave term H_1 is suppressed by large factors[1)], this simple estimate makes it plausible that the three-body interactions give a large positive contribution to J, and a small negative contribution to L. To obtain more quantitative results, it would be interesting to apply the Faddeev method in the framework of effective field theories for nuclear matter.

References
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