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Baryons in the NJL model as solutions of the relativistic Faddeev equation

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Abstract

The relativistic Faddeev equation for three quarks is solved in the $SU(2)_f \times SU(3)_c$ NJL model to get the nucleon and delta states. We truncate the interacting two-body channels to the scalar and axial vector diquark channels, which are expected to be dominant from the non-relativistic analogy. We find that both channels contribute attractively to the nucleon as well as the delta state, and that the principal mechanism for the mass spitting between the nucleon and the delta in this picture is the interaction in the scalar diquark channel, which is not present in the delta state. We show the dependences of the masses of the nucleon and the delta on the specific form of the interaction lagrangian, and derive restrictions on the possible forms of four-fermi interactions.

1. Introduction

The NJL model was first presented as an application of the mechanism of superconductivity to particle physics, where the pion was described as a Goldstone boson of the spontaneously broken chiral symmetry with the nucleon as the elementary fermion [1]. Afterwards it was thought of as an effective theory of low-energy QCD, based on quark degrees of freedom. It exhibits in a mean-field approximation the spontaneous breaking of chiral symmetry, which is one of the most important properties of low-energy QCD, in a particularly clear manner, and allows a simple description of mesons [2] as $q\bar{q}$ bound states in the ladder approximation to the Bethe–Salpeter equation. Another important property of QCD in the low-energy region is the confinement, which is not respected in the NJL model. Many efforts have been (and are still being) made to derive this model from QCD [3], and to extend it so as to include the effects of the confinement [4].

The investigations so far concentrated mainly on mesons, but recently also the structure of baryons has been studied in the NJL model, making use of the mean-field

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approximation [5,6] and the diquark-quark approximation [7]. It is now well known that in the mean-field approximation, there exists no stable solution corresponding to the nucleon state [5,6] but one has to introduce interactions of higher order than the usual four-fermion one [5,8] to obtain a stable solution which qualitatively describes the nucleon properties. The validity of the mean-field description of few-quark systems, such as the nucleon, is, however, not obvious. In the quark-diquark approximation [7], on the other hand, one assumes the same spin-flavor structure of the baryon wave function as in the non-relativistic quark model to add a third quark to the diquark, where the latter is a qq bound state solution of the Bethe-Salpeter equation. That is, the interaction between the quark and the diquark is neglected.

In order to go beyond these pictures and to include explicitly the correlations between the quarks in a relativistically covariant way which is consistent with the description of mesons, we have to solve the relativistic Faddeev equation for the three quarks [9-13]. In the NJL model, because the elementary interaction between quarks is of zero range and thus separable, we are really able to solve the relativistic Faddeev equation including the negative-energy intermediate states in the ladder approximation. Using the helicity formalism [14] it is also possible to perform the spin and parity projection of the Faddeev kernel fully relativistically. For a given form of the four-fermi interaction lagrangian, the number of interacting two-body channels is finite and therefore it is in principle possible to treat the three-body problem without any approximations (except for the ladder approximation). In this work, however, we truncate the interacting two-body diquark channels for simplicity to the scalar $(0^+, T = 0)$ and axial vector $(1^+, T = 1)$ ones which are expected to be dominant from the non-relativistic analogy. We then solve the relativistic Faddeev equation for the nucleon and the delta states in the flavor SU(2) NJL model without any further approximations, concentrating here on the resulting eigenvalues (masses). Instead of assuming a specific form of interaction lagrangian, we treat the coupling constants in the scalar and the axial vector qq channels as parameters. In this way we investigate the dependence of the baryon masses on the form of the interaction lagrangian, in order to constrain the form of the possible effective quark-quark interactions of NJL type suitable for hadron physics. Using our numerical solutions of the Faddeev equation we will discuss possible forms of the four-fermi interaction which can reproduce the pion, nucleon and delta masses. Note that due to the lack of confinement we must describe the delta particle as a bound state, since a resonant state would decay into a quark and a diquark or three quarks instead of the "physical" decay into a nucleon and a pion. A similar difficulty is encountered in the description of heavier mesons like vector mesons or the η' [15,5]. Recent works [4] have shown that to some extent it is possible to mimic confinement by including momentum-dependent quark-quark interactions. It then, however, becomes very difficult to solve the relativistic Faddeev equations exactly. Since the main purpose of our present work is to find exact solutions of the Faddeev equations in the NJL model, we leave the phenomenological incorporation of confinement as a subject for future research.

The results for the nucleon mass in the present Faddeev approach have already been published in two letters [11,12]. The present paper intends to provide the formal details of these calculations, to extend them to describe the delta and in particular the origin of the delta–nucleon mass difference in this model, and to use these numerical results to derive restrictions on the possible forms of chiral-invariant four-fermi interactions. In Section 2 we present the interaction lagrangians, introduce the effective couplings and solve the two-body Bethe–Salpeter equation in the scalar and axial vector diquark channels in the ladder approximation. In order to be self-contained, we derive in Section 3 the relativistic Faddeev equation and give its detailed form for the case of the NJL model including the scalar and axial vector diquark channels. In Section 4 we project the relativistic Faddeev equation on physical baryon states. In particular, we discuss the spin and parity projection in some detail. In Section 5 we explain our numerical methods, and in Section 6 we present our results for the nucleon and delta masses and use them to discriminate between various forms of the interaction lagrangian. We find, for example, that in the original NJL lagrangian the qq interaction is too weak to form a bound nucleon state if the $q\bar{q}$ interaction is constrained by the pion mass. However, using the color current type interaction lagrangian [7,10], which has some foundation as an effective lagrangian of low-energy QCD, we get a realistic nucleon state whose mass is about 900 MeV, but the delta is unbound. We investigate the values of the coupling constants needed to obtain also the delta as a bound state of about 1200 MeV, and give the most simple possible form of the corresponding chiral-invariant interaction lagrangian which is able to reproduce the pion, nucleon and delta masses simultaneously. In Section 7 we summarize our results. Technical details of the calculations are comprised in four appendices.

2. Lagrangians and two-body T-matrices

2.1. Lagrangians

We consider the following $SU(2)_f \times SU(3)_c$ symmetric quark lagrangian:

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_I, \tag{2.1}$$

where $\mathcal{L}_0 = \bar{\psi}(i\partial\!\!\!/ - m)\psi$ is the free quark lagrangian with m the current u-, d-quark mass, and $\mathcal{L}_I$ is a chirally symmetric four-fermion interaction lagrangian of the NJL type. Examples are the original NJL form [1]

$$\mathcal{L}_I = g\left(\left(\bar{\psi}\psi\right)^2 - \left(\bar{\psi}(\gamma_5\boldsymbol{\tau})\psi\right)^2\right), \tag{2.2}$$

or the color current interaction lagrangian used in some recent works [7,10]:

$$\mathcal{L}_I = -g\sum_{c=1}^{8}\left(\bar{\psi}(\gamma_\mu\tfrac{1}{2}\lambda_c)\psi\right)^2, \tag{2.3}$$

where λ_c are the $SU(3)_c$ generators with the normalization $\mathrm{tr}(\lambda_c\lambda_d) = 2\delta_{cd}$. As we mentioned already, instead of choosing a specific form of $\mathcal{L}_I$, we wish to study how the results depend on it, so as to impose useful restrictions on the possible forms of $\mathcal{L}_I$. For this purpose we note that using Fierz rearrangement one can rewrite any given $\mathcal{L}_I$ into a form where the interaction strength in a particular channel can be read off directly [1,7]. (This procedure is explained in Appendix A.) For the $\bar{q}q$ channels this means simply to rewrite $\mathcal{L}_I$ into the Fierz symmetric form $\mathcal{L}_{I,q\bar{q}} = \frac{1}{2}(\mathcal{L}_I + \mathcal{L}_{I,F})$, where $\mathcal{L}_{I,F}$ is the Fierz rearranged form of $\mathcal{L}_I$. (With this lagrangian, exchange terms simply give a factor 2.) In the $(0^+, T = 0)$ and $(0^-, T = 1)$ color singlet $q\bar{q}$ channels we parametrize the relevant piece of $\mathcal{L}_I$ as

$$\mathcal{L}_{I,\pi} = \frac{1}{2}g_\pi \left((\overline{\psi}\psi)^2 - (\overline{\psi}(\gamma_5\boldsymbol{\tau})\psi)^2\right). \tag{2.4}$$

The interaction strength g_π is related to the coupling constants of the original $\mathcal{L}_I$. For example, in the case of (2.2) $g_\pi = \frac{13}{12}g$, and in the case of (2.3) $g_\pi = \frac{2}{9}g$. ($g_\pi > 0$ means attraction in the pionic $q\bar{q}$ channel.) For the qq channels we rewrite $\mathcal{L}_I$ into an equivalent form which consists of terms of the form $(\overline{\psi}A\overline{\psi}^{\mathrm{T}})(\psi^{\mathrm{T}}B\psi)$, where A, B are matrices totally antisymmetric in Dirac, isospin and color indices. Here we are interested only in the color $\bar{3}$ channels, since the color 6 channels do not contribute to the colorless three-quark state. In the scalar $(0^+, T = 0)$ and axial vector $(1^+, T = 1)$ color $\bar{3}\, qq$ channels we parametrize the relevant pieces of $\mathcal{L}_I$ as [1]

$$\mathcal{L}_{I,s} = g_s \left(\overline{\psi}(\gamma_5 C)\tau_2\beta^A\overline{\psi}^{\mathrm{T}}\right)\left(\psi^{\mathrm{T}}(C^{-1}\gamma_5)\tau_2\beta^A\psi\right) \tag{2.5}$$

$$\mathcal{L}_{I,a} = g_a \left(\overline{\psi}(\gamma_\mu C)(\tau_i\tau_2)\beta^A\overline{\psi}\right)\left(\psi^{\mathrm{T}}(C^{-1}\gamma^\mu)(\tau_2\tau_i)\beta^A\psi\right), \tag{2.6}$$

where $\beta^A = \sqrt{\frac{3}{2}}\lambda^A$ $(A = 2, 5, 7)$ projects on the color $\bar{3}$ channel, $C = i\gamma_2\gamma_0$ is the charge conjugation matrix, and $i = 1, 2, 3$. ($g_s > 0$ ($g_a > 0$) means attraction in the scalar (axial vector) qq channel.) Introducing the ratios [2]

$$r_s = \frac{g_s}{g_\pi}, \qquad r_a = \frac{g_a}{g_\pi}, \tag{2.7}$$

we have for the lagrangian (2.2) $r_s = \frac{2}{13}$, $r_a = \frac{1}{13}$, and for the lagrangian (2.2) $r_s = \frac{1}{2}$, $r_a = \frac{1}{4}$. We see that if g_π is fixed by the pion mass the interactions in the 0^+ and 1^+ qq channels are much stronger for the lagrangian (2.3) than for the lagrangian (2.2).

In Table 1 we list g_π, g_s and g_a for some chirally symmetric interaction lagrangians used in the literature. (The lagrangian (2.2) corresponds to the first case in Table 1 and (2.3) to the fourth case with an overall minus sign.) Of course, one can also consider any linear combination of these lagrangians, and we will discuss this possibility in Section 6. The chosen lagrangian should have a positive g_π in order that the interaction in the pionic channel is attractive, and also a positive g_s since the attraction in the scalar qq channel is most important for the binding of the nucleon as we will discuss later.

[1] Due to chiral symmetry there is also an interaction term in the 1^- qq channel which has the form (2.6) with $\gamma_\mu \to \gamma_\mu\gamma_5$. The major component of this channel, however, corresponds to $l = 1$ and is therefore not considered in this paper.

[2] Note that this definition of r_a differs from the one used in Ref. [12].

Table 1
The left row refers to various interactions lagrangians, where $(\Lambda_1, \Lambda_2) \equiv (\bar{\psi}\Lambda_1\psi)(\bar{\psi}\Lambda_1\psi)$ and g the four-fermi coupling constant. The other three rows show the interaction strengths in the pionic, scalar diquark and axial vector diquark channels (see Eqs. (2.4), (2.5) and (2.6))

Lagrangian	g_π/g	g_s/g	g_a/g
$g\left[(1,1) - (\gamma_5\tau_i, \gamma_5\tau_i)\right]$	$\frac{13}{12}$	$\frac{1}{6}$	$\frac{1}{12}$
$g\left[\left(\frac{1}{2}\lambda_c, \frac{1}{2}\lambda_c\right) - \left(\frac{1}{2}\lambda_c\gamma_5\tau_i, \frac{1}{2}\lambda_c\gamma_5\tau_i\right)\right]$	$\frac{1}{9}$	$-\frac{1}{9}$	$-\frac{1}{18}$
$g\left(\gamma_\mu, \gamma^\mu\right)$	$-\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{12}$
$g\left(\frac{1}{2}\lambda_c\gamma_\mu, \frac{1}{2}\lambda_c\gamma^\mu\right)$	$-\frac{2}{9}$	$-\frac{1}{9}$	$-\frac{1}{18}$
$g\left(\gamma_\mu\gamma_5, \gamma^\mu\gamma_5\right)$	$\frac{1}{6}$	$\frac{1}{6}$	$-\frac{1}{12}$
$g\left(\frac{1}{2}\lambda_c\gamma_\mu\gamma_5, \frac{1}{2}\lambda_c\gamma^\mu\gamma_5\right)$	$\frac{2}{9}$	$-\frac{1}{9}$	$\frac{1}{18}$

2.2. Gap equation and two-body T-matrices

We rewrite the lagrangian (2.1) in the form

$$\mathcal{L} = \bar{\psi}(i\partial\!\!\!/ - M)\psi + \left(\mathcal{L}_\mathrm{I} + \delta M\bar{\psi}\psi\right), \tag{2.8}$$

with $\delta M = M - m$. Requiring that the interaction part $\left(\mathcal{L}_\mathrm{I} + \delta M\bar{\psi}\psi\right)$ in (2.8) does not produce further corrections to the quark mass we obtain the gap equation

$$M = m + 2ig_\pi \int \frac{\mathrm{d}^4k}{(2\pi)^4}\, \mathrm{tr}(S_\mathrm{F}(k)). \tag{2.9}$$

M is the constituent quark mass, and

$$S_\mathrm{F}(k) = \frac{1}{k\!\!\!/ - M + i\epsilon}$$

is the Feynman propagator for the constituent quark. The integration in (2.9) is divergent, so we have to regularize it. In this paper we will use a sharp euclidean cut-off Λ after performing the Wick rotation for all divergent integrals like (2.9).

For a momentum-independent interaction kernel K the Bethe–Salpeter (BS) equation in the $\bar{q}q$ channel reads

$$\bar{t}_{\alpha\beta,\gamma\delta}(k) = K_{\alpha\beta,\gamma\delta} + \int \frac{\mathrm{d}^4q}{(2\pi)^4} K_{\alpha\beta,\lambda\varepsilon} S_{\mathrm{F},\varepsilon\varepsilon'}(k+q) S_{\mathrm{F},\lambda'\lambda}(q) \bar{t}_{\varepsilon'\lambda',\gamma\delta}(k), \tag{2.10}$$

where k is the total momentum of the two-body system and the indices refer to Dirac, isospin and color. In the case of the pionic channel we have from (2.4)

$$K_{\alpha\beta,\gamma\delta} = -2ig_\pi(\gamma_5\tau_i)_{\alpha\beta}(\gamma_5\tau_i)_{\gamma\delta}, \tag{2.11}$$

and the solution to (2.10) is

$$\bar{t}_\pi(k)_{\alpha\beta,\gamma\delta} = (\gamma_5\tau_i)_{\alpha\beta}\bar{\tau}_\pi(k)(\gamma_5\tau_i)_{\gamma\delta} \tag{2.12}$$

with

$$\bar{\tau}_\pi(k) = \frac{-2ig_\pi}{1+2g_\pi \Pi_\pi(k^2)}, \tag{2.13}$$

where

$$\Pi_\pi(k^2)\delta_{ii'} = i\int \frac{d^4q}{(2\pi)^4} \operatorname{tr}\left[\gamma_5\tau_i S_F(q)\gamma_5\tau_{i'} S_F(k+q)\right]$$
$$= 6i\delta_{ii'}\int \frac{d^4q}{(2\pi)^4} \operatorname{tr_D}\left[\gamma_5 S_F(q)\gamma_5 S_F(k+q)\right]. \tag{2.14}$$

The pion mass is obtained as the pole of (2.13). Due to the gap equation (2.9), $m_\pi = 0$ if $m = 0$, which corresponds to the Goldstone pole.

For interaction lagrangians like (2.5) and (2.6), the BS equation in the qq channel reads

$$\bar{t}_{\alpha\beta,\gamma\delta}(k) = K_{\alpha\beta,\gamma\delta} + \frac{1}{2}\int \frac{d^4q}{(2\pi)^4} K_{\alpha\beta,\lambda\varepsilon} S_{F,\lambda\lambda'}(k+q) S_{F,\varepsilon\varepsilon'}(-q)\bar{t}_{\lambda'\varepsilon',\gamma\delta}(k), \tag{2.15}$$

where the factor $\frac{1}{2}$ is a symmetry factor due to the two indistinguishable particles in the intermediate state. (It appears since we include a factor 2 due to the exchange term in every factor K in the ladder diagram.) For the scalar channel we have from (2.5)

$$K_{\alpha\beta,\gamma\delta} = 4ig_s\left(\gamma_5 C\tau_2\beta^A\right)_{\alpha\beta}\left(C^{-1}\gamma_5\tau_2\beta^A\right)_{\gamma\delta}, \tag{2.16}$$

and the solution to (2.15) is

$$\bar{t}_s(k)_{\alpha\beta,\gamma\delta} = \left(\gamma_5 C\tau_2\beta^A\right)_{\alpha\beta}\bar{\tau}_s(k)\left(C^{-1}\gamma_5\tau_2\beta^A\right)_{\gamma\delta} \tag{2.17}$$

with

$$\bar{\tau}_s(k) = 2\times\frac{2ig_s}{1+2g_s\Pi_s(k^2)}, \tag{2.18}$$

where

$$\Pi_s(k^2)\delta_{A'A} = i\int \frac{d^4q}{(2\pi)^4} \operatorname{tr}\left[(\gamma_5 C)\tau_2\beta^A S_F(-q)^T(C^{-1}\gamma_5)\tau_2\beta^{A'} S_F(k+q)\right]$$
$$= 6i\delta_{A'A}\int \frac{d^4q}{(2\pi)^4} \operatorname{tr_D}\left[\gamma_5 S_F(q)\gamma_5 S_F(k+q)\right]. \tag{2.19}$$

Here we used the relation $CS_F(-q)^T C^{-1} = S_F(q)$. From (2.14) and (2.19) we see that $\Pi_s = \Pi_\pi$, and therefore if r_s of Eq. (2.7) equals one the pion and the scalar diquark are degenerate: $m_s = m_\pi$ if $r_s = 1$. (Here m_s is the mass of the scalar diquark.)

For the axial vector channel we have from Eq. (2.6)

$$K_{\alpha\beta,\gamma\delta} = 4ig_a\left(\gamma_\mu C\tau_i\tau_2\beta^A\right)_{\alpha\beta}\left(C^{-1}\gamma^\mu\tau_2\tau_i\beta^A\right)_{\gamma\delta}, \tag{2.20}$$

and the solution to (2.15) is

$$\bar{t}_a(k)_{\alpha\beta,\gamma\delta} = \left(\gamma_\mu C\tau_i\tau_2\beta^A\right)_{\alpha\beta}\bar{\tau}_a^{\mu\nu}(k)\left(C^{-1}\gamma_\nu\tau_2\tau_i\beta^A\right)_{\gamma\delta}, \tag{2.21}$$

with

$$\tau_{\rm a}^{\mu\nu}(k) = 2 \times 2ig_{\rm a}\left[g^{\mu\nu} - \frac{2g_{\rm a}\Pi_{\rm a}(k^2)}{1+2g_{\rm a}\Pi_{\rm a}(k^2)}\left(g^{\mu\nu} - \frac{k^\mu k^\nu}{k^2}\right)\right], \tag{2.22}$$

where

$$\begin{aligned}
&\Pi_{\rm a}(k^2)\left(g^{\mu\nu} - \frac{k^\mu k^\nu}{k^2}\right)\delta_{A'A}\,\delta_{i'i} \\
&= i\int\frac{d^4q}{(2\pi)^4}\,{\rm tr}\left[(\gamma^\mu C)(\tau_i\tau_2)\beta^A S_{\rm F}(-q)^{\rm T}(C^{-1}\gamma^\nu)(\tau_2\tau_{i'})\beta^{A'}S_{\rm F}(k+q)\right] \\
&= 6i\delta_{A'A}\delta_{i'i}\int\frac{d^4q}{(2\pi)^4}\,{\rm tr_D}\left[\gamma^\mu S_{\rm F}(q)\gamma^\nu S_{\rm F}(k+q)\right].
\end{aligned} \tag{2.23}$$

The bubble graph $\Pi_{\rm a}$ of Eq. (2.23) is the same as the one involved in the 1^- $\bar{q}q$ channel.

3. Relativistic three-body theory

3.1. General formalism

In this section we review the derivation of the relativistic three-body equations [16–18] (for an introduction to the three-body problem, see Ref. [19]). The ultimate aim is to solve the Dyson equation for the three-body propagator

$$G = G_0 + G_0KG, \tag{3.1}$$

where G_0 is a product of three single-particle propagators and K is the interaction kernel. If we assume only two-body interactions we can write $K = K_1 + K_2 + K_3$, where K_i means that particles (jk) interact and i is the spectator. (Throughout this section, (ijk) is an even permutation of (123).) In terms of the two-body propagator g_i in the three-body Hilbert space which satisfies $g_i = G_0 + G_0K_ig_i$, Eq. (3.1) becomes

$$G = g_i + g_i(K_j + K_k)G. \tag{3.2}$$

We further introduce the two-body T-matrix in the three-body Hilbert space $\tilde{t}_i$ by $g_i = (1 + G_0\tilde{t}_i)G_0$ as well as the Faddeev decomposition $G = G_0 + G_1 + G_2 + G_3$ where $G_i = G_0K_iG$ describes processes where the pair i interacts last (see Eq. (3.1)). Use of these definitions in (3.2) gives

$$G_i = G_0\tilde{t}_iG_0 + G_0\tilde{t}_i(G_j + G_k). \tag{3.3}$$

In terms of the three-body T-matrix $T = \sum_i T_i$ with $G_i = G_0T_iG_0$, one obtains from (3.3) the well-known Faddeev equations

$$T_i = \tilde{t}_i + \tilde{t}_iG_0(T_j + T_k), \tag{3.4}$$

$$T = \sum_i T_i. \tag{3.5}$$

The quantities T_i contain also three-body reducible processes. It is often convenient to separate these from the irreducible ones. For this one introduces quantities Y_{ij} by

$$T_i = \tilde{t}_i + \sum_m \tilde{t}_i Y_{im} \tilde{t}_m. \tag{3.6}$$

The quantities $\tilde{t}_i Y_{ij} \tilde{t}_j$ describe three-body irreducible processes where pair j interacts first and pair i last. It is easy to see that the Y_{ij} must satisfy the equation

$$Y_{ij} = \bar{\delta}_{ij} G_0 + \sum_l \bar{\delta}_{jl} G_0 \tilde{t}_l Y_{li}, \tag{3.7}$$

with $\bar{\delta}_{ij} = (1 - \delta_{ij})$. To see this, we note that if (3.7) is satisfied the second terms on the r.h.s. of (3.6) and (3.4) are identical,

$$\begin{aligned}\sum_m Y_{im}\tilde{t}_m &= G_0 \left[(\tilde{t}_j + \tilde{t}_k) + \sum_{ml} \bar{\delta}_{il}\, \tilde{t}_l Y_{lm} \tilde{t}_m \right] \\ &= G_0 \left[\tilde{t}_j + \tilde{t}_k + \sum_m \tilde{t}_j Y_{jm} \tilde{t}_m + \sum_m \tilde{t}_k Y_{km} \tilde{t}_m \right] \\ &= G_0 (T_j + T_k),\end{aligned} \tag{3.8}$$

i.e., Eq. (3.4) is satisfied. The usual two-body T-matrix in the two-body space t is related to $\tilde{t}$ by $\tilde{t}_l = t_l S_{\mathrm{F}l}^{-1}$, where $S_{\mathrm{F}l}$ is the Feynman propagator of particle l. In terms of the quantities $X_{ji} = S_{\mathrm{F}j}^{-1} Y_{ji} S_{\mathrm{F}i}^{-1}$ Eqs. (3.6) and (3.7) become

$$X_{ji} = \bar{\delta}_{ijk}\, S_{\mathrm{F}k} + \sum_{ll'} \bar{\delta}_{jll'}\, S_{\mathrm{F}l} S_{\mathrm{F}l'} t_l X_{li}, \tag{3.9}$$

$$T_i = t_i S_{\mathrm{F}i}^{-1} + \sum_m t_i X_{im} t_m, \tag{3.10}$$

$$T = \sum_i T_i. \tag{3.11}$$

In Eq. (3.9), $\bar{\delta}_{ijk} = 1$ if $i \neq j \neq k$, and 0 otherwise.

3.2. Faddeev equation in the NJL model

In the case of a separable interaction like in the NJL model, it is possible to reduce Eq. (3.9) to an effective two-body equation describing the scattering of a particle on a pair of particles (quasi-particle) [19]. We wish to derive this equation in momentum space. For three particles with momenta (k_1, k_2, k_3) we define the Jacobi variables (P, q_i, p_i) as follows:

$$k_i = \tfrac{1}{2}P + p_i, \qquad k_j = \tfrac{1}{4}P + q_i - \tfrac{1}{2}p_i, \qquad k_k = \tfrac{1}{4}P - q_i - \tfrac{1}{2}p_i. \tag{3.12}$$

The inverse relations are

$$q_i = \tfrac{1}{2}(k_j - k_k), \qquad p_i = \tfrac{1}{2}(k_i - (k_j + k_k)), \qquad P = k_i + k_j + k_k. \tag{3.13}$$

(Here the indices do *not* refer to the particle number, but simply characterize the three momenta.) There are linear relationships between the three possible sets of variables (q_i, p_i):

$$q_j = -\tfrac{1}{2}q_i \mp \tfrac{3}{4}p_i \mp \tfrac{1}{8}P, \qquad p_j = \pm q_i - \tfrac{1}{2}p_i - \tfrac{1}{4}P, \tag{3.14}$$

where the upper signs hold if $(ij) = (12)$, (23) or (31), and the lower signs otherwise. We further introduce basis states [20] $|qp\rangle_i^{\alpha_1\alpha_2\alpha_3}$ where q denotes the relative momentum between the pair (jk) and p between particle i and the pair (jk), and $\alpha_1, \alpha_2, \alpha_3$ characterize the Dirac, isospin and color indices of particles i, j, k, respectively. Here (q,p) denotes any of the three sets (q_i, p_i). For example, $|q_1p_1\rangle_1^{\alpha_1\alpha_2\alpha_3}$ denotes a state in which particle 1 has momentum k_1 and index α_1, particle 2 has (k_2, α_2) and particle 3 has (k_3, α_3), etc. (This identification follows from Eq. (3.13).) There are relationships like

$$\begin{aligned} |q_1p_1\rangle_1^{\alpha_1\alpha_2\alpha_3} &= |q_2p_2\rangle_2^{\alpha_2\alpha_3\alpha_1} = |q_3p_3\rangle_3^{\alpha_3\alpha_1\alpha_2}, \\ |q_1p_1\rangle_2^{\alpha_1\alpha_2\alpha_3} &= |q_2p_2\rangle_3^{\alpha_2\alpha_1\alpha_3} = |q_3p_3\rangle_1^{\alpha_3\alpha_1\alpha_2}, \end{aligned} \tag{3.15}$$

etc. In this representation a separable T-matrix is expressed as

$$\begin{aligned} {}^{\alpha_i'\alpha_j'\alpha_k'}_{\;i}\langle q_i'p_i'|t_i|q_ip_i\rangle_i^{\alpha_i\alpha_j\alpha_k} &= (2\pi)^4\delta^{(4)}(p_i' - p_i) \\ &\quad \times \delta_{\alpha_i'\alpha_i}\, \Omega_a^{\alpha_j'\alpha_k'}(q_i')\overline{\Omega}_b^{\alpha_j\alpha_k}(q_i)\tau_{ab}^i(\tfrac{1}{2}P - p_i), \end{aligned} \tag{3.16}$$

where Ω and $\overline{\Omega}$ are the two-body vertex function and its conjugate, and a sum over the two-body channels a, b is implied. As will become clear later (see the discussion below Eq. (B.12)), only those parts of the vertex functions in Eq. (3.16) which are antisymmetric with respect to interchange of particles j and k contribute to the three-body T-matrix between antisymmetrized three-body states. We will therefore assume that the vertex functions satisfy

$$\Omega^{\alpha_k\alpha_j}(-q_i) = -\Omega^{\alpha_j\alpha_k}(q_i). \tag{3.17}$$

For the formal developments it is convenient to have an operator representation of t_i besides the matrix representation (3.16). For this we introduce the following state [19]:

$$|p_i, a\rangle_i^{\alpha_i} \equiv \int \frac{d^4q_i}{(2\pi)^4} \sum_{\alpha_j\alpha_k} \Omega_a^{\alpha_j\alpha_k}(q_i)|q_ip_i\rangle_i^{\alpha_i\alpha_j\alpha_k}, \tag{3.18}$$

as well as its conjugate, from which we obtain

$${}^{\alpha_i\alpha_j\alpha_k}_{\;i}\langle q_ip_i|p_i'a\rangle_i^{\alpha_i'} = (2\pi)^4\delta^{(4)}(p_i' - p_i)\,\delta_{\alpha_i\alpha_i'}\,\Omega_a^{\alpha_j\alpha_k}(q_i). \tag{3.19}$$

A convenient representation for t_i is therefore

$$t_i = \int \frac{d^4p_i}{(2\pi)^4} \sum_{\alpha_i} |p_i, a\rangle_i^{\alpha_i}\, {}^{\alpha_i}_{\;i}\langle p_i, b|\tau_{ab}^i(\tfrac{1}{2}P - p_i). \tag{3.20}$$

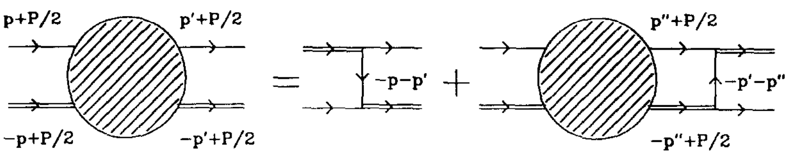

Fig. 1. Graphical representation of the Faddeev equation (3.25). A single line represents a quark and a double line a diquark.

In the NJL model the two-body vertex functions are independent of the momenta, i.e., the $\Omega_a^{\alpha\beta}$ are constant matrices in Dirac, isospin and color space. We now insert the representation (3.20) into the Faddeev equation (3.9) and take matrix elements between states ${}_j^\beta\langle p_j', b|$ and $|p_i, a\rangle_i^\alpha$ to derive an integral equation for the quantity

$$\begin{aligned} X_{ji,ba}^{\beta\alpha}(p_j', p_i) &\equiv {}_j^\beta\langle p_j', b|X_{ji}|p_i, a\rangle_i^\alpha \\ &= \int \frac{\mathrm{d}^4 q_j'}{(2\pi)^4} \int \frac{\mathrm{d}^4 q_i}{(2\pi)^4} \overline{\Omega}_b^{\delta\epsilon}(q_j')\, {}_j^{\beta\delta\epsilon}\langle q_j' p_j'|X_{ji}|p_i q_i\rangle_i^{\alpha\beta\gamma}\, \Omega_a^{\beta\gamma}(q_i). \end{aligned} \tag{3.21}$$

Note that this quantity is already an effective two-body quantity; it depends only on the relative momentum between one particle and the pair as well as on the indices α of the particle and a of the pair. The resulting integral equation reads (see Appendix B)

$$\begin{aligned} X_{ji,ba}^{\beta\alpha}(p_j', p_i) = Z_{ji,ba}^{\beta\alpha}(p_j', p_i) &+ \sum_l \int \frac{\mathrm{d}^4 \tilde{p}_l}{(2\pi)^4} Z_{jl,bc}^{\beta\gamma}(p_j', \tilde{p}_l) S_{\mathrm{F}l}^{\gamma\delta}(\tfrac{1}{2}P + \tilde{p}_l) \\ &\times \tau_{\mathrm{cd}}^l(\tfrac{1}{2}P - \tilde{p}_l) X_{li,da}^{\delta\alpha}(\tilde{p}_l, p_i) \end{aligned} \tag{3.22}$$

with[3]

$$Z_{ji,ba}^{\beta\alpha}(p_j', p_i) = {}_j^\beta\langle p_j', b|\bar{\delta}_{jik} S_{\mathrm{F}k}|p_i, a\rangle_i^\alpha = \bar{\delta}_{ijk}\, \Omega_a^{\beta\delta} S_{\mathrm{F}k}^{\gamma\delta}(-p_i - p_j') \overline{\Omega}_b^{\gamma\alpha} \tag{3.23}$$

Eq. (3.22) can be interpreted as an equation for the scattering amplitude of a particle on a pair of particles (see Fig. 1).

The total three-body T-matrix for a separable interaction can be expressed in terms of the effective two-body quantities (3.21) by using the representation (3.20) in Eq. (3.10) and taking matrix elements between three-body basis states $|qp\rangle_i^{\alpha\beta\gamma}$. We will give the explicit expression below for the case of identical particles.

3.3. Identical particles

For identical particles the indices l or k on the one- and two-body Green-functions S_{F} ant τ in Eqs. (3.22) and (3.23) can be dropped. If we consider X_{ji} simply as a function of two momenta p' and p, the indices on the momenta can also be dropped.

[3] The quantities X and Z of Refs. [11,12] correspond to the quantities $-iX$ and $-iZ$ of this paper.

The only dependence on the particle indices is then due to the factor $\bar{\delta}_{ijk} S_{Fk} = \bar{\delta}_{ij} S_F$ in Eq. (3.23), i.e., we have $Z_{ij} = Z\bar{\delta}_{ij}$. If we define a quantity

$$X_{ba}^{\beta\alpha}(p',p) = \frac{1}{6}\sum_{ij} X_{ji,ba}^{\beta\alpha}(p',p), \tag{3.24}$$

it satisfies the integral equation

$$X_{ba}^{\beta\alpha}(p',p) = Z_{ba}^{\beta\alpha}(p',p) + \int \frac{d^4p''}{(2\pi)^4} Z_{bc}^{\beta\gamma}(p',p'') \times S_F^{\gamma\delta}(\tfrac{1}{2}P + p'')\bar{\tau}_{cd}(\tfrac{1}{2}P - p'') X_{da}^{\delta\alpha}(p'',p) \tag{3.25}$$

with

$$Z_{ba}^{\beta\alpha}(p',p) = \Omega_a^{\beta\delta} S_F^{\gamma\delta}(-p-p')\overline{\Omega}_b^{\gamma\alpha}, \tag{3.26}$$

$$\bar{\tau}_{ab} = 2\tau_{ab}. \tag{3.27}$$

Eq. (3.25) is shown graphically in Fig. 1. As we will show in the next subsection, the quantity $\bar{\tau}$ introduced here coincides with the quantity $\bar{\tau}$ of Subsection 2.2, see Eqs. (2.18) and (2.22). The reason for introducing the quantity (3.24) is that it determines the total three-body T-matrix between antisymmetrized basis states. To see this, we use (3.10), (3.11), (3.20) and (3.21) to obtain

$$\begin{aligned} T = &\sum_i \int \frac{d^4\tilde{p}}{(2\pi)^4} \sum_\alpha |\tilde{p},a\rangle_i^{\alpha}\, {}_i^{\alpha}\langle\tilde{p},b|\tau_{ab}^i(\tfrac{1}{2}P - \tilde{p}) S_{Fi}^{-1} \\ &+ \sum_{ij} \int \frac{d^4\tilde{p}}{(2\pi)^4}\frac{d^4\tilde{\tilde{p}}}{(2\pi)^4} \sum_{\alpha\beta} |\tilde{p},a\rangle_i^{\alpha} \tau_{ab}^i(\tfrac{1}{2}P - \tilde{p}) X_{ij,bc}^{\alpha\beta}(\tilde{p},\tilde{\tilde{p}}) \\ &\times \tau_{cd}^j(\tfrac{1}{2}P - \tilde{\tilde{p}}) {}_j^{\beta}\langle\tilde{\tilde{p}},d|. \end{aligned} \tag{3.28}$$

We sandwich (3.28) between antisymmetric basis states, expressed with respect to any particle l (see the discussion above Eq. (3.15)):

$$|k_1k_2k_3\rangle_a^{\alpha_1\alpha_2\alpha_3} = \sqrt{\tfrac{1}{6}}\sum_i \left(|q_ip_i\rangle_l^{\alpha_i\alpha_j\alpha_k} - |-q_ip_i\rangle_l^{\alpha_i\alpha_k\alpha_j}\right), \tag{3.29}$$

where for fixed i, (ijk) is an even permutation of (123). A simple calculation leads to (see Appendix B)

$$\begin{aligned} &{}_a^{\alpha_1'\alpha_2'\alpha_3'}\langle k_1'k_2'k_3'|T|k_1k_2k_3\rangle_a^{\alpha_1\alpha_2\alpha_3} \\ &= \sum_{\substack{(ijk)\\(lmn)}} (2\pi)^4\delta(p_i' - p_l)\Omega_a^{\alpha_j'\alpha_k'}\bar{\tau}_{ab}(\tfrac{1}{2}P - p_l)\overline{\Omega}_b^{\alpha_m\alpha_n} S_{F\alpha_i'\alpha_l}^{-1}(\tfrac{1}{2}P + p_l) \\ &+ \sum_{\substack{(ijk)\\(lmn)}} \Omega_a^{\alpha_j'\alpha_k'}\bar{\tau}_{ab}(\tfrac{1}{2}P - p_i') X_{bc}^{\alpha_i'\alpha_l}(p_i',p_l)\bar{\tau}_{cd}(\tfrac{1}{2}P - p_l)\overline{\Omega}_d^{\alpha_m\alpha_n}. \end{aligned} \tag{3.30}$$

Here (ijk) and (lmn) are even permutations of (123). If Eq. (3.25) is solved for X, Eq. (3.30) represents the solution of the relativistic three-body problem in the NJL model.

3.4. Faddeev equation including scalar and axial vector two-body channels

The two-body vertex functions Ω, $\overline{\Omega}$ and propagators τ, which are needed in the Faddeev equations (3.25), (3.26), have been discussed for the scalar and axial vector channels in Section 2, see Eqs. (2.17) and (2.21). However, we note that in the derivation of Eq. (3.25) the particles have been treated as distinguishable, though identical in their physical properties. (Note that in Eq. (3.22) the quantity τ still had an index l which was dropped in (3.25) since $\tau^l \equiv \tau$ for all l.) In contrast to this, the matrices $\overline{\tau}$ in Section 2 have been derived for indistinguishable particles. The connection between these quantities is given by Eq. (3.27), as can be seen as follows: If the particles are treated as distinguishable, the interaction lagrangian corresponding e.g; to Eq. (2.5) has the form

$$\widetilde{\mathcal{L}}_{\mathrm{I,s}} = 2g_s \left(\overline{\psi}^{(1)}(\gamma_5 C)\tau_2\beta^A\overline{\psi}^{\mathrm{T}(2)}\right)\left(\psi^{(2)\mathrm{T}}(C^{-1}\gamma_5)\tau_2\beta^A\psi^{(1)}\right). \tag{3.31}$$

(The factor $\frac{1}{2}$ when going from (3.31) to (2.5) has the same origin as the familiar factor $\frac{1}{2}$ which is introduced in the two-body potential operator written in second quantized form for indistinguishable particles.) In this case there is no factor $\frac{1}{2}$ in the Bethe–Salpeter equation (see Eq. (2.15)), and the T-matrix becomes

$$t_s(k) = \left(\gamma_5 C\tau_2\beta^A\right)\tau_s(k)\left(C^{-1}\gamma_5\tau_2\beta^A\right), \qquad 2\tau_s(k) = \overline{\tau}_s(k), \tag{3.32}$$

with $\overline{\tau}_s(k)$ given by Eq. (2.18). The factor 2 in the relation $2\tau_s(k) = \overline{\tau}_s(k)$ comes from the particle exchange in the final (or initial) state of the ladder diagram. An analogous discussion holds also for the axial vector channel, i.e;

$$t_a(k) = \left(\gamma_\mu C\tau_i\tau_2\beta^A\right)\tau_{\mathrm{a}}^{\mu\nu}(k)\left(C^{-1}\gamma_\nu\tau_2\tau_i\beta^A\right), \qquad 2\tau_{\mathrm{a}}^{\mu\nu}(k) = \overline{\tau}_{\mathrm{a}}^{\mu\nu}(k), \tag{3.33}$$

with $\overline{\tau}_{\mathrm{a}}^{\mu\nu}(k)$ given by Eq. (2.22).

Inserting the vertex functions appearing in (3.32), (3.33) into (3.26) we obtain for the quark exchange kernel

$$Z_{a'a}^{\alpha'\alpha}(p',p) = \left[(\beta_{\mathrm{A}}\beta_{\mathrm{A'}})\begin{pmatrix} \gamma^5 S_{\mathrm{F}}(p'+p)\gamma^5 & \tau_m(\gamma^\mu S_{\mathrm{F}}(p'+p)\gamma^5) \\ (\gamma^5 S_{\mathrm{F}}(p'+p)\gamma^{\mu'})\tau_{m'}^\dagger & \tau_m(\gamma^\mu S_{\mathrm{F}}(p'+p)\gamma^{\mu'})\tau_{m'}^\dagger \end{pmatrix}\right]_{\alpha'\alpha}. \tag{3.34}$$

Here $a \equiv (a_{\mathrm{D}}, m, A)$ characterizes the diquark channel with $a_{\mathrm{D}} = (\mu, 5)$ for the Dirac matrix, $m = \pm 1, 0$ for the spherical isospin matrices $\tau_{\pm 1} = \mp\sqrt{\frac{1}{2}}(\tau_x \pm i\tau_y)$, $\tau_0 = \tau_z$, and A for the $\bar{3}$ color matrix. The indices α, α' stand for the Dirac, isospin and color indices of the quark states. In deriving (3.34) we used $C^{-1}S_F^T(-k)C = S_F(k)$, and the isospin part of the axial vector vertex function in Eq. (3.33) was rewritten using $(\tau_i\tau_2)(\tau_2\tau_i) = (\tau_m\tau_2)(\tau_2\tau_m^\dagger)$. The two-body propagator $\tau(k)$ in (3.25) can be written as

$$\overline{\tau}^{a'_D a_D}(k) = \begin{pmatrix} \overline{\tau}_s(k) & 0 \\ 0 & \overline{\tau}_a^{\mu'\mu}(k) \end{pmatrix} \tag{3.35}$$

with $\overline{\tau}_s(k)$ and $\overline{\tau}_a^{\mu'\mu}(k)$ given by Eqs. (2.18), (2.22). Eq. (3.25) together with (3.34), (3.35) forms the basis of our description of baryons in the relativistic Faddeev approach.

3.5. Bound states

Near a three-body bound state of mass M_B the three-body T-matrix behaves as

$$T \to \frac{\Gamma\overline{\Gamma}}{P^2 - M_B^2 + i\varepsilon} \qquad \text{as } P^2 \to M_B^2, \tag{3.36}$$

which defines the three-body vertex function Γ. From Eq. (3.30) we obtain for the antisymmetrized vertex function

$$\Gamma^{\alpha_1\alpha_2\alpha_3}(k_1k_2k_3) = \sum_{(ijk)} \Omega_a^{\alpha_j\alpha_k}\overline{\tau}_{ab}(\tfrac{1}{2}P - p_i)X_b^{\alpha_i}(p_i), \tag{3.37}$$

where X_b^α is the effective two-body vertex function for the system of a particle and a pair:

$$X_{ab}^{\alpha\beta}(p',p) \to \frac{X_a^\alpha(p')\overline{X}_b^\beta(p)}{P^2 - M_B^2 + i\varepsilon} \qquad \text{as } P^2 \to M_B^2. \tag{3.38}$$

It satisfies the homogeneous version of Eq. (3.25):

$$X_a^\alpha(p) = \int \frac{d^4p'}{(2\pi)^4} Z_{ac}^{\alpha\gamma}(p,p')S_F^{\gamma\delta}(\tfrac{1}{2}P + p')\overline{\tau}_{cd}(\tfrac{1}{2}P - p')X_d^\delta(p'). \tag{3.39}$$

3.6. Correspondence to Feynman diagrams

It is sometimes convenient to write the Faddeev equations (3.22) or (3.25) in a form which directly corresponds to the expressions derived from Feynman diagrams. For this we note that for N particles the $2N$ point function

$$G_F \equiv \langle 0|T\left(\psi^{(1)}\ldots\psi^{(N)}\overline{\psi}^{(1)}\ldots\overline{\psi}^{(N)}\right)|0\rangle$$

is related to the usual Green function $G^{(N)}$ and the T-matrix $T^{(N)}$ by

$$G_F^{(N)} = i^N G^{(N)} = i^N\left(G_0^{(N)} + G_0^{(N)}T^{(N)}G_0^{(N)}\right). \tag{3.40}$$

On the other hand, if in a Feynman diagram we amputate the external legs $i^N G_0^{(N)}$ from the interacting part of the $2N$ point function, we are left with a quantity $T_{\rm F}^{(N)}$:

$$G_{\rm F}^{(N)} = i^N G_0^{(N)} + \left(i^N G_0^{(N)}\right) T_{\rm F}^{(N)} \left(i^N G_0^{(N)}\right). \tag{3.41}$$

We therefore have the correspondence

$$T_{\rm F}^{(N)} = i^{-N} T^{(N)} \tag{3.42}$$

between the amputated Feynman graph $T_{\rm F}^{(N)}$ and the usual T-matrix $T^{(N)}$. Using (3.42) for $N = 2$ and $N = 3$ we see that if we write the Faddeev equation (3.25) in the (symbolic) form

$$(iX) = (iZ) + (iZ)(iS_{\rm F})(-\overline{\tau})(iX), \tag{3.43}$$

the quantities iZ, iS and $-\overline{\tau}$ agree with the expressions derived from Feynman rules. This correspondence is useful to check the various factors in the Faddeev equation.

4. Projection to physical baryon states

4.1. Color and isospin projection

The color part of Eq. (3.34) is the matrix element of the quark exchange operator $(\widehat{Z})$ between diquark-quark states $|\bar{3}c, 3i\rangle$, i.e.,

$$\begin{aligned}\langle \bar{3}c', 3i'|\widehat{Z}|\bar{3}c, 3i\rangle &= (\beta_c \beta_{c'})_{i'i} \\ &= -\tfrac{3}{2}\left(\delta_{ci}\delta_{c'i'} - \delta_{c'c}\delta_{i'i}\right),\end{aligned} \tag{4.1}$$

where we changed the color $\bar{3}$ index $A = 2, 5, 7$ used in the previous sections to $c = 1$, 2, 3, and the corresponding color matrices are given by $(\beta_c)_{ij} = i\sqrt{\tfrac{3}{2}}\varepsilon_{cij}$. We wish to calculate the matrix element of $\widehat{Z}$ between states of good total color

$$|(\bar{3}3)C\alpha\rangle = \sqrt{\tfrac{1}{2}}(\lambda_\alpha^{(C)})_{ci}|\bar{3}c, 3i\rangle, \tag{4.2}$$

where $(C = 0, \alpha = 0)$ denotes the color singlet, $(C = 8, \alpha = 1\ldots 8)$ the octet, $\lambda_0^{(0)} = \sqrt{\tfrac{2}{3}}\mathbf{1}$ and $\lambda_\alpha^{(8)}$ are the usual Gell-Mann matrices. Using (4.1) and (4.2) we get

$$\langle(\bar{3}3)C'\alpha'|\widehat{Z}|(\bar{3}3)C\alpha\rangle = \delta_{C'C}\delta_{\alpha'\alpha} \times \begin{cases} -3 & (C = 0), \\ \tfrac{3}{2} & (C = 8). \end{cases} \tag{4.3}$$

From this we see that, if a particular physical baryon state $(C = 0)$ is bound, the corresponding color octet state will be unbound. Since we are interested in color singlets, we have a factor (-3) in the Faddeev kernel.

Similarly, the isospin part of (3.34) is the matrix element between diquark-quark states $|tm, \tfrac{1}{2}\alpha\rangle$, i.e.,

$$\langle t'm', \tfrac{1}{2}\alpha'|\widehat{Z}|tm, \tfrac{1}{2}\alpha\rangle = (\tau_m^{(t)}\tau_{m'}^{(t')+})_{\alpha'\alpha}$$
$$= \sum_\beta \sqrt{(2t+1)(2t'+1)}(\tfrac{1}{2}t, \beta m|\tfrac{1}{2}\alpha')(\tfrac{1}{2}t', \beta m'|\tfrac{1}{2}\alpha). \quad (4.4)$$

Here $\tau_m^{(1)}$ are the usual spherical isospin matrices defined below Eq. (3.34), and $\tau_m^{(0)} \equiv \delta_{m0}\mathbf{1}$. We also used the identity

$$(\tau_m^{(t)})_{\alpha\beta} = \sqrt{2t+1}(\tfrac{1}{2}t, \beta m|\tfrac{1}{2}\alpha). \quad (4.5)$$

Using (4.4) and some relations for Clebsch-Gordan coefficients we obtain for the matrix element between states with good total isospin T, M

$$\langle(\tfrac{1}{2}t')T'M'|\widehat{Z}|(\tfrac{1}{2}t)TM\rangle$$
$$= 2(-1)^{T'+1/2+t+t'}\sum(\tfrac{1}{2}\tfrac{1}{2}, \beta\alpha|t'm')(\tfrac{1}{2}t', \alpha'm'|T'M')(\tfrac{1}{2}\tfrac{1}{2}, \alpha'\beta|tm)(t\tfrac{1}{2}, m\alpha|TM) \quad (4.6)$$
$$= 2(-1)^{T'+1/2+t+t'}\langle\tfrac{1}{2}, (\tfrac{1}{2}\tfrac{1}{2})t', T'M'|(\tfrac{1}{2}\tfrac{1}{2})t, \tfrac{1}{2}, TM\rangle$$
$$= \delta_{T'T}\,\delta_{M'M} \times \begin{cases} \begin{pmatrix} 1 & \sqrt{3} \\ \sqrt{3} & -1 \end{pmatrix} & \text{if } T = \tfrac{1}{2}, \\ 2 & \text{if } T = \tfrac{3}{2}, \end{cases} \quad (4.7)$$

where the sum in Eq. (4.6) refers to $m', m, \alpha', \alpha, \beta$. The result (4.7) is just the familiar recoupling coefficient, where the (2×2) matrix refers to the possible values (0 or 1) of (t', t). Using (4.3) and (4.7), we get for the kernel (3.34) in the color singlet, $T = \frac{1}{2}$ channel

$$Z_{a'a}^{\alpha'\alpha}(p', p) = -3\begin{pmatrix} \gamma^5 S_F(p'+p)\gamma^5 & \sqrt{3}\gamma^\mu S_F(p'+p)\gamma^5 \\ \sqrt{3}\gamma^5 S_F(p'+p)\gamma^{\mu'} & -\gamma^\mu S_F(p'+p)\gamma^{\mu'} \end{pmatrix}_{\alpha'\alpha} \quad (4.8)$$

and for the color singlet, $T = \frac{3}{2}$ channel

$$Z_{a'a}^{\alpha'\alpha}(p', p) = -6\left(\gamma^\mu S_F(p'+p)\gamma^{\mu'}\right)_{\alpha'\alpha}, \quad (4.9)$$

where from now all indices refer to the Dirac structure only. The off-diagonal elements in (4.8) describe the coupling between the scalar and axial vector diquark channels, while in (4.9) only the axial vector diquark contributes. The total Faddeev kernel is then obtained from (3.25) as

$$F_{a'a}^{\alpha'\alpha}(p', p) = Z_{a'b}^{\alpha'\beta}(p', p)G_{ba}^{\beta\alpha}(p) \quad (4.10)$$

with the three-body propagator

$$G_{a'a}^{\alpha'\alpha}(p) = S_F^{\alpha'\alpha}(\tfrac{1}{2}P+p)\overline{\tau}^{a'a}(\tfrac{1}{2}P-p), \quad (4.11)$$

where the two-body propagator $\overline{\tau}^{a'a}$ is given by (3.35). The Faddeev equation (3.25)

$$X_{ba}^{\beta\alpha}(p', p) = Z_{ba}^{\beta\alpha}(p', p) + \int\frac{d^4p''}{(2\pi)^4}F_{bc}^{\beta\gamma}(p', p'')X_{ca}^{\gamma\alpha}(p'', p) \quad (4.12)$$

is thus reduced to a matrix equation in Dirac spinor and diquark channel space with the dimension $(4 \times 5)^2$.

4.2. Spin and parity projection

To perform the spin projection of the Faddeev kernel, it is most convenient to apply the helicity formalism of Jacob and Wick [14]. As it stands, the Faddeev kernel (4.10) in the system where the total momentum of the quark and diquark vanishes is expressed in the basis states $|p, \alpha a\rangle$, where $p = (p_0, \boldsymbol{p})$ is the relative four-momentum, α is the Dirac index for the quark (α = 1, 2, 3, 4) and a is the generalized Lorentz index for the diquark (a = 5, 0, 3, +1, −1, where $a = 5$ refers to the scalar diquark, $a = 0, 3$ to the time- and z-component of the axial vector diquark, and ± 1 to the spherical $\pm$ components of the latter)[4]. The procedure consists of two steps: First, one transforms to a basis with good helicities $|p, s_\alpha, \lambda_a\rangle$ according to

$$
\begin{aligned}
|p, s_\alpha \lambda_a\rangle &= \chi^{\alpha'}_{s_\alpha}(\boldsymbol{p}) \varepsilon^{a'}_{\lambda_a}(\boldsymbol{p}) |p, \alpha' a'\rangle \\
&= \widehat{S}^{\alpha'\beta'}(\omega) \widehat{R}^{a'}{}_{b'}(\omega) \chi^{\beta'}_{s_\alpha} \varepsilon^{b'}_{\lambda_a} |p, \alpha' a'\rangle .
\end{aligned}
\tag{4.13}
$$

Here the symbol s_α ($\alpha = 1, \ldots, 4$) characterizes both the helicity and the intrinsic parity of the quark basis state, i.e., $\chi_{s_1} = (1,0,0,0)$ and $\chi_{s_3} = (0,0,1,0)$ are eigenvectors of $\frac{1}{2}\Sigma_z$ with eigenvalue $+\frac{1}{2}$ and positive and negative parity, respectively, while $\chi_{s_2} = (0,1,0,0)$ and $\chi_{s_4} = (0,0,0,1)$ are eigenvectors of $\frac{1}{2}\Sigma_z$ with eigenvalue $-\frac{1}{2}$ and positive and negative parity, respectively. $\widehat{S} = \mathrm{diag}(S, S)$ with S the usual spin-$\frac{1}{2}$ rotation matrix rotates the spinor by the Euler angles $\omega \equiv (\theta, \phi, \psi)$ into the direction of $\hat{\boldsymbol{p}}$. Hence $\chi_{s_\alpha}(\boldsymbol{p})$ is an eigenstate of the helicity operator $\frac{1}{2}\boldsymbol{\Sigma} \cdot \hat{\boldsymbol{p}}$ with eigenvalue s_α. (We note that with this choice of the basis $\chi_{s_\alpha}(\boldsymbol{p})$ is *not* a solution of the Dirac equation.) Similarly, for the diquark the symbol λ_a ($a = 5, 0, 3, +1, -1$) characterizes both the helicity and the nature of the diquark state, i.e., $\varepsilon_{\lambda_5} = (1, 0, \boldsymbol{0})$ corresponds to the scalar diquark and helicity 0, $\varepsilon_{\lambda_0} = (0, 1, \boldsymbol{0})$ to the time component of the axial vector diquark with helicity 0, and $\varepsilon_{\lambda_a} = (0, 0, \boldsymbol{\varepsilon}_{\lambda_a})$ with $a = 3, \pm 1$ to the space components of the axial vector diquark. These basis vectors are rotated by $\widehat{R} = \mathrm{diag}(1, 1, R)$, with R the rotation matrix for ordinary vectors. Hence $\varepsilon_{\lambda_a}(\boldsymbol{p})$ is an eigenstate of the helicity operator $\widehat{\boldsymbol{S}}_\mathrm{d} \cdot \hat{\boldsymbol{p}} = \mathrm{diag}(0, 0, \boldsymbol{S}_\mathrm{d} \cdot \hat{\boldsymbol{p}})$ with eigenvalue λ_a. ($\boldsymbol{S}_\mathrm{d}$ are the usual spin-1 matrices.) Note that our basis states for the axial vector diquark are different from the "physical" ones which are obtained by a Lorentz transformation and satisfy completeness in Minkowski space [22]. (The reason why we do not make use of this basis is that it is ill-defined on the light cone ($p^2 = 0$), where two of the basis vectors become identical. This non-completeness at $p^2 = 0$ leads to singularities in the helicity representation of the kernel.)

As is clear from the above, our basis states are chosen such that $\chi^\beta_{s_\alpha} = \delta_{\alpha\beta}$, $\varepsilon^b_{\lambda_a} = \delta_{ab}$, and therefore Eq. (4.13) becomes

[4] We suppose that the kernel has already been transformed to the spherical representation of the axial vector diquark states, see Eq. (C.1). In the following, $a, b, \ldots$ denote the indices $(5, 0, 3, +1, -1)$.

$$|p, s_\alpha \lambda_a\rangle = \widehat{S}^{\alpha'\alpha}(\omega)\widehat{R}^{a'}{}_a(\omega)|p, \alpha' a'\rangle. \tag{4.14}$$

Due to this relation, the Faddeev kernel in the helicity representation becomes

$$F^{s_{\alpha'} s_\alpha}_{\lambda_{a'} \lambda_a}(p', p) = \left(\widehat{S}^{-1}(\omega')\right)^{\alpha'\beta'} \left(\widehat{R}^{-1}(\omega')\right)_{a'}{}^{b'} F^{\beta'\beta}_{b'b}(p', p) \left(\widehat{S}(\omega)\right)^{\beta\alpha} \left(\widehat{R}(\omega)\right)^{b}{}_{a}, \tag{4.15}$$

where the kernel on the r.h.s. is given by (4.10). A relation similar to (4.15) holds also for the quantities X and Z, and therefore the Faddeev equation in the helicity representation has a form similar to (4.12) with $\alpha \to s_\alpha$, $a \to \lambda_a$, etc.

The second step is the projection of good total angular momentum according to [14]

$$|p_0\bar{p}, s_\alpha\lambda_a, JM\rangle = N_J \int \mathrm{d}\omega\, D^J_{M\, s_\alpha+\lambda_a}(\omega)|p, s_\alpha\lambda_a\rangle, \tag{4.16}$$

where $D^J_{M\lambda}$ is the Wigner D-function, $N_J = \sqrt{(2J+1)/8\pi^2}$, and

$$\int \mathrm{d}\omega = \int_0^{2\pi} \mathrm{d}\psi \int_0^{\pi} \sin\theta\, \mathrm{d}\theta \int_0^{2\pi} d\phi$$

denotes the integration over the Euler angles corresponding to the direction of $\boldsymbol{p}$, i.e., on the r.h.s. of (4.16) $p = (p_0, R(\omega)\tilde{\boldsymbol{p}})$. (We use the notations $\bar{p} \equiv |\boldsymbol{p}|$ and $\tilde{\boldsymbol{p}} = (0,0,\bar{p}) = R^{-1}(\omega)\boldsymbol{p}$.) As a result, we obtain the spin projected kernel as follows:

$$F^{J'M',JM}_{s_{\alpha'}\lambda_{a'},s_\alpha\lambda_{a'}}(p_0', \bar{p}', p_0, \bar{p}) = N_J N_{J'} \int \mathrm{d}\omega \int \mathrm{d}\omega' D^{*J'}_{M',s_{\alpha'}+\lambda_{a'}}(\omega') \times F^{s_{\alpha'} s_\alpha}_{\lambda_{a'}\lambda_a}(p', p) D^J_{M,s_\alpha+\lambda_a}(\omega) \tag{4.17}$$

with $F^{s_{\alpha'} s_\alpha}_{\lambda_{a'}\lambda_a}$ given by (4.15). If the same procedure is applied also to the quantities X and Z, the Faddeev equation (4.12) can be rewritten as a two-dimensional integral equation (see below).

In Appendix C we derive the following relations for the behavior of the Faddeev kernel in the original representation under rotations and parity transformations:

$$\left(\widehat{S}^{-1}(\omega)\right)^{\alpha\alpha'} \left(\widehat{R}^{-1}(\omega)\right)_{a}{}^{a'} F^{\alpha'\beta'}_{a'b'}(p_0'\boldsymbol{p}', p_0\boldsymbol{p}) \left(\widehat{S}(\omega)\right)^{\beta'\beta} \left(\widehat{R}(\omega)\right)^{b'}{}_{b} = F^{\alpha\beta}_{ab}(p_0'\tilde{\boldsymbol{p}}', p_0\tilde{\boldsymbol{p}}), \tag{4.18}$$

$$\begin{aligned}(\mathcal{P}F\mathcal{P})^{\alpha a}_{\beta b} &= \eta_\alpha\eta_a F^{\alpha\beta}_{ab}(p_0' - \boldsymbol{p}', p_0 - \boldsymbol{p})\eta_\beta\eta_b \\ &= F^{\alpha\beta}_{ab}(p_0'\boldsymbol{p}', p_0\boldsymbol{p}),\end{aligned} \tag{4.19}$$

where

$$\tilde{\boldsymbol{p}} = R^{-1}(\omega)\boldsymbol{p} = (0,0,\bar{p}), \tag{4.20}$$

$$\tilde{\boldsymbol{p}}' = R^{-1}(\omega)\boldsymbol{p}' = R(\omega^{-1}\omega')(0,0,\bar{p}'), \tag{4.21}$$

and

$$\eta_\alpha = (1,1,-1,-1), \qquad \eta_a = (1,-1,1,1,1) \tag{4.22}$$

denote the intrinsic parities of the quark and diquark basis states. In Eq. (4.18) both momenta $\boldsymbol{p}$ and $\boldsymbol{p}'$ are rotated by the same angle such that $\boldsymbol{p} \to \bar{\boldsymbol{p}}$ points along the z-axis. To understand Eq. (4.19), we note that the parity transformation of the eigenfunctions in the original representation is given by [21]

$$\mathcal{P}\psi_\alpha^a(p_0,\boldsymbol{p}) = \eta_\alpha\eta_a\psi_\alpha^a(p_0,-\boldsymbol{p}), \tag{4.23}$$

and (4.19) is the corresponding transformation law for the kernel.

Eq. (4.18) leads to the conservation of the total angular momentum, while (4.19) can be used to derive separate integral equations for the positive- and negative-parity parts. The conservation of the total angular momentum can be seen as follows: From (4.15) and (4.18) we see that

$$F^{s_{\alpha'}s_\alpha}_{\lambda_{a'}\lambda_a}(p_0'\boldsymbol{p}',p_0\boldsymbol{p}) = F^{s_{\alpha'}s_\alpha}_{\lambda_{a'}\lambda_a}(p_0'\bar{\boldsymbol{p}}',p_0\bar{\boldsymbol{p}}). \tag{4.24}$$

(Note that $\bar{\boldsymbol{p}}'$ points along the direction $\omega^{-1}\omega'$ and $\bar{\boldsymbol{p}}$ along the z-axis, see Eq. (4.21).) Eq. (4.24) means that the kernel in the helicity representation depends only on the direction of $\boldsymbol{p}'$ relative to $\boldsymbol{p}$. We now use (4.24) in (4.17) and shift the variable $\omega' \to \omega_1 \equiv \omega^{-1}\omega'$, which is the direction of $\bar{\boldsymbol{p}}'$. Then the integral over ω can be carried out analytically:

$$\begin{aligned}
&\int \mathrm{d}\omega\, D^{*J'}_{M's_{\alpha'}+\lambda_{a'}}(\omega\omega_1)D^{J}_{Ms_\alpha+\lambda_a}(\omega) \\
&= \sum_{m'}\int \mathrm{d}\omega\, D^{*J'}_{M'm'}(\omega)D^{*J}_{m's_{\alpha'}+\lambda_{a'}}(\omega_1)D^{J}_{Ms_\alpha+\lambda_a}(\omega) \\
&= N_J^{-2}\,\delta_{J'J}\delta_{M'M}\,D^{*J}_{s_\alpha+\lambda_a,s_{\alpha'}+\lambda_{a'}}(\omega_1).
\end{aligned} \tag{4.25}$$

Therefore, the kernel (4.17) is diagonal in J, M. Relations similar to (4.17), (4.24) hold also for Z, and therefore also the quark-diquark scattering amplitude X is diagonal in J, M. Therefore, the spin projected Faddeev equation becomes [5]

$$\begin{aligned}
X^{s_{\alpha'}s_\alpha}_{J\lambda_{a'}\lambda_a}(p_0'\bar{p}',p_0\bar{p}) &= Z^{s_{\alpha'}s_\alpha}_{J\lambda_{a'}\lambda_a}(p_0'\bar{p}',p_0\bar{p}) \\
&\quad + \int\frac{\mathrm{d}p_0''\,\bar{p}''^2\,\mathrm{d}\bar{p}''}{(2\pi)^5}F^{s_{\alpha'}s_{\beta'}}_{J\lambda_{a'}\lambda_{b'}}(p_0'\bar{p}',p_0''\bar{p}'')X^{s_{\beta'}s_\beta}_{J\lambda_{b'}\lambda_b}(p_0''\bar{p}'',p_0\bar{p})
\end{aligned} \tag{4.26}$$

with the kernel

$$\begin{aligned}
F^{s_{\alpha'}s_\alpha}_{J\lambda_{a'}\lambda_a}(p_0'\bar{p}',p_0\bar{p}) &= \int \mathrm{d}\omega'\, D^{*J}_{s_\alpha+\lambda_a,s_{\alpha'}+\lambda_{a'}}(\omega') \\
&\quad \times\widehat{S}^{-1}(\omega')^{\alpha'\beta}\widehat{R}^{-1}(\omega')_{a'}{}^{b}F^{\beta\alpha}_{ba}(p_0'\boldsymbol{p}',p_0\bar{\boldsymbol{p}}).
\end{aligned} \tag{4.27}$$

[5] The integration measure in Eq. (4.26) is due to $\int \mathrm{d}^4p''/(2\pi)^4 = \int \mathrm{d}p_0''\,\bar{p}''^2\,\mathrm{d}\bar{p}''/(2\pi)^4 \times \int \mathrm{d}\omega''/2\pi$, since the angular integrals include also an integration over the Euler angle ψ'' besides θ'' and ϕ''.

A relation analogous to (4.27) holds also for Z_J.

Eq. (4.26) is a two-dimensional integral equation, and with respect to the Dirac and Lorentz indices it has the same dimension as the original Eq. (4.12), namely $(4\times5)^2$. This dimension can be reduced to 10×10 by performing the projection to states with definite parity. We first note that from (4.14), (4.16) and (4.23) we get the parity transformation of the eigenfunctions in the JM representation as

$$\mathcal{P}\psi^{JM}_{s_\alpha\lambda_a}(p_0,\vec{p}) = z_{\alpha a}\psi_{s_{\bar\alpha}\lambda_{\bar a}}(p_0,\vec{p}) \tag{4.28}$$

with the phase factor

$$z_{\alpha a} = \eta_\alpha\eta_a(-1)^{J-s-j_a}. \tag{4.29}$$

In Eq. (4.28), the index $\bar\alpha$ is defined such that $s_{\bar\alpha} = -s_\alpha$ but $\eta_{\bar\alpha} = \eta_\alpha$, and similar for $\bar a$. That is, if $\alpha = (1,2,3,4)$ then $\bar\alpha = (2,1,4,3)$, and if $a = (5,0,3,+1,-1)$ then $\bar a = (5,0,3,-1,+1)$. In (4.29), $s = \frac{1}{2}$, and j_a is spin of the diquark characterized by a, i.e., $j_a = (0,0,1,1,1)$. Then the parity invariance of the kernel (4.19) is expressed in the JM representation as

$$(\mathcal{P}F\mathcal{P})^{J'M',JM}_{s_{\alpha'}\lambda_{a'},s_\alpha\lambda_a} \equiv z_{\alpha'a'}F^{J'M',JM}_{s_{\bar\alpha'}\lambda_{\bar a'},s_{\bar\alpha}\lambda_{\bar a}}z_{\alpha a} \tag{4.30}$$

$$= F^{J'M',JM}_{s_{\alpha'}\lambda_{a'},s_\alpha\lambda_a}. \tag{4.31}$$

Eqs. (4.28) and (4.31) are derived in Appendix C. From Eq. (4.28) we see that

$$\psi^{(\pm)JM}_{s_\alpha\lambda_a} \equiv \sqrt{\tfrac{1}{2}}\left(\psi^{JM}_{s_\alpha\lambda_a} \pm z_{\alpha a}\psi^{JM}_{s_{\bar\alpha}\lambda_{\bar a}}\right) \tag{4.32}$$

has parity eigenvalues ±1. Eq. (4.32) is nothing but a unitary transformation

$$\left(\psi^{JM}_{s_\alpha\lambda_a},\psi^{JM}_{s_{\bar\alpha}\lambda_{\bar a'}}\right) \to \left(\psi^{(+)JM}_{s_\alpha\lambda_a},\psi^{(-)JM}_{s_\alpha\lambda_a}\right),$$

which can be expressed as

$$U_{\alpha a} = \sqrt{\tfrac{1}{2}}\begin{pmatrix} 1 & z_{\alpha a} \\ 1 & -z_{\alpha a}\end{pmatrix}.$$

Therefore, if we order the components of our eigenfunction ψ^{JM} such that it is written as an array of pairs

$$\left(\psi^{JM}_{s_\alpha\lambda_a},\psi^{JM}_{s_{\bar\alpha}\lambda_{\bar a}}\right),$$

the unitary transformation to a state whose components have well-defined parity can be represented as a matrix $\widehat{U}$ which has the 2×2 blocks $U_{\alpha a}$ along its diagonal. This unitary transformation diagonalizes the parity operator to $\mathcal{P} = \mathrm{diag}(+1,-1,+1,-1,\ldots)$. The corresponding kernel is then obtained as $\widehat{U}F\widehat{U}^{-1}$, and applying the positive-parity projection $\hat{P}_+ = \frac{1}{2}(1+\mathcal{P})$ then simply means to delete the second, forth, ... rows and columns of $(\widehat{U}F\widehat{U}^{-1})$. In this way one arrives at the kernel for the positive- (negative-) parity states

$$
\begin{aligned}
F^{(\pm)s_{\alpha'}s_\alpha}_{J\,\lambda_{a'}\lambda_a} &= \tfrac{1}{2}\left(F^{s_{\alpha'}s_\alpha}_{J\,\lambda_{a'}\lambda_a} \pm F^{s_{\alpha'}s_{\bar\alpha}}_{J\,\lambda_{a'}\lambda_{\bar a}} z_{\alpha a} \pm z_{\alpha' a'} F^{s_{\bar\alpha'}s_\alpha}_{J\,\lambda_{\bar a'}\lambda_a} + z_{\alpha' a'} F^{s_{\bar\alpha'}s_{\bar\alpha}}_{J\,\lambda_{\bar a'}\lambda_{\bar a}} z_{\alpha a}\right) \\
&= F^{s_{\alpha'}s_\alpha}_{\lambda_{a'}\lambda_a} \pm F^{s_{\alpha'}s_{\bar\alpha}}_{J\,\lambda_{a'}\lambda_{\bar a}} z_{\alpha a},
\end{aligned}
\tag{4.33}
$$

where in the last step we used the parity invariance (4.31). Since in (4.33) only those indices (αa), (βb), ... should be taken which are not related by $(\beta b) = (\bar\alpha\bar a)$ etc., the kernel for a definite parity has now the dimension 10×10. A similar procedure can be applied to the parity projection of Z_J, and therefore the parity projected Faddeev equation can again be written in the form (4.26), i.e; two separate equations for the positive- and negative-parity parts $X^{(+)}$ and $X^{(-)}$.

In the actual calculation we solve the homogeneous version of the positive-parity part of Eq. (4.26):

$$
X^{(+)s_\alpha}_{J\,\lambda_a}(p_0\bar p) = \int \frac{\mathrm{d}p_0'\,\bar p'^2\,\mathrm{d}\bar p'}{(2\pi)^5} F^{(+)s_\alpha s_\beta}_{J\,\lambda_a\lambda_b}(p_0\bar p, p_0'\bar p')\, X^{(+)s_\beta}_{J\,\lambda_b}(p_0'\bar p'). \tag{4.34}
$$

The further evaluation of the kernel is outlined in Appendix D: After expressing the dependence of $F^{\beta\alpha}_{ba}$ on the direction of $\boldsymbol{p}'$ by a further D-function, it is possible to carry out the angular integrals in (4.27) analytically.

5. Numerical procedure

In this section we will explain the choice of the parameters and our numerical method to solve the relativistic Faddeev equation. As we mentioned already, we will not choose a specific form of the interaction lagrangian $\mathcal{L}_\mathrm{I}$ in Eq. (2.1), but we will treat the effective coupling constants in the various channels included in our calculation as parameters. We also mentioned already that we regulate divergent integrals by a sharp euclidean cut-off Λ. Therefore, for the calculation of the pionic properties and the constituent quark mass M there enter the following three parameters: The coupling constant in the pionic channel g_π (see Eq. (2.4)), the current quark mass m and the cut-off Λ. They are determined by imposing the conditions $m_\pi = 140$ MeV, $f_\pi = 93$ MeV and $M = 400$ MeV. As usual, the first condition concerns the pole position of (2.13), and the second condition is obtained from the relevant diagram for pion decay as

$$
f_\pi = -12 i g_\pi M \int \frac{\mathrm{d}^4 q}{(2\pi)^4} \frac{1}{(q^2 - M^2)((k+q)^2 - M^2)} \tag{5.1}
$$

for $k^2 = m_\pi^2$. The third condition on the constituent quark mass M, which is related to the three parameters mentioned above by the gap equation (2.9), could in principle also be dropped, i.e; one could treat M as a parameter. However, if one wants to describe the delta simultaneously with the nucleon as a bound state of about the right mass, one needs $M \gtrsim 400$ MeV, and in order to stay within the usually adopted range for the values of M, we fix it to $M = 400$ MeV. The parameters determined in this way are shown in Table 2. When we solve the Faddeev equation for the baryons, truncating the two-body channels to the scalar and axial vector diquark ones, there enter the additional

Table 2
Parameters determined to give m_π = 140 MeV, f_π = 93 MeV, M = 400 MeV and the resulting scalar diquark masses for several ratios r_s

g_π [GeV^{-2}]	10.4
m [MeV]	9.0
Λ [MeV]	739
m_s [MeV]	
$r_s = 0.4$	764
$r_s = 0.6$	627
$r_s = 0.8$	446
$r_s = 1$	140

parameters g_s and g_a (see Eqs. (2.5) and (2.6)), or equivalently the ratios r_s and r_a of Eq. (2.7). We will investigate how the baryon masses depend on these two parameters. We emphasize once more that different values of r_s and r_a reflect different forms of $\mathcal{L}_I$. In particular, if a given $\mathcal{L}_I$ involves only a single coupling constant g, this leads to definite values of the ratios r_s and r_a, see Table 1. Our aim is to impose restrictions on these parameters (and therefore also on the possible forms of $\mathcal{L}_I$) based on our numerical results.

When we use the parameters as determined above, there exists a pole of the qq T-matrix in the scalar diquark channel (Eq. (2.17)) for $r_s > \frac{1}{3}$. The resulting scalar diquark masses are shown for three r_s in Table 2. As explained earlier, for $r_s = 1$ the scalar diquark becomes degenerate with the pion. On the contrary, there exists no pole of the qq T-matrix in the axial vector diquark channel (Eq. (2.21)) except for very large values of r_a ($r_a > 2$).

Next we explain our numerical procedure to solve the homogeneous relativistic Faddeev equation (4.34). We solve it in the rest frame of the baryon, where $P = (E, \mathbf{0})$ with E the total three-body energy. The kernel involves the following singularities in the integration momentum p_0' (see Eq. (4.10)): $p_0' = -\frac{1}{2}E + E_{p'} - i\epsilon$ and $p_0' = -\frac{1}{2}E - E_{p'} + i\epsilon$ (which will be called singularities 1 and 2) from the quark propagator in Eq. (4.11), $p_0' = \frac{1}{2}E + \omega_{p'} - i\epsilon$ and $p_0' = \frac{1}{2}E - \omega_{p'} + i\epsilon$ with $\omega_p = \sqrt{\mu^2 + \mathbf{p}^2}$ from the two-body propagator in Eq. (4.11) (singularities 3 and 4; here μ is the diquark mass m_d in the case of a two-body bound state, and $\mu = 2M$ corresponds to the two branch points which are always present), and finally at $p_0' = -p_0 + E_{p+p'} - i\epsilon$ and $p_0' = -p_0 - E_{p+p'} + i\epsilon$ (singularities 5 and 6) from the propagator of the exchanged quark in Eq. (4.8). Since in this paper we are interested only in bound states we impose the condition $E < M + m_d$. We also have $m_d \leqslant 2M$, where the equality sign corresponds to the case where no two-body bound state exists. If we denote

$$\alpha = \frac{M - m_d}{2}, \tag{5.2}$$

one easily sees that in the complex p_0' plane the poles 2 and 4 lie always left of $p_0' = \alpha$, and poles 1 and 3 lie always right of $p_0' = \alpha$. Therefore, we will perform a Wick rotation of the integral in (4.34) around the point $p_0' = \alpha$, i.e; to the path $p_0' = \alpha + i\omega'$ ($-\infty < \omega' < \infty$). (The location of the poles 5 and 6 depends on p_0, and since a priori

p_0 is real one has to bend the integration path such that pole 5 lies right and pole 6 left of the path.) Finally, we continue p_0 to complex values $p_0 = \alpha + i\omega$. Then the poles 5 and 6 move to $-\alpha \pm E_{p+p'} - i\omega$, and then it is easy to see that the pole 5 (6) lies always right (left) of the integration path $p_0' = \alpha + i\omega'$. To summarize, we solve the Faddeev equation (4.34) for complex $p_0 = \alpha + i\omega$, where the integration path is $p_0' = \alpha + i\omega'$, i.e; a straight line parallel to the imaginary axis.

If we discretize the integral Eq. (4.34) to a matrix equation, the dimension of the positive-parity kernel is $(10N_p)^2$, where N_p is the number of mesh points for the pair $(p_0 \bar{p})$. (For the case $J = \frac{1}{2}$ the dimension is $(8N_p)^2$, since matrix elements with $\lambda + s = \pm\frac{3}{2}$ do not contribute.) In the numerical calculation we transform $(p_0 \bar{p})$ to two-dimensional polar coordinates (r, φ) with $r \leqslant \Lambda$. The convergence was tested by increasing the number of mesh points up to $N_p = 100$. We directly diagonalize the kernel F to obtain its eigenvalues $\lambda(E)$, where E is treated as a parameter which is restricted to $E < M + m_{\rm d}$. The baryon mass $M_{\rm B}$ is then obtained from the condition

$$\lambda(E = M_{\rm B}) = 1. \tag{5.3}$$

We also mention that in the static approximation, where the mass of the exchanged quark is treated as infinitely heavy, the Faddeev equation becomes separable and can be solved almost analytically [10,11]. This was used to test the computer code. Also, if the axial vector diquark channel is neglected, the angular momentum projection can be done very simply without the use of the helicity formalism [11]. This was used as a further check of the formulation and the code.

6. Results and discussions

In this section we discuss our results for the nucleon and delta masses. In order to study the role of the scalar diquark channel in describing the nucleon state, we first fix the ratio $r_{\rm a} = 0$ and plot the largest eigenvalue $\lambda(E)$ of the Faddeev kernel for the nucleon state as a function of the total energy E for several values of $r_{\rm s}$ in Fig. 2. The nucleon mass $M_{\rm N}$ is then obtained from the condition (5.3). For each value of $r_{\rm s}$ we show the quark-diquark threshold $m_{\rm s} + M$, where $m_{\rm s} < 2M$ is the mass of the scalar diquark, by the dotted vertical lines. We see that the nucleon mass decreases and the binding energy measured from the quark-diquark threshold increases with $r_{\rm s}$, i.e. the contribution of the scalar diquark channel is attractive. We see from Fig. 2 that in the case $r_{\rm a} = 0$ we need $r_{\rm s} \geqslant 0.5$ to obtain a three-body bound state, and $r_{\rm s} \simeq \frac{2}{3}$ to get about the right nucleon mass.

The results of Fig. 2 have been first obtained in Ref. [11], and have been confirmed by an independent calculation in Ref. [13]. In Ref. [13] the results for the nucleon mass are given as a function of $m_{\rm s}$ instead of $r_{\rm s}$. To see that the calculations are consistent, we note that our value $r_{\rm s} \simeq \frac{2}{3}$, which is required to get about the experimental nucleon mass, corresponds to $m_{\rm s} \simeq 570$ MeV, which is very close to the value given in Table I of Ref. [13]. Also, if we use the correspondence between $m_{\rm s}$ and $r_{\rm s}$ shown in Table 2, we

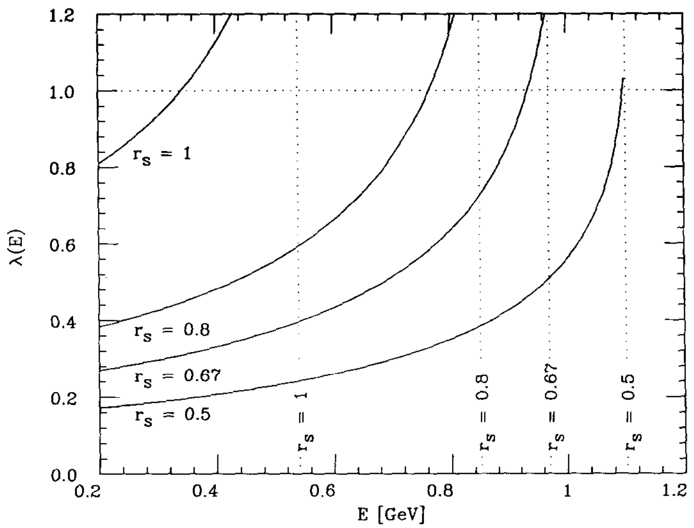

Fig. 2. The largest eigenvalue of the Faddeev kernel for the nucleon state as a function of the total energy E for $r_a = 0$ and several values of r_s. For each value of r_s the quark-diquark threshold is shown by the dotted vertical lines. The intersections of the curves with the dotted horizontal line ($\lambda = 1$) give the nucleon masses for each value of r_s.

see that the results for the nucleon mass given in Fig. 6 of Ref. [13] agree qualitatively with those of Fig. 2 of the present paper; differences arise due to different choices for M (375 MeV in Ref. [13], 400 MeV in the present calculation).

Next we study the contribution of the axial vector diquark channel [12]. The eigenvalue curves are shown in Fig. 3 for several values of r_a with the ratio $r_s = 0.5$ being fixed. We see that the nucleon mass decreases with r_a, and therefore for $g_a > 0$ the contribution of the axial vector diquark channel to the nucleon mass is attractive, too. In particular, in the case of $r_s = 0.5, r_a = 0.25$, which corresponds to the color current interaction lagrangian (2.3), the nucleon mass is about 900 MeV. With the original NJL lagrangian (2.2) ($r_s = \frac{2}{13}, r_a = \frac{1}{13}$), however, it is not possible to obtain a bound state. From the results shown in Fig. 3 it is clear that the axial vector diquark channel plays an important role for the nucleon mass.

In Fig. 4 we show the largest eigenvalue of the Faddeev kernel for the delta state for several values of r_a. In this case the scalar diquark channel does not contribute since the scalar diquark has isospin zero, and therefore the results of Fig. 4 do not depend on r_s. In order to obtain a bound delta state we need $r_a \geqslant 0.44$, which means a rather strong interaction in the axial vector diquark channel. Thus, neither the original NJL lagrangian nor the color current lagrangian give a bound delta state. With our choice of $M = 400$ MeV, we obtain a loosely bound delta state with mass $M_\Delta \lesssim 1190$ MeV if $r_a \gtrsim 0.44$. In order to obtain a more tightly bound delta state with $M_\Delta \simeq 1200$ MeV, one could choose a larger M and correspondingly somewhat larger r_a.

In Fig. 5 we show the pairs of parameters (r_s, r_a) which give a reasonable nucleon

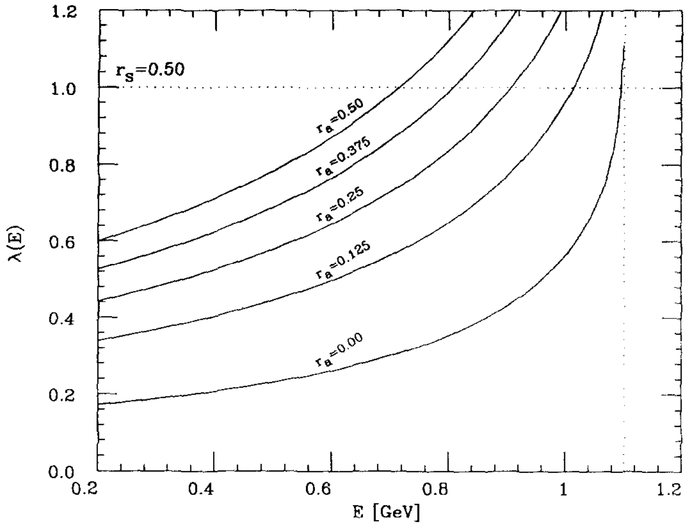

Fig. 3. Same as Fig. 2 for $r_s = 0.5$ and several values of r_a.

mass, where by "reasonable" we mean the range 800 MeV $\leqslant M_N \leqslant$ 1100 MeV. Fig. 5 actually shows the parameters which give M_N = 800, 940 and 1100 MeV. This figure shows how the attraction is shared between the scalar and axial vector diquark channels to obtain a given nucleon mass. If we make r_s smaller than 0.3 we need an interaction in

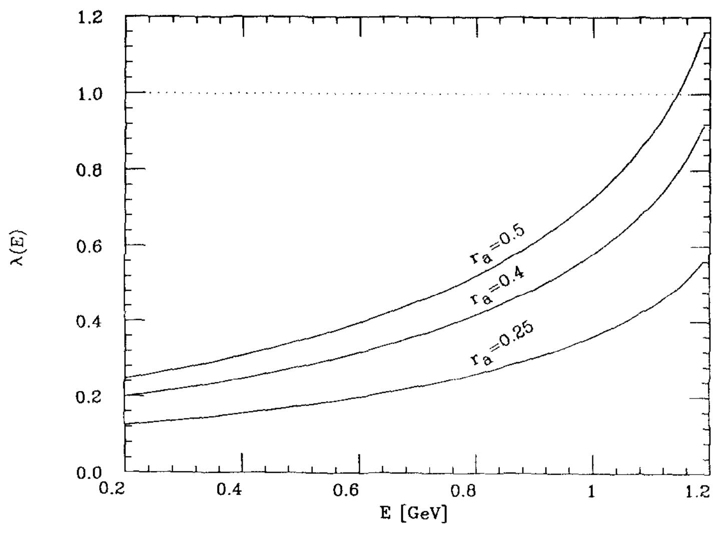

Fig. 4. The largest eigenvalue of the Faddeev kernel for the delta state as a function of the total energy E for several values of r_a. The threshold is $3M$ = 1.2 GeV. The intersections of the curves with the dotted horizontal line ($\lambda = 1$) give the delta masses for each value of r_a.

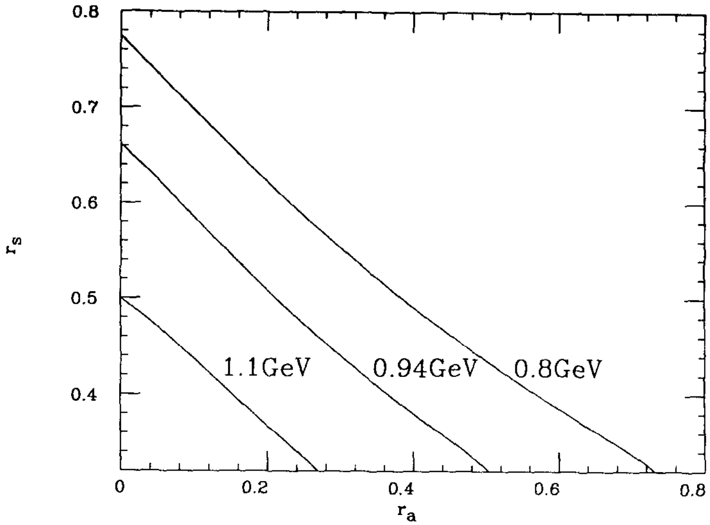

Fig. 5. The three lines connect those values of the parameters (r_s, r_a) which give a nucleon mass of 800, 940 and 1100 MeV.

the axial vector channel which is stronger than in the scalar channel ($r_a > r_s$) in order to obtain a reasonable nucleon mass. On the other hand, r_s must be smaller than 0.78 in order to stay within the indicated range of M_N. Actually we find that the nucleon mass depends roughly linearly on the ratios r_s and r_a:

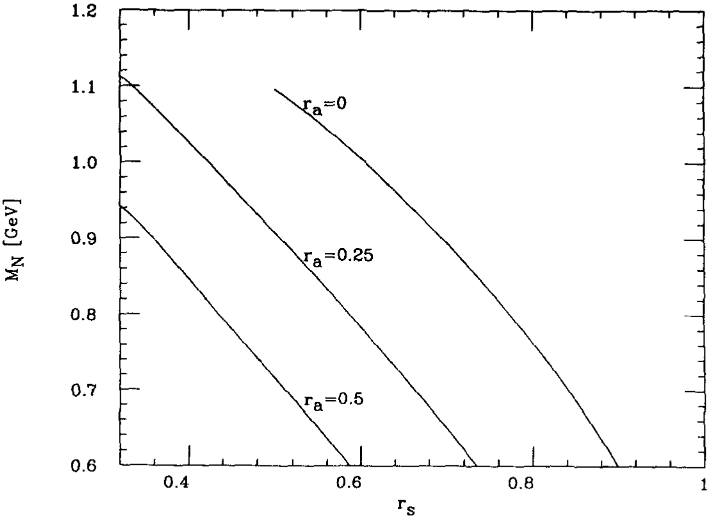

Fig. 6. The nucleon mass as a function of r_s for three values of r_a.

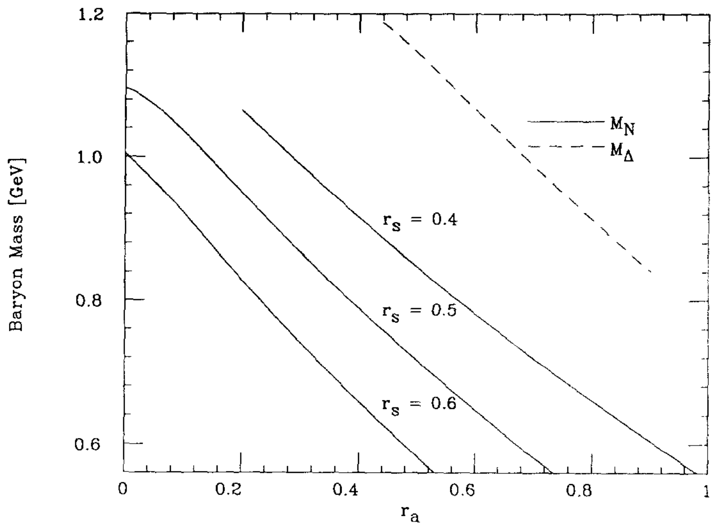

Fig. 7. The solid lines show the nucleon mass as a function of r_a for three values of r_s, and the dashed line shows the delta mass as a function of r_a. (Note that the delta mass is independent of r_s.)

$$M_N \simeq -1.1r_s - 0.7r_a + 1.7 \text{ [GeV]}. \tag{6.1}$$

Thus, any interaction lagrangian which gives values for r_s and r_a satisfying approximately the relation $r_s + 0.6r_a =(0.5$ to $0.8)$ gives a reasonable nucleon mass within the range indicated in Fig. 5.

The approximate linear relationship (6.1) is also demonstrated in Figs. 6 and 7 which show the nucleon mass as a function of one of the ratios (r_s, r_a) for several values of the other one. If a curve in these figures is truncated on its upper left side this means that no bound state can be obtained by further decreasing the ratio shown on the horizontal axis. Fig. 7 also shows the delta mass, which does not depend on r_s, as a function of r_a. As mentioned above, $r_a \geqslant 0.44$ is needed to obtain a bound delta state. Fig. 7 also shows that in order to get simultaneously a reasonable result for the nucleon and the delta mass we need a lagrangian which gives the ratios $r_s \lesssim 0.4$ and $r_a \gtrsim 0.4$. To our knowledge, none of the four-fermi interaction lagrangians used in the literature so far has this property. We will come back to this point below.

If we fit the delta mass shown in Fig. 7 to a linear curve we have

$$M_\Delta \simeq -0.76r_a + 1.52 \text{ [GeV]}. \tag{6.2}$$

Together with Eq. (6.1) this gives for the delta-nucleon mass difference [6]:

$$M_\Delta - M_N \simeq 1.1r_s - 0.06r_a - 0.18 \text{ [GeV]}. \tag{6.3}$$

[6] We emphasize that relations (6.1) to (6.3) are only very rough ones and should not be used for quantitative purposes.

We see that the main mechanism for the delta–nucleon mass difference is the interaction in the scalar diquark channel: Increasing r_s lowers the nucleon mass but leaves the delta mass unchanged. A small contribution comes also from the axial vector diquark channel due to the fact that the delta mass decreases with r_a slightly more rapidly than the nucleon mass.

The above discussion shows that for the case of $M \simeq 400$ MeV a four-fermi interaction lagrangian which corresponds to $g_\pi > 0$, $r_s \lesssim 0.4$, $r_a \gtrsim 0.4$ can reasonably well describe the pion, the nucleon and the delta masses simultaneously. As shown in Table 1, none of the commonly used chiral-invariant interaction lagrangians has the property $r_s \lesssim 0.4$, $r_a \gtrsim 0.4$. The next simplest possibility is to consider a linear combination of two such interaction lagrangians. By using Table 1 one finds that the following interaction lagrangian:

$$\mathcal{L}_I = -g\left[\sum_{c=1}^{8}\left(\bar\psi\gamma^\mu \tfrac{1}{2}\lambda_c\psi\right)^2 + \kappa\left(\bar\psi\gamma^\mu\gamma_5\psi\right)^2\right] \tag{6.4}$$

with $g > 0$ gives $g_\pi/g = \frac{1}{18}(4-3\kappa)$, $r_s = (2-3\kappa)/(4-3\kappa)$, $r_a = (1+\frac{3}{2}\kappa)/(4-3\kappa)$. As long as $\kappa < \frac{4}{3}$, g can be adjusted such that $g_\pi = 10.4\ \mathrm{GeV}^{-2}$ in order to give the experimental pion mass and decay constant. We note that r_s (r_a) is a monotonously decreasing (increasing) function of κ. It is therefore possible to weaken (strengthen) the interaction in the scalar (axial vector) diquark channel relative to the $\kappa = 0$ values $r_s = \frac{1}{2}$ and $r_a = \frac{1}{4}$. In order that $r_s > 0$ we need $\kappa < \frac{2}{3}$, and in order to have a bound delta state ($r_a > 0.44$) we need $\kappa > 0.27$. Therefore, κ lies in the range $0.27 < \kappa < \frac{2}{3}$. Note also that for the lagrangian (6.4) the relation between r_s and r_a is $r_a = 1 - \frac{3}{2}r_s$. For example, for $\kappa = \frac{2}{9}$ we obtain $r_s = r_a = 0.4$, which results in $M_N = 915$ MeV, while the delta is still unbound, see Fig. 7. If κ is increased to $\kappa = \frac{1}{3}$, we have $r_s = 0.33$, $r_a = 0.5$, which gives $M_N = 940$ MeV, $M_\Delta = 1140$ MeV, see Figs. 6 and 7. Between there is the case $\kappa = 0.27$ ($r_s = 0.37$, $r_a = 0.44$), where the delta is close to threshold, $M_\Delta = 1190$ MeV, and $M_N = 920$ MeV. We conclude that the lagrangian (6.4) with $\kappa \lesssim \frac{1}{3}$ can describe simultaneously the masses of the pion, the nucleon and the delta reasonably well. (However, we mention again that for the description of the delta as a bound state, higher values for M might be preferable.)

The lagrangian (6.4) consists of the color current interaction (2.3) supplied with a correction term which has the form of a color and isospin singlet axial vector current interaction. Since the latter one affects the $(1^+, T=0)$, $q\bar q$ channel most strongly, we note that in this channel the Fierz symmetrized interaction has the form $-\frac{1}{2}g_A\,(\bar\psi\gamma^\mu\gamma_5\psi)^2$ with $g_A/g = (\frac{1}{9} + \frac{13}{12}\kappa)$ corresponding to the two terms in (6.4). For $\kappa > 0$ this is a repulsive interaction, i.e; it does not lead to a pole in the T-matrix for the $(1^+, T=0)$, $q\bar q$ channel.

We note that the conditions $g_\pi > 0$ and $r_s \lesssim 0.4$, $r_a \gtrsim 0.4$ of course do not uniquely determine the form of the chiral-invariant four-fermi interaction. For example, it is clear that by considering a linear combination of three terms (instead of two in Eq. (6.4)) it is certainly possible to adjust the coefficients such that the desired values for r_s and r_a

are obtained. The lagrangian (6.4), however, seems to represent the simplest possible form.

We point out that these conditions on the form of the interaction lagrangian have been derived within our present bound state description of the nucleon and the delta, and therefore their validity is restricted to this framework. One must bear in mind that the nucleon and in particular the delta constructed here are loosely bound systems, and therefore the effects of the confinement, which are not included in this calculation, could be important. There has been some progress recently to mimic the effects of the confinement by taking into account momentum-dependent interactions such as to avoid the single quark poles in the Green functions [4]. It would be interesting to extend our present Faddeev approach such as to include the effects of the confinement in a phenomenological way.

7. Summary and conclusion

In this paper we investigated the masses of the nucleon and the delta in the NJL model as an effective theory for QCD in the low-energy region by applying the Faddeev method. Restricting the two-body channels to the scalar and axial vector diquark ones, which are expected to be dominant from the non-relativistic analogy, we solved the relativistic Faddeev equation for the nucleon and the delta state without any further approximations. The spin projection has been carried out completely relativistically by applying methods based on the helicity formalism. For the cut-off scheme we used the covariant sharp euclidean cut-off. In this way we were able to extend the usual description of mesons in the Bethe–Salpeter framework to a completely covariant description of the baryons in the Faddeev framework.

Our purpose was to discuss the dependence of the baryon masses on the form of the four-fermi interaction lagrangian. We therefore treated the coupling constants in the scalar and axial vector diquark channels, or equivalently their ratios to the coupling constant in the pionic channel (r_s and r_a defined by Eq. (2.7)), as parameters. Lagrangians employed so far in the literature, like the original NJL lagrangian or the color current interaction lagrangian, then correspond to certain values of r_s and r_a. Our results are summarized as follows: Both the scalar and axial vector diquark channels contribute attractively to the nucleon mass, and for quantitative arguments the axial vector channel cannot be neglected. We have found that the nucleon mass depends approximately linearly on the two ratios, and that any interaction lagrangian which gives ratios satisfying approximately the relation $r_s + 0.6 r_a =$ (0.5 to 0.8) gives a reasonable nucleon mass (i.e; between 0.8 and 1.1 GeV). In particular, the color current interaction lagrangian gives a nucleon mass of 0.9 GeV, but with the original NJL interaction lagrangian one cannot obtain a bound nucleon state. Concerning the delta mass, which does not depend on r_s, we found that its dependence on r_a is similar to the case of the nucleon. Moreover, one needs a rather strong interaction in the axial vector channel to obtain a bound delta state. For example, neither with the color current interaction lagrangian nor with the original

NJL interaction lagrangian one can obtain a bound delta state. In order to reproduce simultaneously the nucleon and the delta mass we need an interaction lagrangian which gives the ratios $r_s \lesssim 0.4$ and $r_a \gtrsim 0.4$. We have pointed out that in this scenario the dominant mechanism to generate the delta-nucleon mass difference is the interaction in the scalar diquark channel, which is easily understood from the fact that with increasing interaction strength in this channel the nucleon mass decreases but the delta mass remains unchanged. We have also given the most simple form of the four-fermi interaction lagrangian which gives the required values for r_s and r_a, i.e; which can describe the masses of the pion, the nucleon and the delta simultaneously. The main part of this lagrangian is the color current type interaction which is supplemented by a correction term of axial vector current type.

Using the methods developed in this paper, solutions of the relativistic three-quark Faddeev equation with four-fermi interactions of the NJL type can be obtained without great numerical difficulties. Thus we have demonstrated that an extension of the usual Bethe–Salpeter framework for the $q\bar{q}$ system to the Faddeev framework for the qqq system is feasible. Since this framework treats the quark–quark interactions explicitly, it seems superior to the mean-field (soliton) description of baryons which has been pursued most actively so far. (We note, however, that the soliton description might include sea quark effects which are not taken into account in the Faddeev approach, and therefore a detailed comparison of the two descriptions is necessary before one can draw firm conclusions.) In order to really establish the Faddeev approach as a successful method for baryons one should investigate properties of the nucleon like the magnetic moment, the axial vector coupling constant, etc., as it has been done recently in the simple additive quark–diquark picture [23]. Such investigations are carried out in Ref. [24]. Eventually, the effects of the confinement should be included into the model in a phenomenological way. Since the interactions necessary to mimic the confinement might be momentum dependent and non-separable, this requires the development of techniques which go beyond the present paper. The methods developed in this paper might nevertheless form the basis of such more ambitious approaches.

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Appendix A. Fierz transformations

In this appendix we explain the use of Fierz transformations in order to bring any four-fermi interaction lagrangian into a form where the interaction strength in a particular channel can be read off directly. We will restrict the discussion to the flavor SU(2) case.

A.1. $q\bar{q}$ channels

In order to obtain the interaction strength in a particular $q\bar{q}$ channel, we use the Fierz identity [21]

$$\left(\overline{\psi}_3\Gamma^1\psi_1\right)\left(\overline{\psi}_4\Gamma^2\psi_2\right)=\frac{-1}{4\times3\times2}\sum_\alpha\left(\overline{\psi}_3\Gamma_\alpha\psi_2\right)\left(\overline{\psi}_4\Gamma^2\Gamma^\alpha\Gamma^1\psi_1\right), \tag{A.1}$$

where Γ^α denotes a product of the following Dirac, color and flavor (isospin) matrices:

$$\Gamma^\alpha_{\rm D}=\{1,\gamma^5,i\gamma^5\gamma^\mu,\gamma^\mu,\sigma^{\mu\nu}\}, \tag{A.2}$$

$$\Gamma^\alpha_{\rm C}=\{1,\beta^a\}, \tag{A.3}$$

$$\Gamma^\alpha_{\rm I}=\{1,\tau^i\}. \tag{A.4}$$

In (A.3), $\beta^a=\sqrt{\frac{3}{2}}\lambda^a$ $(a=1,\ldots,8)$ with λ^a the usual Gell-Mann matrices, such that $\mathrm{tr}(\beta^{a'}\beta^a)=3\delta^{a'a}$. In (A.1) the subscripts on the fermion fields just indicate their original positions and will be left out in the following. The minus sign on the r.h.s. is due to Fermi statistics.

For the product $\Gamma^2\Gamma^\alpha\Gamma^1$ on the r.h.s. of (A.1) we give the following identities:

$$\begin{aligned}\gamma_5\Gamma^\alpha_{\rm D}\gamma^5&=\{1,\gamma^5,-i\gamma^5\gamma^\mu,-\gamma^\mu,\sigma^{\mu\nu}\},\\ \gamma^5\gamma_\mu\Gamma^\alpha_{\rm D}\gamma^5\gamma^\mu&=\{-4,4\gamma^5,2i\gamma^5\gamma^\mu,-2\gamma^\mu,0\},\\ \gamma_\mu\Gamma^\alpha_{\rm D}\gamma^\mu&=\{4,-4\gamma^5,2i\gamma^5\gamma^\mu,-2\gamma^\mu,0\};\end{aligned} \tag{A.5}$$

$$\beta^b\Gamma^a_{\rm C}\beta^b=\{8,-\beta^a\}, \tag{A.6}$$

$$\tau^k\Gamma^i_{\rm I}\tau^k=\{3,-\tau^i\}. \tag{A.7}$$

One can distinguish between four possibilities for Γ^1 and Γ^2 in Eq. (A.1): (i) They are just Dirac matrices (i.e; unit matrices in color and isospin space); (ii) They are the product of a Dirac matrix and an isospin matrix τ^i; (iii) They are the product of a Dirac matrix and a color matrix β^i; (iv) They involve all three types of matrices. To write down the result for Eq. (A.1) for these four cases, we introduce the notation

$$\begin{aligned}(a_1a_2a_3a_4a_5)\equiv{}&a_1\left(\overline{\psi}\psi\right)^2+a_2\left(\overline{\psi}\gamma^5\psi\right)^2+a_3\left(\overline{\psi}i\gamma^5\gamma^\mu\psi\right)^2\\&+a_4\left(\overline{\psi}\gamma^\mu\psi\right)^2+a_5\left(\overline{\psi}\sigma^{\mu\nu}\psi\right)^2,\\(a_1a_2a_3a_4a_5)_\tau\equiv{}&a_1\left(\overline{\psi}\boldsymbol{\tau}\psi\right)^2+a_2\left(\overline{\psi}\gamma^5\boldsymbol{\tau}\psi\right)^2+a_3\left(\overline{\psi}i\gamma^5\gamma^\mu\boldsymbol{\tau}\psi\right)^2\\&+a_4\left(\overline{\psi}\gamma^\mu\boldsymbol{\tau}\psi\right)^2+a_5\left(\overline{\psi}\sigma^{\mu\nu}\boldsymbol{\tau}\psi\right)^2.\end{aligned} \tag{A.8}$$

Then we obtain

$$\begin{aligned}\left(\overline{\psi}\Gamma^1_{\rm D}\psi\right)\left(\overline{\psi}\Gamma^2_{\rm D}\psi\right)&=-\tfrac{1}{24}\left((a_1\ldots a_5)^{2,1}+(a_1\ldots a_5)^{2,1}_\tau\right)+\text{color }8,\\ \left(\overline{\psi}\Gamma^1_{\rm D}\boldsymbol{\tau}\psi\right)\cdot\left(\overline{\psi}\Gamma^2_{\rm D}\boldsymbol{\tau}\psi\right)&=-\tfrac{1}{24}\left(3(a_1\ldots a_5)^{2,1}-(a_1\ldots a_5)^{2,1}_\tau\right)+\text{color }8,\\ \left(\overline{\psi}\Gamma^1_{\rm D}\beta^a\psi\right)\left(\overline{\psi}\Gamma^2_{\rm D}\beta^a\psi\right)&=-\tfrac{1}{3}\left((a_1\ldots a_5)^{2,1}+(a_1\ldots a_5)^{2,1}_\tau\right)+\text{color }8,\end{aligned}$$

$$\left(\overline{\psi}\Gamma_{\rm D}^1\boldsymbol{\tau}\beta^a\psi\right)\cdot\left(\overline{\psi}\Gamma_{\rm D}^2\boldsymbol{\tau}\beta^a\psi\right)=-\tfrac{1}{3}\left(3(a_1\dots a_5)^{2,1}-(a_1\dots a_5)_\tau^{2,1}\right)+\text{color 8}. \tag{A.9}$$

Here the notation $(\dots)^{2,1}$ indicates that the coefficients in (A.8) depend on the Dirac matrices $\Gamma_{\rm D}^1$ and $\Gamma_{\rm D}^2$. The cases of interest are, using Eq. (A.5),

$$\begin{aligned}
&\Gamma_{\rm D}^1=1,\ \Gamma_{\rm D}^2=1:\ (a_1\dots a_5)^{1,1}=(a_1\dots a_5)_\tau^{11}=(11111)\\
&\Gamma_{\rm D}^1=\gamma^5,\ \Gamma_{\rm D}^2=\gamma^5:\ (a_1\dots a_5)^{\gamma_5,\gamma_5}=(a_1\dots a_5)_\tau^{\gamma_5,\gamma_5}=(11-1-11)\\
&\Gamma_{\rm D}^1=\gamma_\mu\gamma_5,\ \Gamma_{\rm D}^2=\gamma^\mu\gamma^5:\ (a_1\dots a_5)^{\gamma_\mu\gamma_5,\gamma^\mu\gamma_5}=(a_1\dots a_5)_\tau^{\gamma_\mu\gamma_5,\gamma^\mu\gamma^5}=(-442-20)\\
&\Gamma_{\rm D}^1=\gamma_\mu,\ \Gamma_{\rm D}^2=\gamma^\mu:\ (a_1\dots a_5)^{\gamma_\mu,\gamma^\mu}=(a_1\dots a_5)_\tau^{\gamma_\mu,\gamma^\mu}=(4-42-20).
\end{aligned} \tag{A.10}$$

Using Eqs. (A.9), (A.10) one can rewrite identically a given $\mathcal{L}_{\rm I}$ into the Fierz transformed form $\mathcal{L}_{\rm I,F}$. Therefore, if one uses the Fierz symmetric form

$$\mathcal{L}_{{\rm I},q\bar{q}}=\tfrac{1}{2}\left(\mathcal{L}_{\rm I}+\mathcal{L}_{\rm I,F}\right), \tag{A.11}$$

one can calculate the direct terms only and multiply a factor 2 due to the exchange terms. It is therefore convenient to define the strength of the $q\bar{q}$ interaction in various channels as the coupling constants in the lagrangian (A.11), as we have done it in Eq. (2.4) for the $(0^+,T=0)$ and $(0^-,T=1)$ channels.

A.2. qq channels

In order to obtain the interaction strength in a particular qq channel, we first use the identity [1]

$$\left(\overline{\psi}_3\Gamma^1\psi_1\right)\left(\overline{\psi}_4\Gamma^2\psi_2\right)=-\left(\overline{\psi}_3\Gamma^1\psi_1\right)\left(\psi_2^{\rm T}\Gamma^{2\rm T}\overline{\psi}_4^{\rm T}\right)=\left(\overline{\psi}_3\Gamma^1\psi_1\right)\left(\overline{\psi}_2'\Gamma'^2\psi_4'\right), \tag{A.12}$$

where we introduced the charge conjugated fields and operators by

$$\begin{aligned}
&\psi'=C\tau_2\overline{\psi}^{\rm T}\quad(C=i\gamma_2\gamma_0),\\
&\overline{\psi}'=-\psi^{\rm T}C^{-1}\tau_2,\\
&\Gamma'=C\tau_2\Gamma^{\rm T}C^{-1}\tau_2.
\end{aligned} \tag{A.13}$$

For the Dirac, color and isospin matrices (A.2)-(A.4) we have

$$\Gamma_{\rm D}'^\alpha=\{1,\gamma^5,i\gamma^5\gamma^\mu,-\gamma^\mu,-\sigma^{\mu\nu}\}, \tag{A.14}$$

$$\Gamma_{\rm C}'^\alpha=\{1,\pm\beta^a\}, \tag{A.15}$$

$$\Gamma_{\rm I}'^\alpha=\{1,-\tau^i\}, \tag{A.16}$$

where the $+(-)$ sign in (A.15) holds for a = 1, 3, 4, 6, 8 (a = 2, 5, 7). Then we perform the Fierz transformation of the r.h.s. of Eq. (A.12):

$$\left(\overline{\psi}_3\Gamma^1\psi_1\right)\left(\overline{\psi}'_2\Gamma'^2\psi'_4\right)=\frac{-1}{4\cdot 3\cdot 2}\sum_\alpha\left(\overline{\psi}_3\Gamma_\alpha\psi'_4\right)\left(\overline{\psi}'_2\Gamma'^2\Gamma^\alpha\Gamma^1\psi_1\right). \tag{A.17}$$

On the r.h.s. of (A.17) one uses the identities (cf. Eqs. (A.5)-(A.7))

$$\begin{aligned}
\gamma'_5\Gamma^\alpha_{\rm D}\gamma^5 &= \gamma_5\Gamma^\alpha_{\rm D}\gamma^5 = \{1,\gamma^5,-i\gamma^5\gamma^\mu,-\gamma^\mu,\sigma^{\mu\nu}\},\\
(\gamma^5\gamma_\mu)'\Gamma^\alpha_{\rm D}(\gamma^5\gamma^\mu) &= \gamma^5\gamma_\mu\Gamma^\alpha_{\rm D}\gamma^5\gamma^\mu = \{-4,4\gamma^5,2i\gamma^5\gamma^\mu,-2\gamma^\mu,0\},\\
\gamma'_\mu\Gamma^\alpha_{\rm D}\gamma^\mu &= -\gamma_\mu\Gamma^\alpha_{\rm D}\gamma^\mu = \{-4,4\gamma^5,-2i\gamma^5\gamma^\mu,2\gamma^\mu,0\}, \qquad (A.18)\\
\beta'^b\Gamma^A_{\rm C}\beta^b &= (\beta^b)^{\rm T}\Gamma^A_{\rm C}\beta^b = -4\Gamma^A_{\rm C} \quad (A=2,5,7), \qquad (A.19)\\
\tau'^k\Gamma^i_{\rm I}\tau^k &= -\tau^k\Gamma^i_{\rm I}\tau^k = \{-3,\tau^i\}. \qquad (A.20)
\end{aligned}$$

Here we are interested in the color $\bar{3}$ channels only, since the color 6 channels will eventually not contribute to a color singlet three-quark state. Therefore, it is sufficient to consider only those terms on the r.h.s. of (A.17) which involve the color matrices β^A $(A=2,\ 5,\ 7)$. Then only those combinations of Dirac and isospin matrices survive which are totally symmetric. To write down the results, we will again distinguish the same four cases as in Eqs. (A.9), and introduce the notation

$$\begin{aligned}
[a_1a_2a_3] &\equiv a_1\left(\overline{\psi}\beta^A\psi'\right)\left(\overline{\psi}'\beta^A\psi\right)+a_2\left(\overline{\psi}\beta^A\gamma^5\psi'\right)\left(\overline{\psi}'\beta^A\gamma^5\psi\right)\\
&\quad +a_3\left(\overline{\psi}\beta^A\gamma^5\gamma_\mu\psi'\right)\left(\overline{\psi}'\beta^A\gamma^5\gamma^\mu\psi\right)\\
[a_4a_5]_\tau &\equiv a_4\left(\overline{\psi}\boldsymbol{\tau}\beta^A\gamma_\mu\psi'\right)\cdot\left(\overline{\psi}'\boldsymbol{\tau}\beta^A\gamma^\mu\psi\right)+a_5\left(\overline{\psi}\boldsymbol{\tau}\beta^A\sigma^{\mu\nu}\psi'\right)\cdot\left(\overline{\psi}'\boldsymbol{\tau}\beta^A\sigma_{\mu\nu}\psi\right).
\end{aligned} \tag{A.21}$$

In these definitions we included only the totally antisymmetric combinations of matrices. Then we obtain

$$\begin{aligned}
\left(\overline{\psi}\Gamma^1_{\rm D}\psi\right)\left(\overline{\psi}\Gamma^2_{\rm D}\psi\right) &= -\tfrac{1}{24}\left([a_1a_2a_3]^{2,1}+[a_4a_5]^{2,1}_\tau\right)+\text{color }6,\\
\left(\overline{\psi}\Gamma^1_{\rm D}\boldsymbol{\tau}\psi\right)\cdot\left(\overline{\psi}\Gamma^2_{\rm D}\boldsymbol{\tau}\psi\right) &= \tfrac{1}{24}\left(3[a_1a_2a_3]^{2,1}-[a_4a_5]^{2,1}_\tau\right)+\text{color }6,\\
\left(\overline{\psi}\Gamma^1_{\rm D}\beta^a\psi\right)\left(\overline{\psi}\Gamma^2_{\rm D}\beta^a\psi\right) &= \tfrac{1}{6}\left([a_1a_2a_3]^{2,1}+[a_4a_5]^{2,1}_\tau\right)+\text{color }6,\\
\left(\overline{\psi}\Gamma^1_{\rm D}\boldsymbol{\tau}\beta^a\psi\right)\cdot\left(\overline{\psi}\Gamma^2_{\rm D}\boldsymbol{\tau}\beta^a\psi\right) &= -\tfrac{1}{6}\left(3[a_1a_2a_3]^{2,1}-[a_4a_5]^{2,1}_\tau\right)+\text{color }6.
\end{aligned} \tag{A.22}$$

Here the notation $[\ldots]^{2,1}$ indicates that the coefficients in (A.21) depend on the Dirac matrices $\Gamma^1_{\rm D}$ and $\Gamma^2_{\rm D}$. The cases of interest are, using Eq. (A.18),

$$\begin{aligned}
&\Gamma^1_{\rm D}=1,\ \Gamma^2_{\rm D}=1:\ [a_1a_2a_3]^{1,1}=[111];\ [a_4a_5]^{11}_\tau=[11],\\
&\Gamma^1_{\rm D}=\gamma^5,\ \Gamma^2_{\rm D}=\gamma^5:\ [a_1a_2a_3]^{\gamma_5,\gamma_5}=[11-1];\ [a_4a_5]^{\gamma_5,\gamma_5}_\tau=[-11],\\
&\Gamma^1_{\rm D}=\gamma_\mu\gamma_5,\ \Gamma^2_{\rm D}=\gamma^\mu\gamma^5:\ [a_1a_2a_3]^{\gamma_\mu\gamma_5,\gamma^\mu\gamma_5}=[-442];\ [a_4a_5]^{\gamma_\mu\gamma_5,\gamma^\mu\gamma_5}_\tau=[-20],\\
&\Gamma^1_{\rm D}=\gamma_\mu,\ \Gamma^2_{\rm D}=\gamma^\mu:\ [a_1a_2a_3]^{\gamma_\mu,\gamma^\mu}=[-44-2];\ [a_4a_5]^{\gamma_\mu,\gamma^\mu}_\tau=[20].
\end{aligned} \tag{A.23}$$

Using Eqs. (A.22), (A.23) one can rewrite identically a given $\mathcal{L}_{\rm I}$ into a form $\mathcal{L}_{I,qq}$, from which the interaction strengths in the various channels can be identified, see Eqs. (2.5), (2.6) for the $(0^+, T = 0)$ and $(1^+, T = 1)$ channels. Using $\mathcal{L}_{I,qq}$, exchange terms give a factor 4, but with every qq-loop there is associated a symmetry factor $\frac{1}{2}$.

Using Eqs. (A.9), (A.10) and (A.22), (A.23) one easily obtains the various coefficients listed in Table 1.

Appendix B. Three-body equations

B.1. Derivation of Eqs. (3.22) and (3.23)

Inserting (3.20) into (3.9) and sandwiching the resulting equation between ${}^{\beta}_{j}\langle p'_j, b|$ and $|p_i, a\rangle^{\alpha}_{i}$ we obtain

$$X^{\beta\alpha}_{ji,ba}(p'_j, p_i) = {}^{\beta}_{j}\langle p'_j, b|\bar{\delta}_{ijk}\, S_{\mathrm{F}k}|p_i, a\rangle^{\alpha}_{i} + \sum_{ll'} \int \frac{d^4\tilde{p}_l}{(2\pi)^4} \bar{\delta}_{jll'}\, {}^{\beta}_{j}\langle p'_j, b|S_{\mathrm{F}l} S_{\mathrm{F}l'}|\tilde{p}_l, c\rangle^{\beta'}_{l} \times \tau^{l}_{\mathrm{cd}}(\tfrac{1}{2}P - \tilde{p}_l) X^{\beta'\alpha}_{li,da}(\tilde{p}_l, p_i), \tag{B.1}$$

where we used the definition (3.21). The driving term in this equation is, using (3.18),

$$Z^{\beta\alpha}_{ji,ba}(p'_j, p_i) \equiv {}^{\beta}_{j}\langle p'_j, b|\bar{\delta}_{ijk}\, S_{\mathrm{F}k}|p_i, a\rangle^{\alpha}_{i} = \int \frac{d^4 q'_j}{(2\pi)^4} \int \frac{d^4 q_i}{(2\pi)^4} \overline{\Omega}^{\gamma'\delta'}_{b}(q'_j)\, {}^{\beta\gamma'\delta'}_{j}\langle q'_j p'_j|\bar{\delta}_{ijk}\, S_{\mathrm{F}k}|q_i p_i\rangle^{\alpha\gamma\delta}_{i}\, \Omega^{\gamma\delta}_{a}(q_i). \tag{B.2}$$

If (ijk) is an even permutation of (123), we use Eqs. (3.12) and (3.15) to write for the matrix element of $S_{\mathrm{F}k}$

$$\begin{aligned} {}^{\beta\gamma'\delta'}_{j}\langle q'_j p'_j|S_{\mathrm{F}k}|q_j p_j\rangle^{\alpha\gamma\delta}_{i} &= {}^{\beta\gamma'\delta'}_{j}\langle q'_j p'_j|S_{\mathrm{F}k}|q_j p_j\rangle^{\gamma\delta\alpha}_{j} \\ &= (2\pi)^8 \delta^{(4)}(q'_j - q_j)\, \delta^{(4)}(p'_j - p_j)\, \delta_{\beta\gamma}\, \delta_{\alpha\delta'} \\ &\quad \times S^{\gamma'\delta}_{\mathrm{F}k}(\tfrac{1}{4}P - q_i - \tfrac{1}{2}p_i), \end{aligned} \tag{B.3}$$

and (B.2) becomes

$$Z^{\beta\alpha}_{ji,ba}(p'_j, p_i) = \bar{\delta}_{ijk} \int d^4 q_i\, \overline{\Omega}^{\gamma'\alpha}_{b}(q_j) S^{\gamma'\delta}_{\mathrm{F}k}(\tfrac{1}{4}P - q_i - \tfrac{1}{2}p_i)\, \Omega^{\beta\delta}_{a}(q_i)\, \delta^{(4)}(p'_j - p_j). \tag{B.4}$$

We wish to express the r.h.s. of Eq. (B.4) in terms of p'_j and p_i. For this we express q_j and p_j by Eq. (3.14) (with the upper signs). The δ-function in (B.4) then gives $q_i = p'_j + \frac{1}{2}p_i + \frac{1}{4}P$, and for constant Ω's (NJL model) we get

$$Z^{\beta\alpha}_{ji,ba}(p'_i, p_i) = \bar{\delta}_{ijk}\, \Omega^{\beta\delta}_{a} S^{\gamma\delta}_{\mathrm{F}k}(-p_i - p'_j) \overline{\Omega}^{\gamma\alpha}_{b}, \tag{B.5}$$

which is Eq. (3.23). The same result holds also if (ijk) is an odd permutation of (123). In this case, the argument of S_{Fk} in (B.4) is $\frac{1}{4}P - q_j - \frac{1}{2}p_j$, and use of Eq. (3.14) with the lower signs gives again (B.5).

Consider now the matrix element of $S_{Fl}S_{Fl'}$ in Eq. (B.1). If (jll') is an even permutation of (123) we obtain

$$\bar{\delta}_{jll'}\,{}^{\beta}_{j}\langle p'_j b|S_{Fl}S_{Fl'}|\tilde{p}_l, c\rangle^{\beta'}_{l} = \bar{\delta}_{jll'}\int\frac{d^4q'_j}{(2\pi)^4}\int\frac{d^4\tilde{q}_l}{(2\pi)^4}\overline{\Omega}^{\gamma'\delta'}_{b}(q'_j) \times {}^{\beta\gamma'\delta'}_{\quad j}\langle q'_j p'_j|S_{Fl}S_{Fl'}|\tilde{q}_l\tilde{p}_l\rangle^{\beta'\gamma\delta}_{l}\,\Omega^{\gamma\delta}_{c}(\tilde{q}_l). \tag{B.6}$$

Since the matrix element of $S_{Fl}S_{Fl'}$ is

$$\begin{aligned}{}^{\beta\gamma'\delta'}_{\quad j}\langle q'_j p'_j|S_{Fl}S_{Fl'}|\tilde{q}_l\tilde{p}_l\rangle^{\beta'\gamma\delta}_{l} &= {}^{\beta\gamma'\delta'}_{\quad j}\langle q'_j p'_j|S_{Fl}S_{Fl'}|\tilde{q}_j\tilde{p}_j\rangle^{\delta\beta'\gamma}_{j} \\ &= (2\pi)^8\delta^{(4)}(q'_j - \tilde{q}_j)\,\delta^{(4)}(p'_j - \tilde{p}_j) \\ &\quad\times S^{\gamma'\beta'}_{Fl}(\tfrac{1}{4}P + \tilde{q}_j - \tfrac{1}{2}\tilde{p}_j)S^{\delta'\gamma}_{Fl'}(\tfrac{1}{4}P - \tilde{q}_j - \tfrac{1}{2}\tilde{p}_j)\delta_{\beta\delta},\end{aligned} \tag{B.7}$$

(B.6) becomes

$$\bar{\delta}_{jll'}\,{}^{\beta}_{j}\langle p'_j, b|S_{Fl}S_{Fl'}|\tilde{p}_l, c\rangle^{\beta'}_{l} = \bar{\delta}_{jll'}\int d^4\tilde{q}_l\,\delta^{(4)}(p'_j - \tilde{p}_j)\overline{\Omega}^{\gamma'\delta'}_{b}(\tilde{q}_j) \times S^{\gamma'\beta'}_{Fl}(\tfrac{1}{4}P + \tilde{q}_j - \tfrac{1}{2}\tilde{p}_j)S^{\delta'\gamma}_{Fl'}(\tfrac{1}{4}P - \tilde{q}_j - \tfrac{1}{2}\tilde{p}_j)\Omega^{\gamma\beta}_{c}(\tilde{q}_l). \tag{B.8}$$

Since we want to express this by p'_j and $\tilde{p}_l$, we write $(\tilde{q}_j, \tilde{p}_j)$ in terms of $(\tilde{q}_l, \tilde{p}_l)$ using Eq. (3.14) (with the lower signs), and then the δ-function in (B.8) gives $\tilde{q}_l = -p'_j - \frac{1}{2}\tilde{p}_l - \frac{1}{4}P$. For constant Ω's (B.8) becomes

$$\begin{aligned}\bar{\delta}_{jll'}\,{}^{\beta}_{j}\langle p'_j, b|S_{Fl}S_{Fl'}|\tilde{p}_l, c\rangle^{\beta'}_{l} &= \bar{\delta}_{jll'}\,\overline{\Omega}^{\gamma'\delta'}_{b}\,S^{\gamma'\beta'}_{Fl}(\tfrac{1}{2}P + \tilde{p}_l)S^{\delta'\gamma}_{Fl'}(-p'_j - \tilde{p}_l)\Omega^{\gamma\beta}_{c} \\ &= Z^{\beta\gamma'}_{jl,bc}(p'_j, \tilde{p}_l)S^{\gamma'\beta'}_{Fl}(\tfrac{1}{2}P + \tilde{p}_l),\end{aligned} \tag{B.9}$$

where in the last step we used (B.5). If (jll') is an odd permutation of (123), the r.h.s. of (B.9) is replaced by $Z^{\beta\gamma'}_{jl',bc}(p'_j, \tilde{p}_l)S_{Fl'}(\frac{1}{2}P + \tilde{p}_l)$. Using this in Eq. (B.1) we arrive at Eq. (3.22).

B.2. *Derivation of Eq. (3.30)*

For identical particles, the matrix element of (3.28) between the antisymmetric states (3.29) is, due to $X_{ii} = X_{11}$ for all i and $X_{ij} = X_{12}$ for all $i \neq j$

$${}^{\alpha'_1\alpha'_2\alpha'_3}_{\quad a}\langle k'_1k'_2k'_3|T|k_1k_2k_3\rangle^{\alpha_1\alpha_2\alpha_3}_{a}$$

$$
\begin{aligned}
&= 3\sum_{\alpha}\int\frac{d^4\tilde{p}}{(2\pi)^4}\,{}^{\alpha_1'\alpha_2'\alpha_3'}_{\;a}\langle k_1'k_2'k_3'|\tilde{p},a\rangle^{\alpha}_{1}\,{}^{\alpha}_{1}\langle\tilde{p},b|S_{F1}^{-1}|k_1k_2k_3\rangle^{\alpha_1\alpha_2\alpha_3}_{a}\tau_{ab}(\tfrac{1}{2}P-\tilde{p})\\
&+3\sum_{\alpha\beta}\int\frac{d^4\tilde{p}}{(2\pi)^4}\int\frac{d^4\tilde{\tilde{p}}}{(2\pi)^4}\,{}^{\alpha_1'\alpha_2'\alpha_3'}_{\;a}\langle k_1'k_2'k_3'|\tilde{p},a\rangle^{\alpha}_{1}\tau_{ab}(\tfrac{1}{2}P-\tilde{p})\\
&\times\{X^{\alpha\beta}_{11,bc}(\tilde{p},\tilde{\tilde{p}})\,{}^{\beta}_{1}\langle\tilde{\tilde{p}},d|k_1k_2k_3\rangle^{\alpha_1\alpha_2\alpha_3}_{a}\\
&+2X^{\alpha\beta}_{12,bc}(\tilde{p},\tilde{\tilde{p}})\,{}^{\beta}_{2}\langle\tilde{\tilde{p}},d|k_1k_2k_3\rangle^{\alpha_1\alpha_2\alpha_3}_{a}\}\tau_{cd}(\tfrac{1}{2}P-\tilde{\tilde{p}}).
\end{aligned}
\tag{B.10}
$$

Using (3.29) for $l=1$ and (3.18) we get for the antisymmetric vertex functions (see Eq. (3.17))

$$
{}^{\alpha_1'\alpha_2'\alpha_3'}_{\;a}\langle k_1'k_2'k_3'|\tilde{p},a\rangle^{\alpha}_{1}=\frac{2}{\sqrt{6}}(2\pi)^4\sum_i\delta^{(4)}(\tilde{p}-p_i')\,\delta_{\alpha,\alpha_i'}\,\Omega_a^{\alpha_j'\alpha_k'}(q_i'),
\tag{B.11}
$$

$$
{}^{\alpha}_{1}\langle\tilde{p},b|S_{F1}^{-1}|k_1k_2k_3\rangle^{\alpha_1\alpha_2\alpha_3}_{a}=\frac{2}{\sqrt{6}}\sum_l(2\pi)^4\delta^{(4)}(\tilde{p}-p_l)\,\overline{\Omega}_b^{\alpha_m\alpha_n}(q_l)\,S^{-1}_{F\alpha\alpha_l}(\tfrac{1}{2}P+p_l),
\tag{B.12}
$$

while for the symmetric vertex functions these expressions vanish as does the total three-body T-matrix (B.10). Here (ijk) and (lmn) are even permutation of (123). Using also Eq. (3.27), this gives the first term in (3.30). If we use $l=2$ in (3.29) we see that in the last term of (B.10) we can replace ${}^{\beta}_{2}\langle\tilde{\tilde{p}},d|$ by ${}^{\beta}_{1}\langle\tilde{\tilde{p}},d|$, and therefore the term in the braces in (B.10) involves the quantity $X_{11}+2X_{12}=2X$, see Eq. (3.24). If we use the above relations (B.11), (B.12) (without the factor S_{F1}^{-1}) and replace τ with $\overline{\tau}$ using Eq. (3.27), we obtain the second term in (3.30).

Appendix C. Rotational and parity invariance

In Section 4 of the main text, the Faddeev kernel $F^{\alpha'\beta'}_{a'b'}$ in Eqs. (4.15) or (4.18) is assumed to be given in the spherical diquark basis. That is, any matrix $M_{a'a}$ with spherical diquark indices $a,b,c=(5,0,3,+1,-1)$ is related to the representation with cartesian indices $l,m,n=(5,0,3,1,2)\equiv(5,\mu)$ by

$$
M_{a'a}=\hat{A}^{+}_{a'l'}M_{l'l}\hat{A}_{la}
\tag{C.1}
$$

with $\hat{A}_{ia}=\mathrm{diag}(1,1,1,A_{ia})$, $A_{ia}=\sqrt{\frac{1}{2}}\begin{pmatrix}-1&1\\-i&-i\end{pmatrix}$.

C.1. Derivation of Eqs. (4.18) and (4.19)

To show Eq. (4.18), we will derive the relations

$$
\widehat{S}^{-1}(\omega)\widehat{R}^{-1}(\omega)Z(p',p)\widehat{S}(\omega)\widehat{R}(\omega)=Z(\tilde{p}',\tilde{p}),
\tag{C.2}
$$

$$
\widehat{S}^{-1}(\omega)\widehat{R}^{-1}(\omega)G(p)\widehat{S}(\omega)\widehat{R}(\omega)=G(\tilde{p})
\tag{C.3}
$$

in the cartesian representation with $\tilde{p} = (p_0, \tilde{\boldsymbol{p}})$, $\tilde{p}' = (p_0', \tilde{\boldsymbol{p}}')$ and $\tilde{\boldsymbol{p}}$, $\tilde{\boldsymbol{p}}'$ defined in Eq. (4.21). Since the kernel F has the form Eq. (4.10), Eq. (4.18) follows from the above relations after transforming to the spherical representation by using Eq. (C.1). Z and G are written as (see Eqs. (4.8), (4.11) and (3.35))

$$Z_{l'l}^{\alpha'\alpha}(p',p) = -3 \begin{bmatrix} \gamma_5 S_F(p'+p)\gamma_5 & \sqrt{3}\gamma_0 S_F(p'+p)\gamma_5 & \sqrt{3}\gamma_i S_F(p'+p)\gamma_5 \\ \sqrt{3}\gamma_5 S_F(p'+p)\gamma_0 & -\gamma_0 S_F(p'+p)\gamma_0 & -\gamma_i S_F(p'+p)\gamma_0 \\ \sqrt{3}\gamma_5 S_F(p'+p)\gamma_{i'} & -\gamma_0 S_F(p'+p)\gamma_{i'} & -\gamma_i S_F(p'+p)\gamma_{i'} \end{bmatrix}_{\alpha'\alpha} \tag{C.4}$$

$$\begin{aligned} G_{l'l}^{\alpha'\alpha}(p) &= S_F^{\alpha'\alpha}(\tfrac{1}{2}P+p) \begin{bmatrix} \overline{\tau}_{55}(\tfrac{1}{2}P-p) & 0 & 0 \\ 0 & \overline{\tau}_{00}(\tfrac{1}{2}P-p) & \overline{\tau}_{0i}(\tfrac{1}{2}P-p) \\ 0 & \overline{\tau}_{i'0}(\tfrac{1}{2}P-p) & \overline{\tau}_{i'i}(\tfrac{1}{2}P-p) \end{bmatrix} \\ &\equiv S_F^{\alpha'\alpha}(\tfrac{1}{2}P+p)\overline{\tau}_{l'l}(\tfrac{1}{2}P-p), \end{aligned} \tag{C.5}$$

with $\overline{\tau}^{55} = \overline{\tau}_s$ and $\overline{\tau}^{\mu\nu}$ given by (2.18), (2.22). Eq. (C.4) refers to the $T = \frac{1}{2}$ channel.

Using the block form of $\widehat{S}$ and $\widehat{R}$, i.e; $\widehat{S} = \mathrm{diag}(S,S)$ and $\widehat{R} = \mathrm{diag}(1,1,R)$ as explained below Eq. (4.13), and the relations

$$\widehat{S}^{-1}(\omega)\gamma^l\widehat{S}(\omega) = \widehat{R}^l{}_m(\omega)\gamma^m \Rightarrow \widehat{S}^{-1}(\omega)S_F(\tfrac{1}{2}P+p)\widehat{S}(\omega) = S_F(\tfrac{1}{2}P+\tilde{p}), \tag{C.6}$$

$$p^m\widehat{R}_m{}^l(\omega) = \tilde{p}^l, \tag{C.7}$$

we easily obtain (C.3). To show also Eq. (C.2), we note that the l.h.s. of (C.2) is obtained by multiplying to each γ_i in (C.4) a factor $R^i{}_j(\omega)$, to each $\gamma_{i'}$ a factor $\left(R^{-1}(\omega)\right)^{i'}{}_{j'}$, and set the whole expression between $\widehat{S}^{-1}(\omega)$ and $\widehat{S}(\omega)$. Let us consider for example the term resulting from $\gamma_i S_F(p'+p)\gamma_0$ in (C.4). It becomes

$$\begin{aligned} \widehat{S}^{-1}(\omega)\gamma_i R^i{}_j(\omega)S_F(p'+p)\gamma_0\widehat{S}(\omega) &= \gamma_j S^{-1}(\omega)S_F(p'+p)\widehat{S}(\omega)\gamma_0 \\ &= \gamma_j S_F(\tilde{p}'+\tilde{p})\gamma_0, \end{aligned} \tag{C.8}$$

where we used (C.6). Similar manipulations can be performed for the other terms, and we arrive at (C.2). An analogous discussion holds for the quark exchange kernel in the $T = \frac{3}{2}$ channel, see Eq. (4.9).

To derive Eq. (4.19), we show the relations

$$\mathcal{P}Z(p',p)\mathcal{P} = Z((-p'),(-p)), \tag{C.9}$$

$$\mathcal{P}G(p)\mathcal{P} = G((-p)) \tag{C.10}$$

with $(-p)^\mu \equiv (p_0, -\boldsymbol{p})$ and $(-p')^\mu \equiv (p_0', -\boldsymbol{p}')$. To obtain (C.9) we note that the l.h.s. of this equation is obtained by multiplying a factor (-1) to each component of (C.4) which has a single γ_0, which gives the factor $\eta_a\eta_b$ in (4.19), and by multiplying each component of (C.4) by γ_0 from left and right, which gives the factor $\eta_\alpha\eta_\beta$ in Eq. (4.19). (The intrinsic parity factors η_α and η_a are defined in Eq. (4.22).) Using $\gamma_0 S_F(p'+p)\gamma_0 = S_F((-p')+(-p))$, one gets (C.9). Similarly, the l.h.s. of (C.10) is obtained from (C.5) by multiplying $S_F(\frac{1}{2}P+p)$ by γ_0 from left and right, and by multiplying a factor (-1) to $\overline{\tau}_{0i}$ and $\overline{\tau}_{i'0}$. Using $\gamma_0 S_F(\frac{1}{2}P+p)\gamma_0 = S_F(\frac{1}{2}P+(-p))$ in the system where $\boldsymbol{P}=0$, and the form of $\overline{\tau}^{\mu\nu}$, Eq. (2.22), one gets (C.10).

Similar arguments hold for the quark exchange kernel in the $T=\frac{3}{2}$ channel, see Eq. (4.9).

C.2. Derivation of Eqs. (4.28) and (4.31)

To show Eq. (4.28), we use (4.14) and (4.16) to obtain the relation between the wave function in the original representation and in the JM-representation as

$$\psi^a_\alpha(p) = \mathcal{N}_J \sum_{JM}\sum_{\alpha' a'} \widehat{\mathcal{D}}^{*1/2}_{s_\alpha s_{\alpha'}}(\omega)\widehat{\mathcal{D}}^{*1}_{\lambda_a\lambda_{a'}}(\omega) D^J_{M\, s_{\alpha'}+\lambda_{a'}}(\omega)\psi^{JM}_{s_{\alpha'}\lambda_{a'}}(p_0,\bar{p}). \tag{C.11}$$

Here we expressed the rotational matrices in terms of Wigner D-functions according to $\widehat{S}_{\alpha'\alpha} = \widehat{D}^{*1/2}_{s_{\alpha'}s_\alpha}$ and $\widehat{R}^{a'}{}_a = \widehat{D}^{*1}_{\lambda_{a'}\lambda_a}$, and the $\widehat{D}$'s have the same block structure as $\widehat{S}$ and $\widehat{R}$, i.e., $\widehat{D}^{1/2} = \mathrm{diag}(D^{1/2}, D^{1/2})$, $\widehat{D}^1 = \mathrm{diag}(1,1,D^1)$ with D^j the usual D-functions. We apply the parity operator to (C.11), use Eq. (4.23) and the relation

$$D^j_{mm'}(-\omega) = (-1)^{j-m'} D^j_{m-m'}(\omega) \tag{C.12}$$

to rewrite the result again in the form of Eq. (C.11). This gives

$$\begin{aligned}\mathcal{P}\psi^a_\alpha(p) &= \mathcal{N}_J \sum_{JM}\sum_{\alpha' a'} \eta_\alpha\eta_a(-1)^{J-s-j_a}\widehat{D}^{*1/2}_{s_\alpha s_{\alpha'}}(\omega)\widehat{D}^{*1}_{\lambda_a\lambda_{a'}}(\omega)\\ &\quad\times D^J_{M\, s_{\alpha'}+\lambda_{a'}}(\omega)\psi^{JM}_{s_{\bar{\alpha}'}\lambda_{\bar{a}'}}(p_0,\bar{p})\\ &= \mathcal{N}_J \sum_{JM}\sum_{\alpha' a'} \widehat{D}^{*1/2}_{s_\alpha s_{\alpha'}}(\omega)\widehat{D}^{*1}_{\lambda_a\lambda_{a'}}(\omega) D^J_{M\, s_{\alpha'}+\lambda_{a'}}(\omega)\\ &\quad\times \eta_{\alpha'}\eta_{a'}(-1)^{J-s-j_{a'}}\psi^{JM}_{s_{\bar{\alpha}'}\lambda_{\bar{a}'}}(p_0,\bar{p}).\end{aligned} \tag{C.13}$$

The notations used here are explained below Eq. (4.29), and we used $\eta_\alpha = \eta_{\alpha'}$, $\eta_a = \eta_{a'}$, $j_a = j_{a'}$ due to the block structure of the $\widehat{D}$'s. Eq. (C.13) is just the relation (C.11) between the parity transformed wave functions, and we arrive at Eq. (4.28).

To show Eq. (4.31), we note that the kernel in the JM representation is related to the one in the original representation by Eq. (4.17) and (4.15). Rewriting the rotational matrices again in terms of $\widehat{D}$'s as above, this relation becomes

$$\begin{aligned}F^{J'M',JM}_{s_{\alpha'}\lambda_{a'},s_\alpha\lambda_a} &= \mathcal{N}_J\mathcal{N}_{J'}\int \mathrm{d}\omega \int \mathrm{d}\omega' D^{*J'}_{M'\, s_{\alpha'}+\lambda_{a'}}(\omega')\widehat{D}^{1/2}_{s_{\beta'}s_{\alpha'}}(\omega')\widehat{D}^1_{\lambda_{b'}\lambda_{a'}}(\omega')\\ &\quad\times F^{\beta'\beta}_{b'b}(p',p)\widehat{D}^{*1/2}_{s_\beta s_\alpha}(\omega)\widehat{D}^{*1}_{\lambda_b\lambda_a}(\omega)D^J_{M\, s_\alpha+\lambda_a}(\omega).\end{aligned} \tag{C.14}$$

We now apply the parity operation (4.30) to Eq. (C.14), shift $\omega \to -\omega$, $\omega' \to -\omega'$ and use Eq. (C.12). Then we note that $F_{\alpha' a'} = F_{\beta' b'}$ and $F_{\alpha a} = F_{\beta b}$ due to the block structure of the $\widehat{D}$'s. If we use then Eq. (4.19) for the parity transformation in the original representation we arrive immediately at Eq. (4.31).

Appendix D. Computation of the Faddeev kernel

In this appendix we sketch the method to compute the Faddeev kernel (4.27). We first derive an expression for the r.h.s. of this equation in the cartesian representation for the diquark basis (indices $l, m, n = (5,0,3,1,2) \equiv (5,\mu)$) and then transform to the spherical representation (indices $a, b, c = (5,0,3,+1,-1)$) using Eq. (C.1). We will refer to the kernel in the $T = \frac{1}{2}$ channel, but the same methods can be used also for the $T = \frac{3}{2}$ channel. First we note from Eqs. (4.8) and (4.10) that the kernel $F_{l'l}^{\alpha'\alpha}$ can be written as

$$F_{l'l}^{\alpha'\alpha}(p',p) = -3c_{l'l}\left[\gamma_n S_{\mathrm{F}}(p+p')\gamma_{l'} S_{\mathrm{F}}(\tfrac{1}{2}P+p)\right]_{\alpha'\alpha} \bar{\tau}^{n}{}_{l}(\tfrac{1}{2}P-p), \tag{D.1}$$

where $c_{l'l} = 1$ if $(l,l') = (5,5)$; -1 if $(l,l') = (\mu,\mu')$; $\sqrt{3}$ if $(l,l') = (\mu,5)$ or $(5,\mu')$, and zero otherwise. We take $\boldsymbol{p}$ to lie on the z-axis (see Eq. (4.27), we omit the tilde here), and therefore the only non-vanishing elements of the two-body propagator $\bar{\tau}^{nl}$ are $\bar{\tau}^{55}$, $\bar{\tau}^{00}$, $\bar{\tau}^{30} = \bar{\tau}^{03}$, $\bar{\tau}^{33}$ and $\bar{\tau}^{11} = \bar{\tau}^{22}$. Their explicit expressions are easily obtained from Eqs. (2.18) and (2.22). Then the quantity on the r.h.s. of Eq. (4.27) can be written as follows (To simplify the notation, we will omit the Dirac indices in the following and indicate only the diquark indices.):

$$\begin{aligned}\widehat{S}^{-1}(\omega')\widehat{R}^{-1}(\omega')_{l'}{}^{m}F_{ml}(p',p) = {}&-3c_{l'l}\widehat{S}^{-1}(\omega')\widehat{R}^{-1}(\omega')_{l'}{}^{m}\\ &\times\left[\gamma_n S_{\mathrm{F}}(p+p')\gamma_m S_{\mathrm{F}}(\tfrac{1}{2}P+p)\right]\bar{\tau}^{n}{}_{l}(\tfrac{1}{2}P-p),\end{aligned} \tag{D.2}$$

where ω' is the direction of $\boldsymbol{p}'$. Here we used the fact that c_{ml} can be replaced by $c_{l'l}$ due to the block structure of $\widehat{R}$. We write

$$\begin{aligned}\gamma_n S_{\mathrm{F}}(p+p')\gamma_m = {}&\frac{1}{(p+p')^2-M^2}\\ &\times\left[\gamma_m\gamma_n\not{p} - 2g_{nm}\not{p} + 2\gamma_n p_m + \not{p}'\gamma_m\gamma_n - 2\not{p}'g_{nm} + 2p'_n\gamma_m - M\gamma_m\gamma_n + 2Mg_{nm}\right],\end{aligned} \tag{D.3}$$

where $g^{mn} = \mathrm{diag}(g^{55}, g^{\mu\nu})$ with $g^{55} = 1$, and $p^5 = 0$. Into this expression we insert

$$p'^n = \widehat{R}(\omega')^{n}{}_{m}\tilde{p}'^m = \tilde{p}'^m\widehat{R}^{-1}(\omega')_{m}{}^{n}, \tag{D.4}$$

where $\tilde{\boldsymbol{p}}'$ points along the z-axis. After this we use relation (C.6) in the form

$$\widehat{S}^{-1}(\omega')\widehat{R}^{-1}(\omega')_{l}{}^{l'}\gamma_{l'} = \gamma_l\widehat{S}^{-1}(\omega') \tag{D.5}$$

to arrive at

$$\begin{aligned}&\widehat{S}^{-1}(\omega')\widehat{R}^{-1}(\omega')_{l'}{}^{m}F_{ml}(p',p)\\ &= -3c_{l'l}\frac{1}{(p+p')^2-M^2}\left[\gamma_{l'}\widehat{S}^{-1}(\omega')\gamma_n\not{p} + \tilde{\not{p}}'\gamma_{l'}\widehat{S}^{-1}(\omega')\gamma_n\right.\\ &\quad -M\gamma_{l'}\widehat{S}^{-1}(\omega')\gamma_n - 2\widehat{S}^{-1}(\omega')\widehat{R}^{-1}(\omega')_{l'n}\not{p}\end{aligned}$$

$$
\begin{aligned}
&-2\tilde{\not p}'\widehat{S}^{-1}(\omega')\widehat{R}^{-1}(\omega')_{l'n}+2M\widehat{S}^{-1}(\omega')\widehat{R}^{-1}(\omega')_{l'n}\\
&+2\widehat{S}^{-1}(\omega')\widehat{R}^{-1}(\omega')_{l'}{}^{m}p_m\gamma_n+2\gamma_{l'}\widehat{S}^{-1}(\omega')\tilde{p}'_m\widehat{R}^{-1}(\omega')^{m}{}_{n}]\\
&\times S_{\mathrm{F}}\left(\tfrac{1}{2}P+p\right)\overline{\tau}^{n}{}_{l}(\tfrac{1}{2}P-p).
\end{aligned}
\tag{D.6}
$$

Using Eq. (C.1) to transform to the spherical representation we get for the Faddeev kernel (4.27)

$$
\begin{aligned}
F^{s_{\alpha'}s_\alpha}_{J\lambda_{a'}\lambda_a}(p_0'\bar{p}',p_0\bar{p}) = &-3c_{a'a}\int \mathrm{d}\omega'\, D^{*J}_{s_\alpha+\lambda_a,s_{\alpha'}+\lambda_{a'}}(\omega')\frac{1}{(p+p')^2-M^2}\\
&\times\{[\gamma^*_{a'}\widehat{S}^{-1}(\omega')\gamma^b\not p+\tilde{\not p}'\gamma^*_{a'}\widehat{S}^{-1}(\omega')\gamma^b\\
&-M\gamma^*_{a'}\widehat{S}^{-1}(\omega')\gamma^b-2\widehat{S}^{-1}(\omega')\widehat{R}^{-1}(\omega')_{a'}{}^{b}\not p\\
&-2\tilde{\not p}'\widehat{S}^{-1}(\omega')\widehat{R}^{-1}(\omega')_{a'}{}^{b}+2M\widehat{S}^{-1}(\omega')\widehat{R}^{-1}(\omega')_{a'}{}^{b}\\
&+2\widehat{S}^{-1}(\omega')\widehat{R}^{-1}(\omega')_{a'}{}^{c}p_c\gamma^b+2\gamma^*_{a'}\widehat{S}^{-1}(\omega')\tilde{p}'^{c}\widehat{R}^{-1}(\omega')_{c}{}^{b}]\\
&\times S_{\mathrm{F}}\left(\tfrac{1}{2}P+p\right)\}_{\alpha'\alpha}\overline{\tau}_{ba}(\tfrac{1}{2}P-p).
\end{aligned}
\tag{D.7}
$$

Here the star $*$ means complex conjugation only with respect to the explicit i factors in the expressions for the spherical vector components $a_{+1}=-\sqrt{\frac{1}{2}}(a_1+ia_2)$ and $a_{-1}=\sqrt{\frac{1}{2}}(a_1-ia_2)$.

One now separates the dependence on the Euler angles ϕ' and ψ' by writing the rotational matrices in terms of D-functions as done in Eq. (C.11), i.e;

$$
\begin{aligned}
\widehat{S}^{-1}(\omega')_{\alpha'\alpha}&=\mathrm{e}^{is_{\alpha'}\psi'}\,\hat{d}^{1/2}(-\theta')_{s_{\alpha'}s_\alpha}\,\mathrm{e}^{is_\alpha\phi'},\\
\widehat{R}^{-1}(\omega')^{a'}{}_{a}&=\mathrm{e}^{i\lambda_{a'}\psi'}\,\hat{d}^{1}(-\theta')_{\lambda_{a'}\lambda_a}\,\mathrm{e}^{i\lambda_a\phi'},
\end{aligned}
\tag{D.8}
$$

where $\hat{d}^{1/2}=\mathrm{diag}(d^{1/2},d^{1/2})$ and $\hat{d}^1=\mathrm{diag}(1,1,1,d^1)$ with d^j the usual d-functions. (Note that for $\hat{d}^1$ we do not distinguish between upper and lower indices.) Using the facts that (i) $(\gamma^a)_{\alpha\beta}$ is non-zero only if $\lambda_a+s_\beta=s_\alpha$ and $(\gamma^a)^*_{\alpha\beta}$ is non-zero only if $-\lambda_a+s_\beta=s_\alpha$, (ii) Dirac matrices like $\not p$, $\tilde{\not p}'$, $S_{\mathrm{F}}(\frac{1}{2}P+p)$ or products of these are diagonal in the helicity indices since the vectors $\boldsymbol{p}$ and $\tilde{\boldsymbol{p}}'$ point along the z-axis and $\boldsymbol{P}=0$, and (iii) $\overline{\tau}_{ba}$ is non-zero only if $\lambda_b=\lambda_a$, it is easy to check that all phases involving the angles ψ or ϕ cancel in Eq. (D.7). Therefore, Eq. (D.7) reduces to

$$
\begin{aligned}
F^{s_{\alpha'}s_\alpha}_{J\lambda_{a'}\lambda_a}(p_0'\bar{p}',p_0\bar{p}) = &-3c_{a'a}(2\pi)^2\int_0^\pi \sin\theta\,\mathrm{d}\theta\, d^J_{s_\alpha+\lambda_a,s_{\alpha'}+\lambda_{a'}}(\theta)\\
&\times\frac{1}{(p_0+p_0')^2-(\bar{p}^2+\bar{p}'^2+2\bar{p}\bar{p}'\cos\theta)-M^2}\\
&\times\{[\gamma^*_{a'}\hat{d}^{1/2}(-\theta)\gamma^b\not p+\tilde{\not p}'\gamma^*_{a'}\hat{d}^{1/2}(-\theta)\gamma^b\\
&-M\gamma^*_{a'}\hat{d}^{1/2}(-\theta)\gamma^b-2\hat{d}^{1/2}(-\theta)\hat{d}^1(-\theta)_{\lambda_{a'}\lambda_b}\not p\\
&-2\tilde{\not p}'\hat{d}^{1/2}(-\theta)\hat{d}^1(-\theta)_{\lambda_{a'}\lambda_b}+2M\hat{d}^{1/2}(-\theta)\hat{d}^1(-\theta)_{\lambda_{a'}\lambda_b}
\end{aligned}
$$

$$+2\hat{d}^{1/2}(-\theta)\hat{d}^{1}(-\theta)_{\lambda_{a'}\lambda_c}p_c\gamma^b + 2\gamma^*_{a'}\hat{d}^{1/2}(-\theta)\tilde{p}'^c\hat{d}^1(-\theta)_{\lambda_c\lambda_b}]$$
$$\times S_F\left(\tfrac{1}{2}P + p\right)\}_{\alpha'\alpha}\overline{\tau}_{ba}(\tfrac{1}{2}P - p). \tag{D.9}$$

From the form of the $d^{1/2}$ functions it follows that the Dirac matrix $\hat{d}^{1/2}$ can be written as follows:

$$\hat{d}^{1/2}(\theta) = \cos\tfrac{1}{2}\theta - i\gamma_5\gamma_0\gamma_2\sin\tfrac{1}{2}\theta. \tag{D.10}$$

Using this relation in Eq. (D.9) the Dirac matrix structure can be made explicit. Finally, the integral over θ can also be performed analytically, and the explicit evaluation of the required 10×10 matrix elements of the kernel (see the discussion below Eq. (4.33)) is straightforward.

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