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Isoscalar Spin Matrix Elements in *s-d* Shell Nuclei

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The quenching of isovector spin matrix elements in *s-d* shell nuclei is well established experimentally as well as theoretically [1,2,3]. The isoscalar spin gyromagnetic ratios g_s^{IS} of nuclei with one nucleon or hole outside of *LS* closed shells are also quenched by the same mechanism. On the other hand, their isoscalar orbital gyromagnetic ratios g_l^{IS} are slightly enhanced by meson exchange currents [1,2]. Then we are interested very much in the following question: Are the isoscalar spin matrix elements generally quenched in *s-d* shell nuclei? We will try to answer this question in this paper.

KEYWORDS: Isoscalar spin matrix element, *s-d* shell nuclei

1. Quenching Phenomena in Nuclear Physics

Gamow-Teller transitions of nuclei with *LS* closed shell $\pm$ one nucleon have been studied very well by both observations and theoretical calculations [1,2], see Fig. 1.

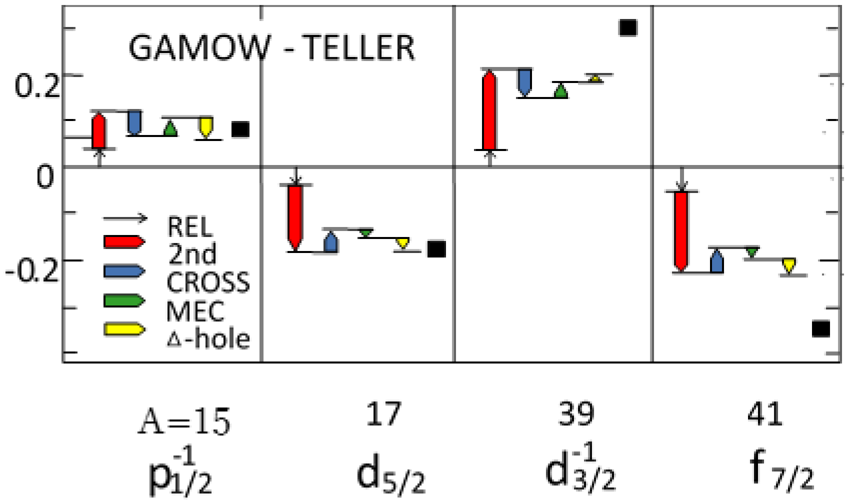

Fig. 1. Gamow-Teller matrix elements in *A*=15, 17, 39, 41 nuclei, from Ref. [2].

In order to explain those GT transitions, we need to take into account relativistic effects, 2nd order configuration mixing effects, meson exchange currents and so on. As we can see in Fig. 1, the most important correction comes from the second order configuration mixing of $2\hbar\omega$ and higher excitations.

The Δ-hole effect was believed to be the main explanation of the quenching of those transitions during many years between 1970 and 1990. However, Wakasa, Sakai and collaborators [3] observed the GT transition strength in the $^{90}\mathrm{Zr}(p,n)$ reaction up to 70 MeV excitation energy (Fig. 2).

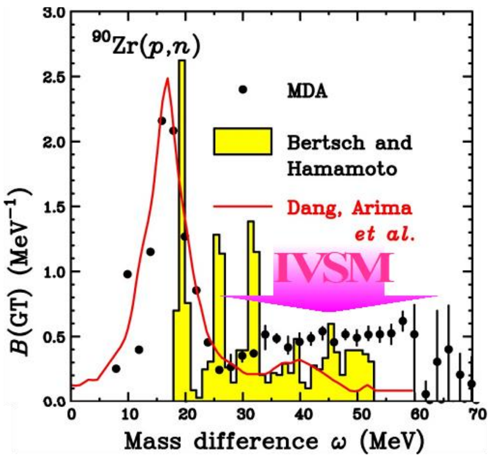

Fig. 2. GT strength observed in the $^{90}\mathrm{Zr}(p,n)$ reaction, in comparison to theoretical calculations.

As Bertsch and Hamamoto [4] have predicted, the GT strengths are spread over excited states with $2\hbar\omega$ or higher excitation energies. This possibility was already suggested by Shimizu, Ichimura and Arima [5], see Ref. [1] for detailed discussions. Dang and collaborators [6] calculated the GT transition in the $^{90}\mathrm{Zr}(p,n)$ reaction by taking into account the higher excited states, as shown in Fig. 2. We can see from this figure that those theoretical calculations [4,6] can explain the observations of Wakasa et al. [3] very well. We should be aware of the fact that the Δ-hole excitation does not seem to be a main cause of quenching of GT transitions.

2. Isoscalar (IS) magnetic moments (μ^{IS}) of T=1/2 mirror s-d shell nuclei

Let us define the "observed spin" $\langle S\rangle_{\mathrm{obs}}$ by the observed IS magnetic moments as follows:

$$\mu_{\mathrm{obs}}^{\mathrm{IS}} = \frac{1}{2}\left(\mu_{\mathrm{obs}}(Z,N) + \mu_{\mathrm{obs}}(N,Z)\right)$$
$$\equiv 0.38\,\langle S\rangle_{\mathrm{obs}} + 0.5\,J\,. \qquad (1)$$

Remember that the IS magnetic moment operator is expressed as

$$\mu^{\mathrm{IS}} = 0.88\,S + 0.5\,L = 0.38\,S + 0.5\,J\,,$$

where S and L are the spin and orbital angular momentum operators, and $J = L + S$ is the total angular momentum operator. (Here μ, S, L, J refer to the z–components of the corresponding vectors.) Further we define the "free spin" $\langle S\rangle_{\mathrm{free}}$ by the magnetic moments which are calculated with free *g*-factors in the $0\hbar\omega$ (*s*-*d*) shell model:

$$\mu^{\mathrm{IS}}_{\mathrm{free}} \equiv 0.38\,\langle S\rangle_{\mathrm{free}} + 0.5\,J\,. \tag{2}$$

Table I. Magnetic moments and expectation values of the spin operator in odd-*A* *s*-*d* shell nuclei.

Nuclei	J	magn. moments	$\mu^{\mathrm{IS}}_{\mathrm{obs}}$	$\langle S\rangle_{\mathrm{obs}}$	$\langle S\rangle_{\mathrm{free}}$
$({}^{17}_{8}\mathrm{O}_9\,,\,{}^{17}_{9}\mathrm{F}_8)$	5/2	$(-1.894, 4.721)$	1.414	0.431	0.500
$({}^{19}_{9}\mathrm{F}_{10}\,,\,{}^{19}_{10}\mathrm{Ne}_9)$	1/2	$(2.629, -1.885)$	0.372	0.320	0.466
$({}^{21}_{10}\mathrm{Ne}_{11}\,,\,{}^{21}_{11}\mathrm{Na}_{10})$	3/2	$(-0.662, 2.386)$	0.862	0.295	0.303
$({}^{23}_{11}\mathrm{Na}_{12}\,,\,{}^{23}_{12}\mathrm{Mg}_{11})$	3/2	$(2.218, -0.536)$	0.841	0.238	0.267
$({}^{25}_{12}\mathrm{Mg}_{13}\,,\,{}^{25}_{13}\mathrm{Al}_{12})$	5/2	$(-0.856, 3.646)$	1.395	0.382	0.399
$({}^{27}_{13}\mathrm{Al}_{14}\,,\,{}^{27}_{14}\mathrm{Si}_{13})$	5/2	$(3.642, -0.865)$	1.388	0.364	0.373
$({}^{29}_{14}\mathrm{Si}_{15}\,,\,{}^{29}_{15}\mathrm{P}_{14})$	1/2	$(-0.555, 1.235)$	0.340	0.236	0.131
$({}^{31}_{15}\mathrm{P}_{16}\,,\,{}^{31}_{16}\mathrm{S}_{15})$	1/2	$(1.132, -0.488)$	0.322	0.189	0.161
$({}^{33}_{16}\mathrm{S}_{17}\,,\,{}^{33}_{17}\mathrm{Cl}_{16})$	3/2	$(0.644, 0.752)$	0.698	-0.137	-0.192
$({}^{35}_{17}\mathrm{Cl}_{18}\,,\,{}^{35}_{18}\mathrm{Ar}_{17})$	3/2	$(0.822, 0.632)$	0.727	-0.060	-0.180
$({}^{37}_{18}\mathrm{Ar}_{19}\,,\,{}^{37}_{19}\mathrm{K}_{18})$	3/2	$(1.145, 0.203)$	0.674	-0.200	-0.323
$({}^{39}_{19}\mathrm{K}_{20}\,,\,{}^{39}_{20}\mathrm{Ca}_{19})$	3/2	$(0.392, 1.022)$	0.707	-0.114	-0.300

Table I and Fig. 3 show the observed spin $\langle S\rangle_{\mathrm{obs}}$ and free spin $\langle S\rangle_{\mathrm{free}}$ in *s*-*d* shell nuclei, where $\langle S\rangle_{\mathrm{obs}}$ are obtained from Eq. (1) by using the experimental magnetic moments compiled by Stone [7], and $\langle S\rangle_{\mathrm{free}}$ are calculated by Brown and Wildenthal [8] in the *s*-*d* shell model assuming free *g*-factors.

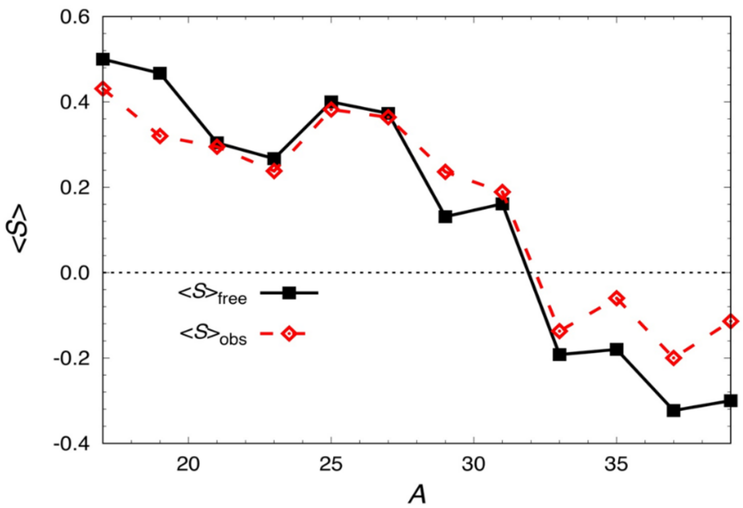

Fig. 3. Spin expectation values in *s*-*d* shell nuclei

As we see from Fig.3, the quenching of $\langle S \rangle$ is clearly seen only at the beginning and the end of the shell, but not in the middle of the shell. One of the reasons for this is that the effect of configuration mixings inside the *s*-*d* shell is a dominant mechanism in the middle of the shell. Let us look at $\langle S \rangle_{\mathrm{free}}$ and $\langle S \rangle_{\mathrm{obs}}$ of ${}^{27}_{13}\mathrm{Al}_{14}$ and ${}^{27}_{14}\mathrm{Si}_{13}$ as an example: $\langle S \rangle_{\mathrm{free}} = 0.373$ is very close to $\langle S \rangle_{\mathrm{obs}} = 0.364$. For a pure configuration $(d_{5/2})^5$ we would have $\langle S \rangle_{\mathrm{free}} = 0.5$. Therefore, the spin matrix element is greatly reduced by the configuration mixings inside the *s*-*d* shell, as illustrated in Fig. 4.

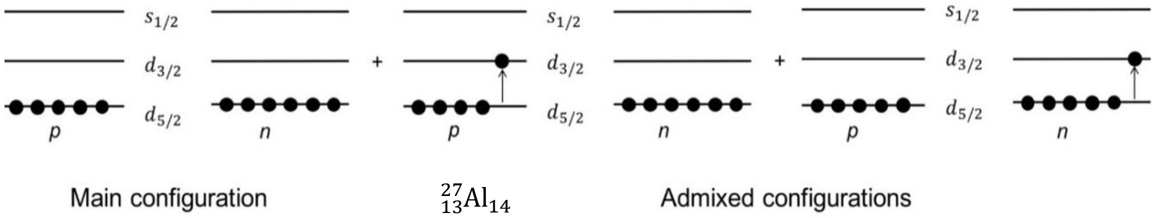

Fig. 4. Configuration mixings inside the *s*-*d* shell for ${}^{27}_{13}\mathrm{Al}_{14}$

This effect is analogous to the core polarization mechanism, and is one of the main reasons to reduce $\langle S \rangle_{\mathrm{free}}$ in the middle of the *s*-*d* shell. Nevertheless, we still have a question about the results shown in Fig.3: Why is $|\langle S \rangle_{\mathrm{obs}}|$ smaller than $|\langle S \rangle_{\mathrm{free}}|$ at the shell edges (*A*=17 and *A*=39) but not in the middle of the shell? We shall come back to this question shortly.

We define the "effective spin" $\langle S \rangle_{\mathrm{eff}}$ by a global fit to the data, such that

$$\langle S \rangle_{\mathrm{eff}} \equiv (1+\delta_s)\langle S \rangle_{\mathrm{free}} + \delta_l \langle L \rangle_{\mathrm{free}} + \text{tensor terms} \tag{3}$$

reproduces $\langle S \rangle_{\mathrm{obs}}$ as good as possible for all *s*-*d* shell nuclei. Here δ_s and δ_l are fit parameters. For a good fit $\langle S \rangle_{\mathrm{eff}} \approx \langle S \rangle_{\mathrm{obs}}$ as shown in Fig. 5.

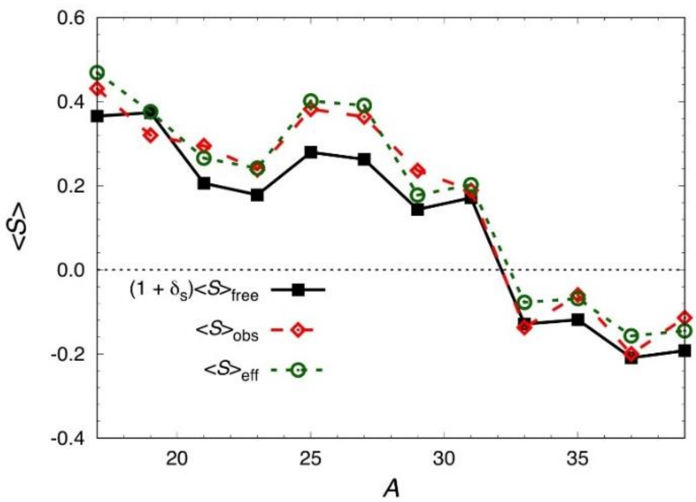

Fig. 5. Spin matrix elements in the *s*-*d* shell. The values of $\langle S \rangle_{\mathrm{eff}}$ correspond the fit given in the sixth line of Table IV of Brown and Wildenthal [8].

Assuming that the tensor terms in Eq. (3) are small, we can express the effective spin operator as

$$S_{\mathrm{eff}} = (1+\delta_s)S + \delta_l L = (1+\delta_s-\delta_l)S + \delta_l J\,. \tag{4}$$

All fits and calculations have shown that $\delta_s < 0$. One of the reasons for this is the depletion of the single particle probabilities by the tensor correlations [5] as shown in Fig. 6. It gives rise to a quenching of the IS spin *g*-factor:

$$g_s^{\mathrm{IS}} = 0.88 + 0.38\delta_s < 0.88\,. \tag{5}$$

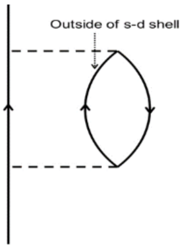

Fig. 6. 2nd order configuration mixing [5] (tensor correlations) outside of *s*-*d* shell.

A similar argument applied to $L_{\mathrm{eff}} \equiv (1+\Delta_l)L + \Delta_s S$ shows that also $\Delta_l < 0$, although its magnitude tends to be smaller than that of δ_s. Then angular momentum conservation gives $\delta_l + \Delta_l = 0$, namely $\delta_l > 0$, and also $\delta_s + \Delta_s = 0$. This corresponds to a (small) enhancement of the IS orbital *g*-factor:

$$g_l^{\mathrm{IS}} = 0.5 + 0.38\delta_l > 0.5\,. \tag{6}$$

As a result, we see the following: In the IS spin matrix elements, there can be cancellations between the terms $(1+\delta_s)S$ and δ_l in Eq. (4). Hence, it is the IS spin *g*-factor g_s^{IS} of Eq. (4) which is quenched, not necessarily the spin matrix element. The IS orbital *g*-factor g_l^{IS} , on the other hand, is slightly enhanced, see Eq. (6).

Having established that it is the IS spin *g*-factor, rather than the spin matrix element which is necessarily quenched, we come back to the question: Why is the spin matrix element quenched for *A*=17 and *A*=39, but not in the middle of the *s*-*d* shell? The fit of Brown and Wildenthal [8] shown in the 6th line of their Table IV shows that for $0d$ nucleon $\delta_s = -0.32$ and $\delta_l = 0.05$. For *A*=17 we then get

$$\langle S\rangle_{\mathrm{free}} = 0.5$$

$$\langle S\rangle_{\mathrm{eff}} = (1-0.32)\times\frac{1}{2} + 0.05\times 2 = 0.5 - 0.16 + 0.1 = 0.44\,,$$

which should be compared with the observed value 0.43. We see a good agreement as shown in Fig. 7. It is confirmed that the spin matrix elements of *A*=17 is quenched experimentally and theoretically.

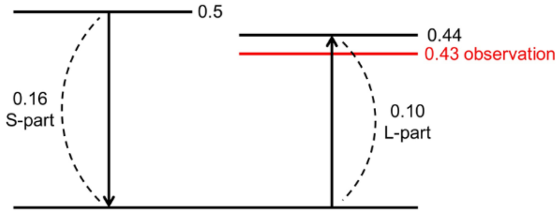

Fig. 7. Contributions to the spin expectation value for A=17.

Now what happens in the middle of the shell? As an example, let us take the nuclei with A=25 and J=5/2:

$$\langle S\rangle_{\text{free}} = 0.399, \qquad \langle S\rangle_{\text{obs}} = 0.382$$

$$\langle S_{\text{eff}}\rangle = (1+\delta_s-\delta_l)\langle S\rangle_{\text{free}} + \delta_l\, J = \langle S\rangle_{\text{free}} - 0.37\langle S\rangle_{\text{free}} + 0.05\, J$$

$$= \langle S\rangle_{\text{free}} - 0.15 + 0.13 = \langle \mathrm{S}\rangle_{\text{free}} - 0.02$$

$$= 0.379 \approx \langle S\rangle_{\text{free}}\,.$$

We see that the quenching is cancelled by the term $\delta_l\, J$. This again points out the fact that there are cancellations between the terms $(1+\delta_s)S$ and $\delta_l\, L$, or equivalently between the terms $(1+\delta_s-\delta_l)S$ and $\delta_l\, J$, in the matrix element of Eq. (4). It therefore is the IS spin g-factor $g_s^{\text{IS}} = 0.88 + 0.38\delta_s$ which is always quenched, but not necessarily the spin matrix element.

However, for spin transition matrix elements the term $\delta_l\, J$ in Eq.(4) cannot contribute. Because we have established that $(1+\delta_s-\delta_l) < 1$, we see that the transition matrix elements of the IS spin operator must be quenched!

Table II. Results for the IS spin and orbital g-factors obtained from the fit shown in the 6th line of Table VI of Brown and Wildenthal [8] and the calculations of Arima et al. [1].

	Brown et al (fit 5)	Arima et al (conf. mix.)	Arima et al (incl. MEC)	free
A=17				
g_s^{IS}	0.778	0.751	0.773	0.88
g_ℓ^{IS}	0.516	0.511	0.524	0.5
A=39				
g_s^{IS}	0.744	0.680	0.717	0.88
g_ℓ^{IS}	0.521	0.510	0.528	0.5

Before going on to discuss transition matrix elements, we wish to show that the quenching of $g_s^{\text{IS}} = 0.88 + 0.38\delta_s$ and the enhancement of $g_l^{\text{IS}} = 0.5 + 0.38\delta_l$

obtained from the fit of Brown and Wildenthal [8] are consistent with the calculations performed by Arima et al. [1] for *A*=17 and *A*=39. This comparison is shown in Table II. Arima et al. [1] have taken into account configuration mixings with $2\hbar\omega$ and higher excited states, as well as meson exchange currents.

3. The (p,p') reactions

In order to study the IS spin transition matrix elements, the (p,p') reaction is one of the possibilities. It has been studies already in 1989 by Crawley et al. [9] and by Matsubara et al. [10] in 2015. They used the *N*=*Z* target nuclei ^{24}Mg, ^{28}Si, ^{32}S and ^{36}Ar.

The quantity extracted from the differential cross sections is

$$|M(\sigma)|^2 \equiv \left|\langle 1^+, T=0 \| \sigma \| 0^+, T=0\rangle\right|^2 = 12|\langle 1^+, T=0|S|0^+, T=0\rangle|^2 \,.$$

Both works used a conversion factor (*C*) or unit cross section (UCS) to extract $|M(\sigma)|^2$ from the measured forward scattering cross sections

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}(\theta \approx 0) = C|M(\sigma)|^2 \,.$$

Crawley et al. [9] used theoretical DWIA and *s*-*d* shell model calculations to determine *C* for each excited state as

$$C = \left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)^{(\mathrm{DWIA})} \Big/ |M^{(s-d)}(\sigma)|^2 \,.$$

Matsubara et al. [10] used data on other nuclei to extract the UCS. For the IS case, however, there was only one datum available (for ^{11}B) as shown in Fig. 8.

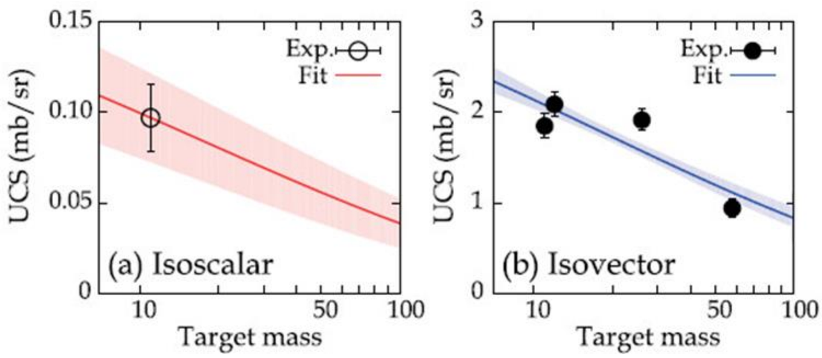

Fig. 8. Linear fit of the UCS as function of target mass, used in Ref. [10].

Let us compare the results of the two analyses for ^{28}Si as an example. Crawley et al. [9] observed two $(1^+, T=0)$ excited states, and Matsubara et al. observed four states. The results of the two analyses differ from each other by large factors as shown in Fig. 9.

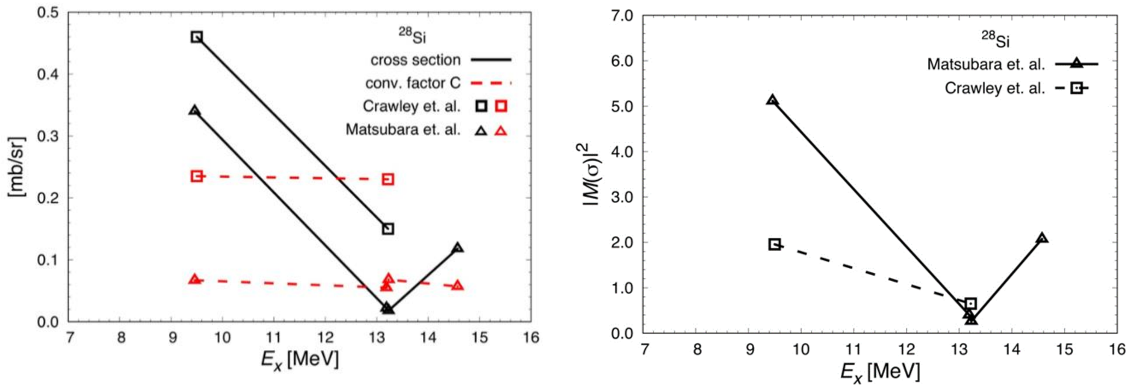

Fig. 9. Results of Crawley et al. [9] and Matsubara et al. [10] for differential cross sections and conversion factors (left panel) and spin transition matrix elements (right panel) in ^{28}Si as functions of the excitation energy.

In Fig. 10 the results of the two experimental analyses for the accumulated IS spin sums are compared to *s*-*d* shell model results using the USD interaction [11]. The results of Matsubara et al. could be interpreted in favor of "non-quenching". However, their analysis of experimental data involved rather uncertain procedures to extract the UCS. We are still waiting for other experiments and more convincing analysis of observed data.

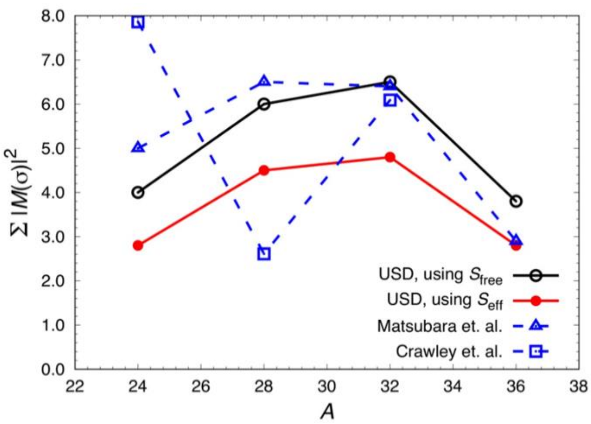

Fig. 10. Results of Crawley et al. [9] and Matsubara et al. [10] for the accumulated sums of the IS spin transition matrix elements, in comparison to *s*-*d* shell model calculations using the USD interaction with free spin matrix elements and effective spin matrix elements.

4. Conclusions

As conclusions, we would like to emphasize the following points:

1. From the IS magnetic moments of T=1/2 mirror nuclei: The quantity which is quenched is the IS spin g-factor g_s^{IS} , and not necessarily the diagonal spin matrix elements.
2. From the form of the effective IS spin operator: The transition matrix elements of the IS spin operator must be quenched.

3. However, (p, p') experiments performed so far could not observe this quenching of IS spin transition matrix elements. At present, no conclusion about quenching or non-quenching can be drawn from the (p, p') experiments, because of (i) theoretical uncertainties in the underlying relation between cross sections and spin transition matrix elements, and (ii) large differences between data and methods of analyses.

Acknowledgement

We would like to thank Dr. M. Ichimura for his cooperation and discussions concerning the magnetic moments and transitions.

This talk is dedicated to the celebration of Dr. T. Otsuka's retirement from the University of Tokyo. We would like to thank him for his cooperation for many years and prey that he will be continuously healthy and active in future.

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