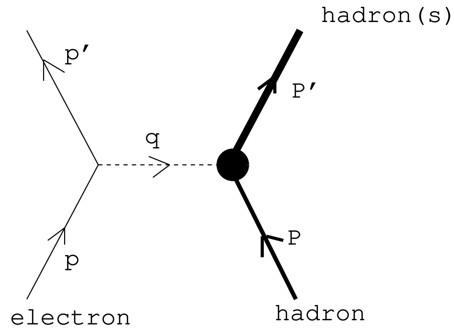
9 Kinematics for elastic and inelastic electron-hadron scattering



Energy-momentum conservation:

$$p + P = p' + P' \tag{9.1}$$

The 4-momentum transfer is defined by $q = p - p' = P' - P$. The on-mass-shell conditions are [1]

$$p^2 = p'^2 = m^2 \quad (\simeq 0 \quad \text{here}); \qquad P^2 = M^2\ ; \qquad P'^2 \geq M^2 \tag{9.2}$$

(Elastic case: $P'^2 = M^2$, inelastic case: $P'^2 > M^2$.)

On the hadron side of this diagram, one can choose 2 Lorentz invariant variables, for example q^2 and $q \cdot P$. We will choose the following variables [2]:

$$Q^2 \equiv -q^2 = \vec{q}^{\,2} - q_0^2 \quad (> 0) \tag{9.3}$$

$$x \equiv \frac{Q^2}{2P \cdot q} \quad (> 0) \tag{9.4}$$

The variable x is called the Bjorken variable. The invariant mass of the final hadronic state is then expressed as

$$P'^2 = (P+q)^2 = M^2 - Q^2 + \frac{Q^2}{x} = M^2 + Q^2\,\frac{1-x}{x} \tag{9.5}$$

The condition $P'^2 \geq M^2$ then gives

$$0 < x \leq 1 \tag{9.6}$$

[1] The electron mass ($m = 0.51$ MeV) can be neglected for scattering energies above $\simeq 100$ MeV.

[2] Remember that for electron scattering $q^2 < 0$ (see Sect. 8). Also, $P \cdot q > 0$ is easily seen in the rest frame of the inital hadron.

(Elastic case: $x = 1$, inelastic case: $x < 1$.)

Relation to the scattering angle of the electron: Neglecting the electron mass, we have

$$Q^2 = -\left(p' - p\right)^2 = 2EE'\left(1 - \cos\theta\right) = 4EE'\sin^2\frac{\theta}{2} \tag{9.7}$$

Relation to the energy loss of the electron in the lab-frame: Using $x = Q^2/(2Mq_0)$ (in the lab-frame), we get for the energy loss

$$q_0 = E - E' = \frac{Q^2}{2Mx} = \frac{2EE'\sin^2\frac{\theta}{2}}{Mx}$$

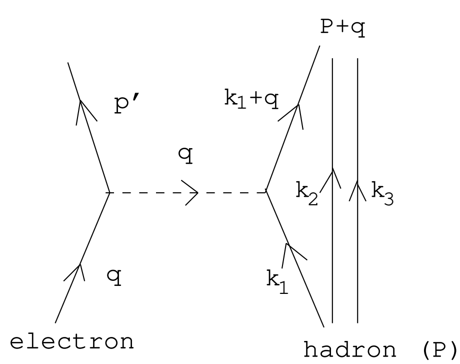
From this relation, we can express the final electron energy E' as follows:

$$E' = \frac{E}{1 + \frac{2E}{Mx}\sin^2\frac{\theta}{2}} \tag{9.8}$$

(Elastic case: $x = 1$.)

The relations (9.7) and (9.8) express the scattering angle θ and the final electron energy E' in terms of the basic variables Q^2 and x.

Important case: If the system (nucleus, or nucleon) is made of constituents (nucleons, or quarks), the scattering can be elastic on the constituents but inelastic on the whole system, i.e; $x < 1$.



The Bjorken variable in this case can be expressed as

$$x = \frac{Q^2}{2P\cdot q} = \frac{Q^2}{2k_i\cdot q}\,\frac{k_i\cdot q}{P\cdot q} = \frac{k_i\cdot q}{P\cdot q} < 1 \tag{9.9}$$

where we used $\frac{Q^2}{2k_i\cdot q} = 1$, because we assumed that the scattering is elastic on parton i. The quantity (9.9) looks like a "momentum fraction" k_i/P of the constituent i. In the "infinite momentum frame" this is indeed the case: Let us assume that the initial hadron moves along the z-axis with very large

3-momentum $|\vec{P}| \to \infty$. The 3-momentum of the constituent i can be expressed as $\vec{k}_i = z_i \vec{P} + \vec{k}_{i\perp}$, where z_i is the momentum fraction along the z-direction, and $\vec{k}_{i\perp}$ is perpendicular to the z-axis. In the infinite momentum frame we have $|\vec{P}| \to \infty$, and therefore $|\vec{k}_{i\perp}| << |\vec{P}|$. In this case,

$$\begin{aligned} P^\mu &= \left(\sqrt{M^2 + \vec{P}^2}, 0, 0, P\right) \stackrel{|\vec{P}|\to\infty}{\longrightarrow} \left(|\vec{P}|, 0, 0, |\vec{P}|\right) \\ k_i^\mu &= \left(\sqrt{m_i^2 + k_i^2}, \vec{k}_i\right) \stackrel{|\vec{P}|\to\infty}{\longrightarrow} \left(z_i|\vec{P}|, 0, 0, z_i|\vec{P}|\right) \end{aligned}$$

Then, with (9.9), we get in the infinite momentum frame

$$x = \frac{k_i \cdot q}{P \cdot q} = z_i \tag{9.10}$$

So, in the infinite momentum frame the Bjorken variable x has the physical meaning of the momentum fraction of the constituent which is interacting with the electron. This is the basis of Feynman's parton model.

Some terminology on different types of scattering processes:

- Elastic: $x = 1$.
- Quasi-elastic: $x < 1$, but elastic on one of the constituents. This is the case discussed above.
- Deep inelastic: This refers to the high energy limit limit $Q^2 \to \infty$ and $P \cdot q \to \infty$, but $x = Q^2/(2P \cdot q) < 1$ fixed.
- Inclusive: Only the electron is observed in the final state (no hadrons are observed).
- Semi-inclusive: Some types of hadrons (e.g., nucleons or pions) in the final state are also observed.
- Exclusive: All particles in the final state are observed.