12 Relation to Feynman's parton model

Remember (Sect. 11):

The cross section for inelastic inclusive electron-proton scattering was expressed in Eq.(11.1) of Sect. 11, in terms of the hadronic tensor $W^{\mu\nu}$ (defined in Eq.(11.3)). Then $W^{\mu\nu}$ was parametrized in terms of structure functions, see Eqs.(11.11), (11.12) (or (11.15), (11.16)) of Sect. 11, to get more explicit forms for the cross section.

Here we introduce a model for the structure functions due to Feynman, which is called the parton model, and compare predictions of this model with experimental data.

(1) **What is the form of the hadronic tensor for a point particle?**

First, for simplicity, we consider the spin averaged case. Use Eq.(11.3) of Sect. 11 for the spin averaged case, and insert the current J^{μ}_{FI} of a point particle, which has the same form as for the electron [1]: $J^{\mu(\text{point})}_{FI} = \sqrt{\frac{M}{E_p}}\sqrt{\frac{M}{E_{p'}}}\,\overline{u}(\vec{p}', s')\gamma^{\mu}u(\vec{p}, s)$. We get

$$\left(\frac{1}{2}\sum_s W^{\mu\nu}\right)_{\text{point}} = \frac{E_p}{2}\int \mathrm{d}^3p'\,\delta^{(4)}(p'-p-q)\,\frac{M^2}{E_p\,E_{p'}}\sum_{s,s'}\left(\overline{u}(\vec{p}', s')\gamma^{\mu}u(\vec{p}, s)\right)\left(\overline{u}(\vec{p}', s')\gamma^{\nu}u(\vec{p}, s)\right)^* \tag{12.1}$$

To perform the integration, use the identity given on p.5 of of Sect. 10:

$$\int \frac{\mathrm{d}^3p'}{2E_{p'}} = \int \mathrm{d}^4p'\,\delta\left(p'^2 - M^2\right)\theta(p'_0)$$

By using the definition of the Bjorken variable $x = -q^2/(2p\cdot q)$ we can rewrite the argument of the delta function as $p'^2 - M^2 = 2p\cdot q(1-x)$. Performing then the spin sums (as we did already many times) by using the method of Dirac traces, Eq.(12.1) becomes

$$\begin{aligned}\left(\frac{1}{2}\sum_s W^{\mu\nu}\right)_{\text{point}} &= \frac{M^2}{2p\cdot q}\delta(x-1)\sum_{s,s'}\left(\overline{u}(\vec{p}', s')\gamma^{\mu}u(\vec{p}, s)\right)\left(\overline{u}(\vec{p}', s')\gamma^{\nu}u(\vec{p}, s)\right)^* \\ &= \frac{1}{2\,p\cdot q}\,\delta(x-1)\left(p'^{\mu}p^{\nu} + p'^{\nu}p^{\mu} - g^{\mu\nu}\left(p'\cdot p - M^2\right)\right) \\ &= \frac{1}{2}\,\delta(x-1)\left(-g^{\mu\nu} + \frac{p'^{\mu}p^{\nu} + p'^{\nu}p^{\mu}}{p\cdot q}\right)\end{aligned} \tag{12.2}$$

where we used $p'\cdot p - M^2 = p\cdot q$ in the last step. (Note that the scattering on a point particle must be elastic, therefore $p^2 = M^2$.) Now compare (12.2) with the parametrization in terms of

[1] Remember from Sect. 11.2 that we denote the momenta of the target particle by p and $p' = p + q$, and its mass by M.

structure functions, see Eq.(11.15) of Sect. 11:

$$\left(\frac{1}{2}\sum_s W^{\mu\nu}\right)_{\text{point}} = F_1^{\text{point}}(x,Q^2)\left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right) + \frac{F_2^{\text{point}}(x,Q^2)}{p\cdot q}\left(p^\mu - \frac{p\cdot q}{q^2}q^\mu\right)\left(p^\nu - \frac{p\cdot q}{q^2}q^\nu\right)$$

Using here $p\cdot q = -q^2/2$ (because the scattering on point particle is elastic), and $q^\mu = p'^\mu - p^\mu$, this becomes

$$\begin{aligned}\left(\frac{1}{2}\sum_s W^{\mu\nu}\right)_{\text{point}} &= -g^{\mu\nu}F_1^{\text{point}}(x,Q^2) + \frac{p'^\mu p^\nu + p'^\nu p^\mu}{2(p\cdot q)}\left(F_1^{\text{point}}(x,Q^2) + \frac{1}{2}F_2^{\text{point}}(x,Q^2)\right) \\ &+ \frac{p'^\mu p'^\nu + p^\mu p^\nu}{2(p\cdot q)}\left(-F_1^{\text{point}}(x,Q^2) + \frac{1}{2}F_2^{\text{point}}(x,Q^2)\right) \qquad (12.3)\end{aligned}$$

Comparing (12.2) and (12.3), we obtain for the structure functions of a point particle [2]:

$$F_1^{\text{point}}(x,Q^2) = \frac{1}{2}\,\delta(x-1)\,, \qquad F_2^{\text{point}}(x,Q^2) = \delta(x-1) = 2x\,F_1^{\text{point}}(x,Q^2) \qquad (12.4)$$

Note: The delta-function $\delta(x-1)$ just expresses the condition of elastic scattering on the point particle. Because a point particle has no structure, the structure functions are independent of Q^2.

The same calculation as above can be done for the full hadronic tensor $(W^{\mu\nu})_{\text{point}}$ of a point particle (not spin averaged). Starting again from Eq.(11.3) of Sect. 11, and using the method of the spin projection operator explained in Sect. 11 to rewrite the spin sum as a Dirac trace, we have

$$\begin{aligned}(W^{\mu\nu})_{\text{point}} &= E_p \int \mathrm{d}^3p'\,\delta^{(4)}(p'-p-q)\,\frac{M^2}{E_p\,E_{p'}}\sum_{s'}\left(\overline{u}(\vec{p}',s')\gamma^\mu u(\vec{p},s)\right)\left(\overline{u}(\vec{p}',s')\gamma^\nu u(\vec{p},s)\right)^* \\ &= \frac{M^2}{p\cdot q}\,\delta(x-1)\sum_{s'}\left(\overline{u}(\vec{p}',s')\gamma^\mu u(\vec{p},s)\right)\left(\overline{u}(\vec{p},s)\gamma^\nu u(\vec{p}',s')\right) \\ &= \frac{1}{2}\delta(x-1)\left(-g^{\mu\nu} + \frac{p'^\mu p^\nu + p'^\nu p^\mu}{p\cdot q} + iM\,\epsilon^{\mu\nu\sigma\lambda}\,\frac{q_\sigma S_\lambda}{p\cdot q}\right) \qquad (12.5)\end{aligned}$$

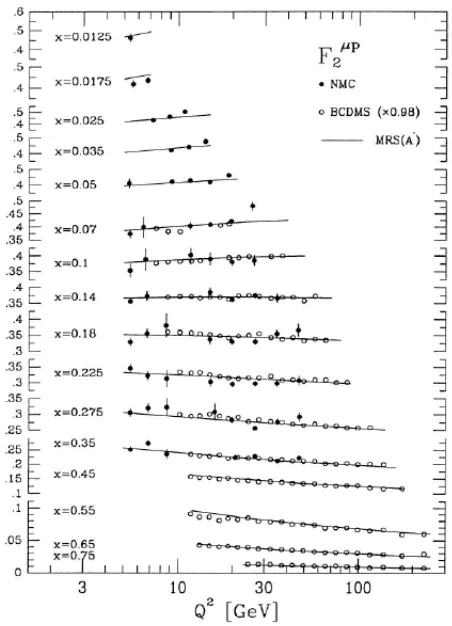
We compare this with the parametrization in terms of the structure functions F_1^{point}, F_2^{point}, g_1^{point}, g_2^{point}, see Eq.(11.15) and (11.16) of Sect.11. The results for F_1^{point} and F_2^{point} are given in Eq.(12.4), and for g_1^{point}, g_2^{point} we obtain

$$g_1^{\text{point}}(x,Q^2) = \frac{1}{2}\delta(x-1)\,, \qquad g_2^{\text{point}}(x,Q^2) = 0 \qquad (12.6)$$

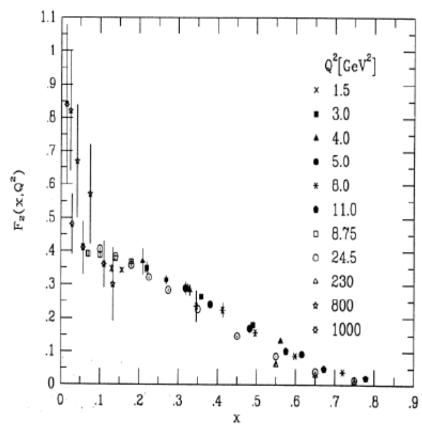
[2]The relation $F_2 = 2x\,F_1$ expresses that the "point particle" considered here has spin 1/2.

(2) Experimental data on the proton structure function $F_2(x, Q^2)$ for $Q^2 > 1.5$ GeV^2:

As function of Q^2 for several x :



As function of x for several Q^2 :



(from ref. 6)

<u>We see:</u> For large enough Q^2 (high energy scattering), F_2 is almost independent of Q^2, i.e., $F_2(x, Q^2) \simeq F_2(x)$. This phenomenon is called the <u>Bjorken scaling</u>.
The data for F_1 (not shown here) indicate that also F_1 depends only on x for large Q^2, and that there is the following relation between the proton structure functions $F_1(x)$ and $F_2(x)$:

$$F_2(x) \simeq 2x\, F_1(x) \tag{12.7}$$

This looks like the relation (12.4) for a point particle, although we know that the proton is not a point particle!

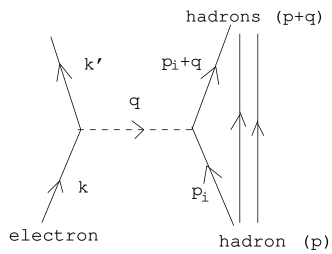
We will show in later lectures that Bjorken scaling and the relation (12.7) can be explained naturally in the <u>Bjorken limit</u>: The Bjorken limit (B.L.) is defined by

$$Q^2 \to \infty\,, \qquad p \cdot q \to \infty\,, \qquad x = \frac{Q^2}{2(p \cdot q)} \text{ fixed} \tag{12.8}$$

As we will see later,the B.L. is only a "theoretical tool" to derive Feynman's <u>parton model</u>, which gives simple expressions and physical interpretations of structure functions. The experimental data show the validity of scaling and the relation (12.7) already for $Q^2 > 2$ GeV^2 and $p \cdot q > 1$ GeV^2. For very small values of x, however, the data show a strong violation of scaling: As the figure on the right side above shows, F_2 becomes very large (may be infinity) as $x \to 0$, and depends on Q^2 in this region.

(3) Idea of Feynman's parton model (most naive version):

The dominant process for inelastic electron-hadron scattering at large Q^2 (high energies) is elastic scattering on point-like constituents (partons) [3].



For elastic scattering on quark i (with 4-momentum p_i and mass $p_i^2 = m_i^2$) we have [4]

$$(p_i + q)^2 = m_i^2 \Rightarrow \frac{Q^2}{2(p_i \cdot q)} = 1 \tag{12.9}$$

This means: The Bjorken variable of the quark is equal to 1.

If z_i is the 4-momentum fraction carried by parton i, then $p_i = z_i p$, where $p = \sum_i p_i$ is the total 4-momentum of the hadron. Then we get from (12.9)

$$\frac{Q^2}{2z_i(p \cdot q)} = 1 \Rightarrow z_i = \frac{Q^2}{2(p \cdot q)} = x \tag{12.10}$$

This means: Only the quark which carries momentum fraction $z_i = x$ can interact with the electron! (Remember that, in the laboratory system, the value of x is determined by the energy and momentum of the initial and final electron in the scattering process, and is independent of the target proton.)

Because the quark is point-like, the structure function (F_2) of quark i with charge e_i is (see (12.4)) [5]

$$F_{2i} = e_i^2\, \delta\left(\frac{Q^2}{2(p_i \cdot q)} - 1\right) = e_i^2\, \delta\left(\frac{x}{z_i} - 1\right) = e_i^2\, z_i\, \delta(x - z_i) = x\, e_i^2\, \delta(x - z_i) \tag{12.11}$$

[3] This principle is similar to Daruma Otoshi: If you hit one piece by giving it a large momentum, this piece takes up the whole momentum and energy, and the other pieces do not participate in the scattering process. Earlier (in Sect. 9) we called such a process "quasi-elastic scattering" - In our lectures, "parton" means the same as "quark". More generally, it means "quark or gluon".

[4] Here i labels the flavor of the quark: $i = u$ (for up-quark), or $i = d$ (for down-quark), etc.

[5] The square of the proton charge (e^2) was not included in the definition of structure functions so far, but was taken out as a factor in the formula for the cross section. But if we assume that the proton consists of 3 partons (quarks), we must include the square of the quark charge (e_i^2) in the definition of the structure functions for each quark separately.

Then the total structure function F_2 of the proton must be given by

$$F_2(x, Q^2) = \sum_i \int_0^1 \mathrm{d}z_i\, q_i(z_i)\, F_{2i} \tag{12.12}$$

where $q_i(z)$ is the probability to find quark i with momentum fraction z inside the proton, i.e., the "momentum distribution function" of the quark i. By using (12.11), we get for the structure function of the proton:

$$F_2 = \sum_i \int_0^1 \mathrm{d}z\, q_i(z)\, x\, e_i^2\, \delta(x-z) = x \sum_i e_i^2\, q_i(x) \tag{12.13}$$

The same calculation can be done also for the structure function F_1: From Eq.(12.4), for quark i we have $F_{1i} = \frac{1}{2z_i} F_{2i} = \frac{1}{2} e_i^2\, \delta(x - z_i)$, and therefore F_1 of the proton is given by

$$F_1(x, Q^2) = \sum_i \int_0^1 \mathrm{d}z_i\, q_i(z_i)\, F_{1i} = \frac{1}{2} \sum_i e_i^2\, q_i(x) \tag{12.14}$$

We see: The structure functions in the parton model, given by Eqs. (12.13) and (12.14), are independent of Q^2, which means Bjorken scaling, and satisfy the relation

$$F_2(x) = 2x\, F_1(x) \tag{12.15}$$

Extension of the naive parton model:

First, let us denote the u-quark distribution simply by $u(x) \equiv q_u(x)$ and the d-quark distribution by $d(x) \equiv q_d(x)$.

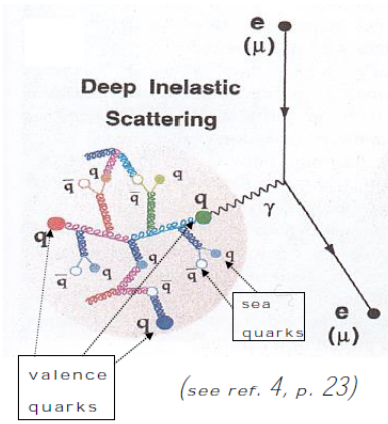
The proton (p) consists of two "valence (v) u-quarks" [6] with charge $e_u = 2/3$, and one "valence d-quark" with charge $e_d = -1/3$. We then get the following structure function of the proton in the extended parton model:

$$F_2^{(p)}(x) = x \left(\frac{4}{9} u_v^{(p)}(x) + \frac{1}{9} d_v^{(p)}(x) \right) + (\text{sea quark contributions}) \tag{12.16}$$

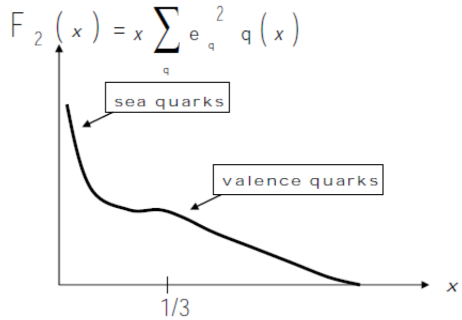
where $u_v^{(p)}(x)$ is the momentum distribution of the valence u - quark in the proton, and $d_v^{(p)}(x)$ is the momentum distribution of the valence d - quark in the proton. The normalizations are

$$\int_0^1 \mathrm{d}x\, u_v^{(p)}(x) = 2\,, \qquad \int_0^1 \mathrm{d}x\, d_v^{(p)}(x) = 1 \tag{12.17}$$

[6] In addition to the three "valence" quarks (momentum distribution $q_v(x)$), there are also "sea" quarks and antiquarks in the proton (momentum distribution $q_s(x)$ and $\overline{q}_s(x)$), because of the spontaneous creation and annihilation of quark-antiquark pairs from the vacuum. (Generally, $q_s(x) = \overline{q}_s(x)$.) Therefore, the total quark distributions are $q(x) = q_v(x) + q_s(x)$, and the antiquark distributions are $\overline{q}(x) = q_s(x)$. We will see later that it is the functions $q(x)$ and $\overline{q}(x)$ which are defined precisely in quantum field theory (in terms of field operators), and therefore $q_v(x)$ is defined by $q_v = q(x) - \overline{q}(x)$.



(see ref. 4, p. 23)



<u>Note:</u> "Valence" quark contributions are dominant for large momentum fractions ($x > 0.5$)), while "sea" quark and gluon contributions are dominant for small x. Intuitively, this means that the 3 valence quarks move together like a "cluster", but the sea quarks and gluons "flatter around" with small momentum fractions.

Relations (12.17) are called the <u>"number sum rules"</u>: Sea quarks and gluons do not contribute to baryon number and charge, and therefore the relations (12.17) are satisfied <u>only</u> by the valence quark distributions. However, (sea quarks + gluons) carry a part of the total momentum of the proton, i.e., they contribute to the <u>"momentum sum rule"</u>:

$$\int_0^1 \mathrm{d}x\, x\, \left(u_v^{(p)}(x) + d_v^{(p)}(x)\right) + (\text{sea quark contributions}) + (\text{gluon contributions}) = 1 \quad (12.18)$$

This sum rule simply says that 100% of the proton momentum is carried by either valence quarks, sea quarks, or gluons, because there is nothing more inside the proton.

For the <u>neutron</u> (one valence u-quark and two valence d-quarks) we obtain

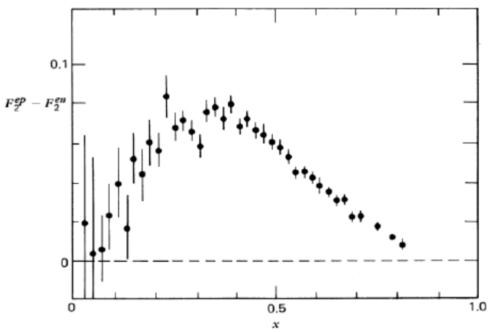
$$\begin{aligned} F_2^{(n)}(x) &= x\left(\frac{4}{9}u_v^{(n)}(x) + \frac{1}{9}d_v^{(n)}(x)\right) + (\text{sea quark contributions}) \\ &= x\left(\frac{4}{9}d_v^{(p)}(x) + \frac{1}{9}u_v^{(p)}(x)\right) + (\text{sea quark contributions}) \end{aligned} \quad (12.19)$$

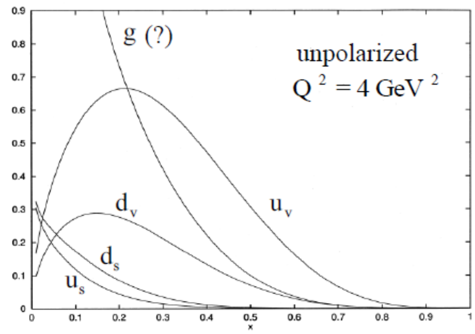
In the last relation above, we assumed "flavor symmetry":

$$u_v^{(p)}(x) = d_v^{(n)}(x)\,, \qquad d_v^{(p)}(x) = u_v^{(n)}(x) \quad (12.20)$$

which means that u and d quarks differ only by their charge, but otherwise behave in the same way. (Remember that, in the strong interactions, also the proton and neutron differ only by their charge.)

The contributions of "sea" quarks to Eq.(12.16) and Eq.(12.19) are almost the same, and cancel if we take the difference $F_2^{(p)}(x) - F_2^{(n)}(x)$:





Because $F_2^{(p)} - F_2^{(n)} = \frac{1}{3}\,x\,\left(u_v^{(p)}(x) - d_v^{(p)}(x)\right)$, the left graph clearly shows that the valence quark distributions have a peak around $x \simeq 1/3$. This is consistent with the naive expectation that each quark carries 1/3 of the total momentum.

From the experimental data of $F_2(x)$ for the proton and neutron (and other data), one can extract all quark distribution functions (right graph). The gluon distribution function ($g(x)$) is still very uncertain, but seems to be very large at small values of x.