物理学概論

東海大学　理学部
物理学科

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Tokai University



・磁場（インダクタンス）
の持つエネルギー

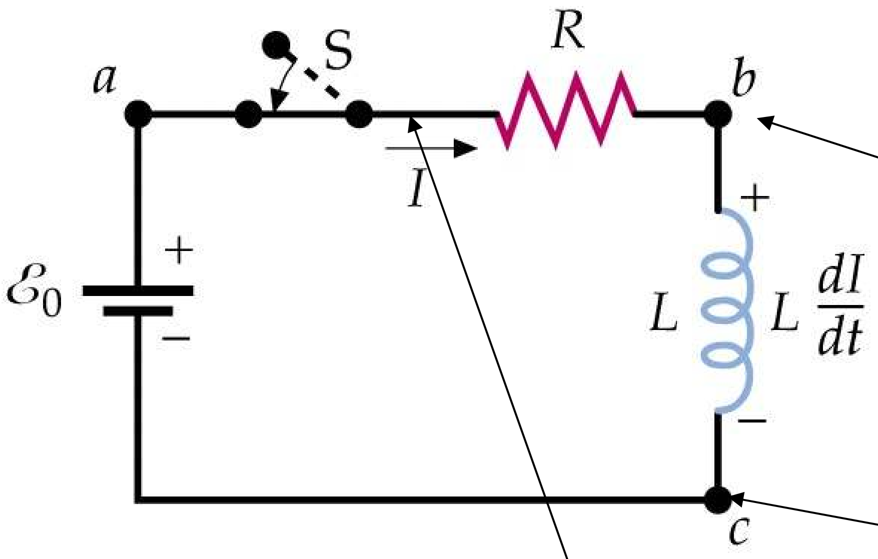
・コンデンサ、
インダクタンスをもつ回路
と過渡応答

$$Q = CV \qquad N\Phi_m = LI$$

$$\tau = CR \qquad \tau = L / R$$

$$\boxed{I = C\frac{dV}{dt}} \qquad \boxed{V = L\frac{dI}{dt}}$$

$$W = \frac{1}{2}CV^2 \qquad W = \frac{1}{2}LI^2$$



$$b : L\frac{dI}{dt}$$

$$a : I \cdot R$$

$$c : 0$$

$$E_0 = I \cdot R + L\frac{dI}{dt}$$

$$E_0 = I \cdot R + L\frac{dI}{dt}$$

$$L\frac{dI}{dt} = E_0 - I \cdot R$$

$$E_0 = I \cdot R + L\frac{dI}{dt}$$

$$L\frac{dI}{dt} = E_0 - I \cdot R$$

$$(L/R)\frac{dI}{dt} = (E_0/R) - I$$

$$(L/R)\frac{dI}{dt} = (E_0/R) - I$$

$$X = \left(E_0 \big/ R\right) - I$$

$$dX = -dI$$

$$X = \left({E_0}/{R}\right) - I \qquad dX = -dI$$

$$(L/R)\frac{dI}{dt} = (E_0/R) - I$$

$$-(L/R)\frac{dX}{dt} = X$$

$$X = \left(E_0/R\right) - I \qquad dX = -dI$$

$$\frac{1}{X}dX = -(R/L)dt$$

$$\ln(X) = -(R/L)t + C$$

$$X = (E_0/R) - I$$

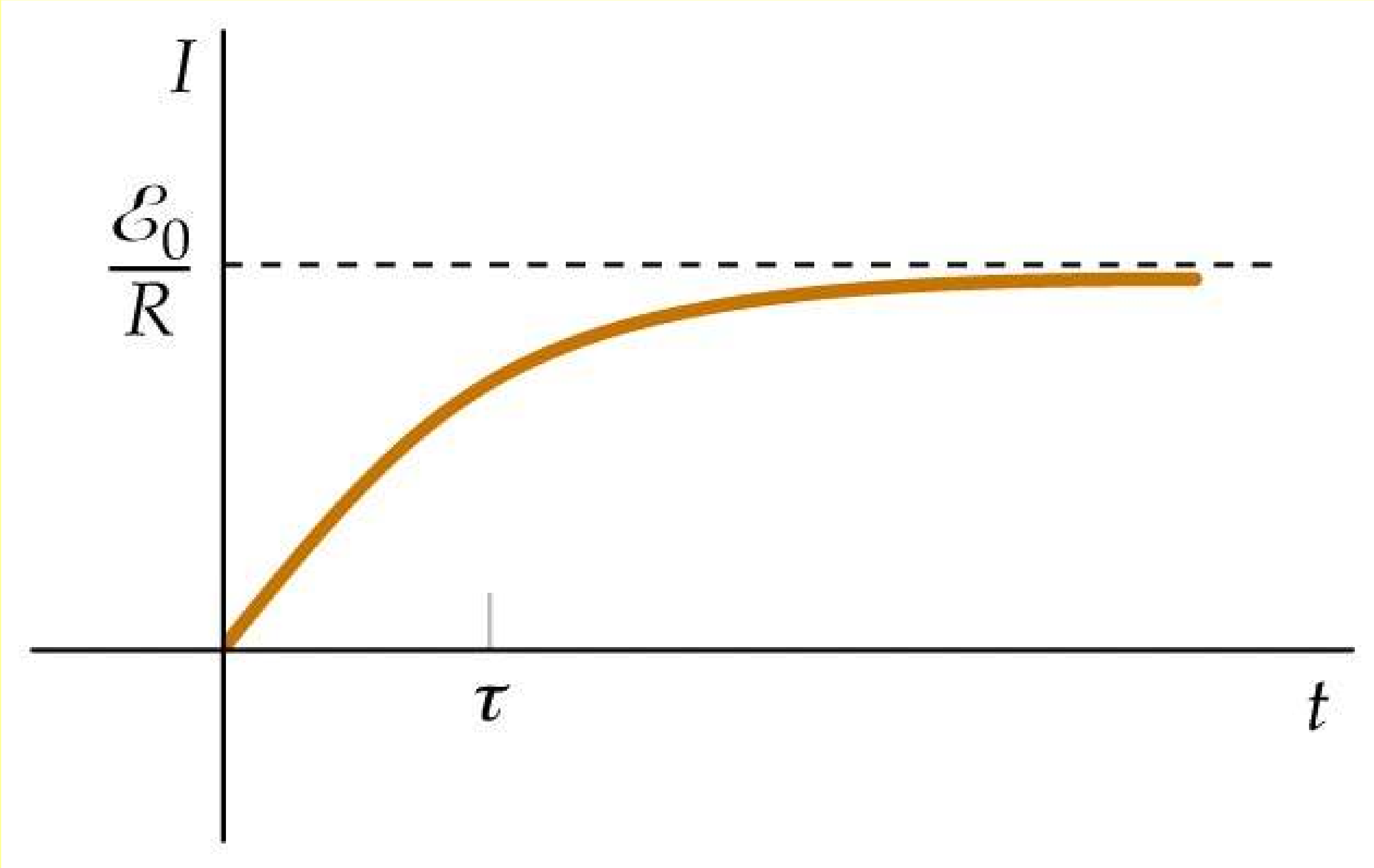
$$\ln(X) = -(R/L)t + C$$

$$(E_0/R) - I = A\exp[-(R/L)t]$$

$$I = (E_0/R) - A\exp[-(R/L)t]$$

$$I_t = \frac{E}{R}\left(1 - e^{-\frac{Rt}{L}}\right)$$

$$\tau = \frac{L}{R}$$



I
$\frac{\mathcal{E}_0}{R}$
τ
t

$$\frac{dQ}{dt} \cdot R + \frac{Q}{C} = V$$

$$\frac{dQ}{dt} = \frac{1}{RC}(CV - Q)$$

$$\ln(Q - CV) = -\frac{t}{RC}$$

$$Q - CV = A\exp[-\frac{t}{RC}]$$

$$t = 0, Q = 0 \Rightarrow A = -CV$$

$$Q = CV\{1 - \exp(-\frac{t}{RC})\}$$

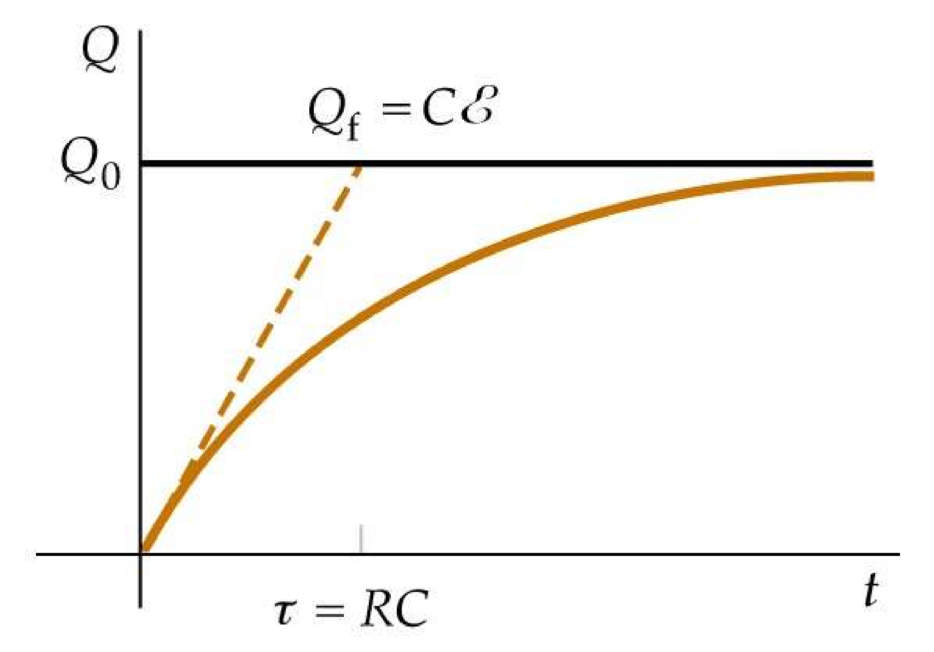
$$I = \frac{d}{dt}\left[CV \{1 - \exp(-\frac{t}{RC})\} \right]$$

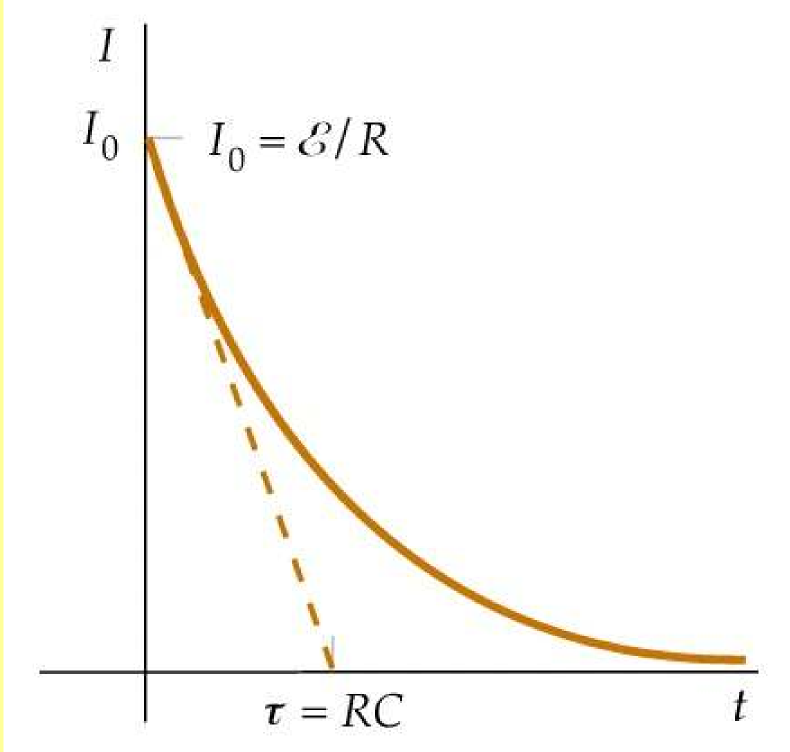
$$t = 0, I = V / R$$

$$I = \left[\frac{CV}{RC} \exp(-\frac{t}{RC})\} \right]$$

$$Q = CV\ \{1 - \exp(-\frac{t}{RC})\}$$

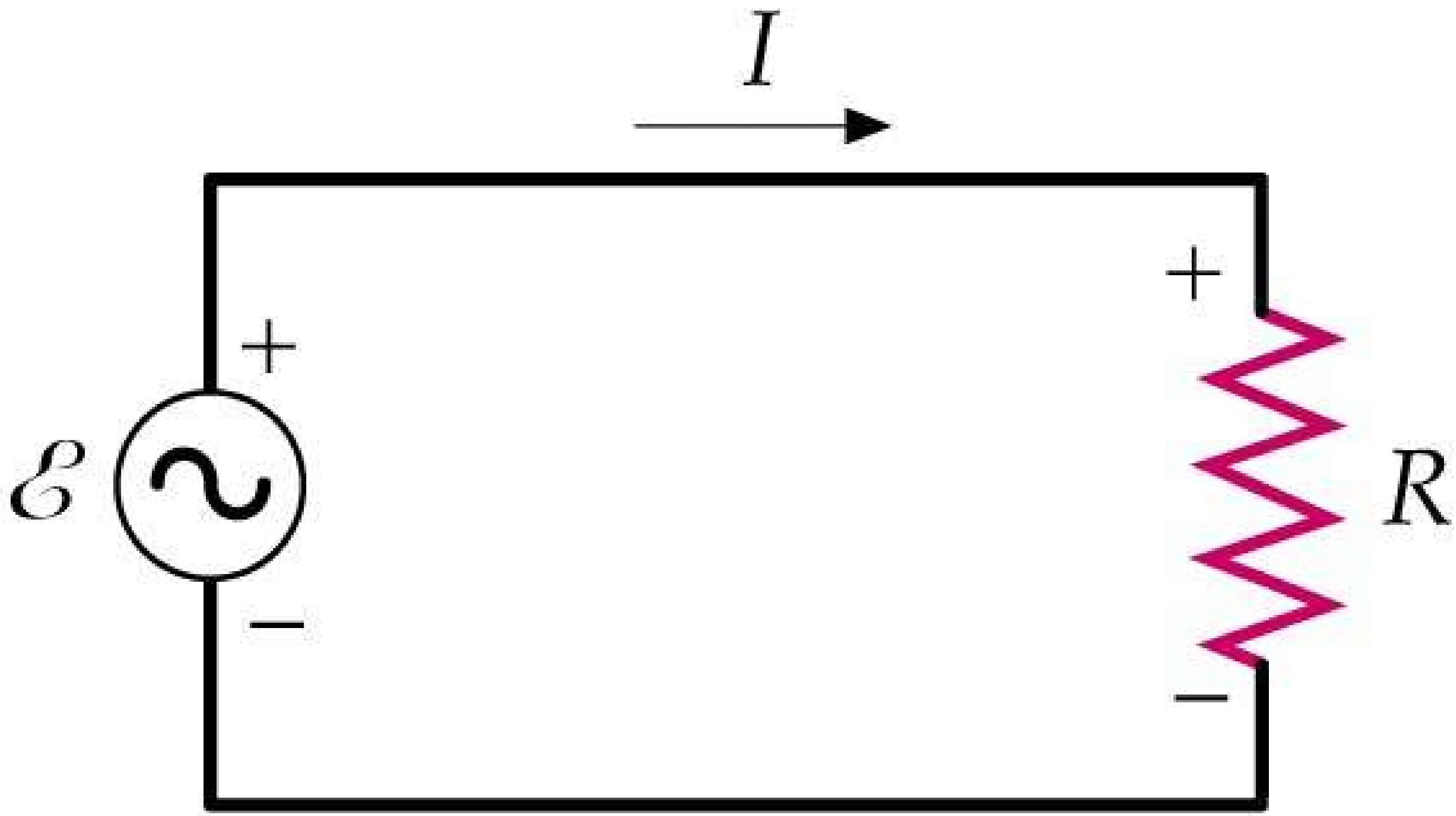
$$I = \left[\frac{V}{R}\exp(-\frac{t}{RC})\}\right]$$

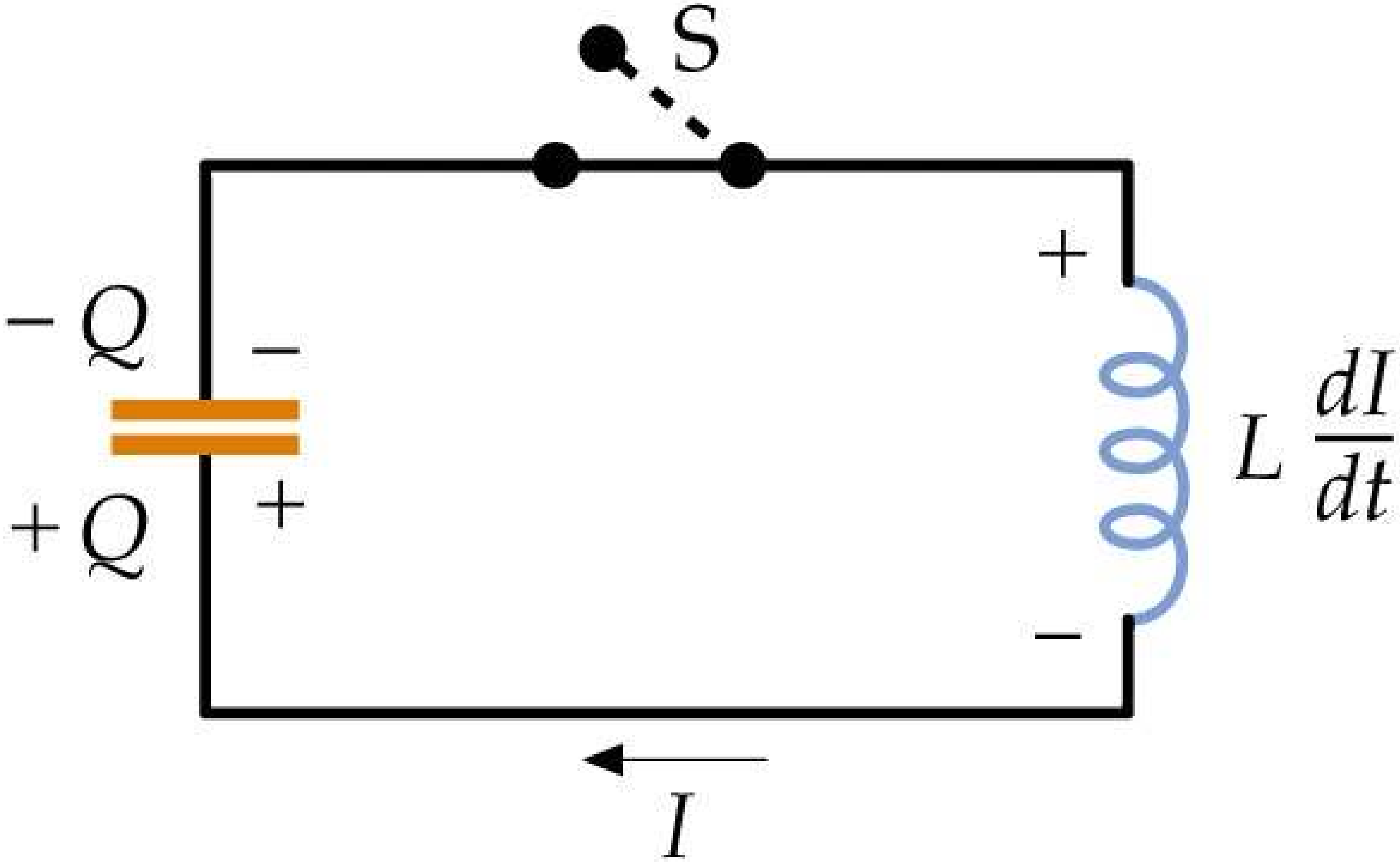

Q
$Q_f = C\mathcal{E}$
Q_0
t
$\tau = RC$

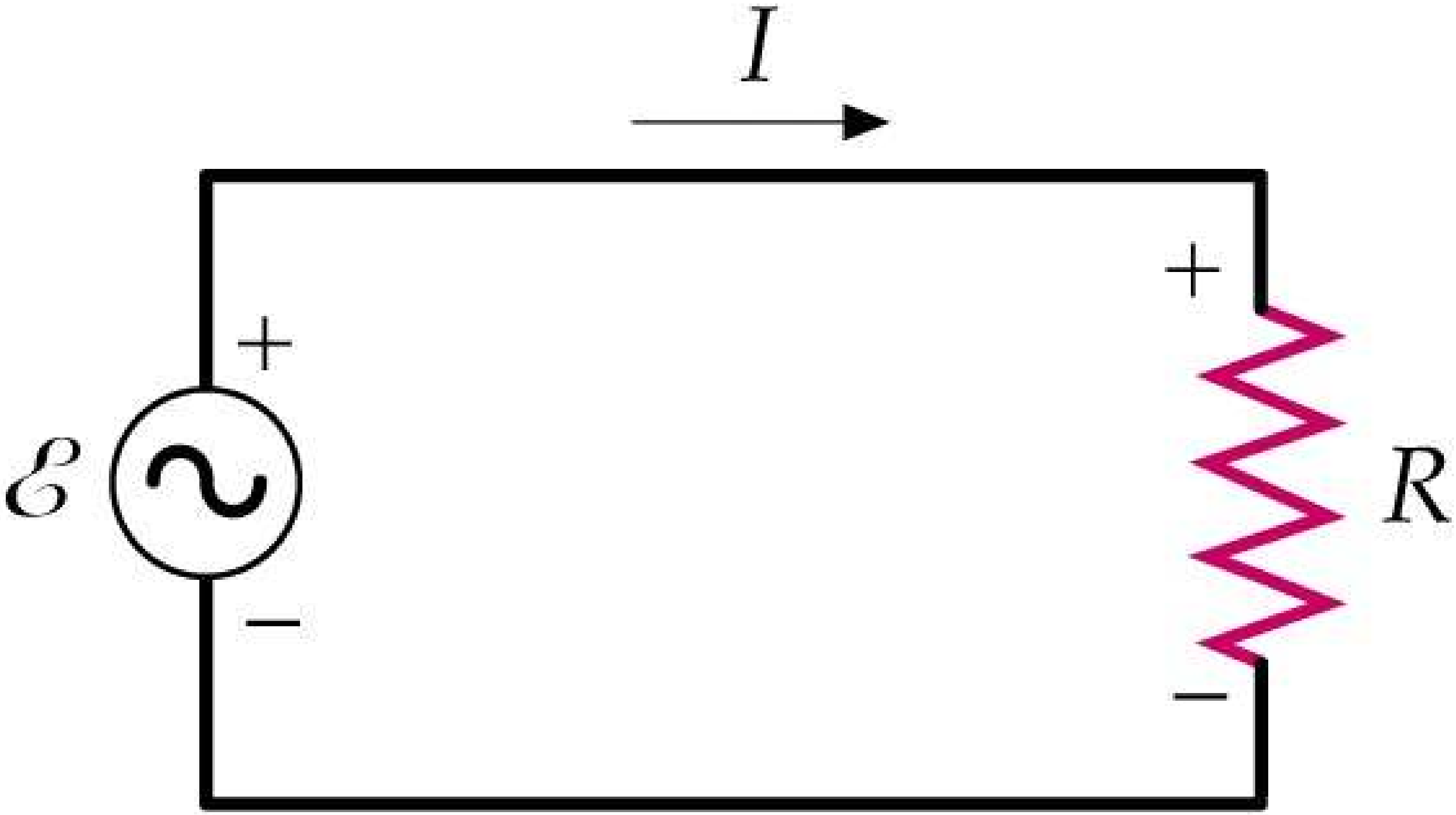

I
I_0
$I_0 = \mathcal{E}/R$
t
$\tau = RC$

本日のメニュー

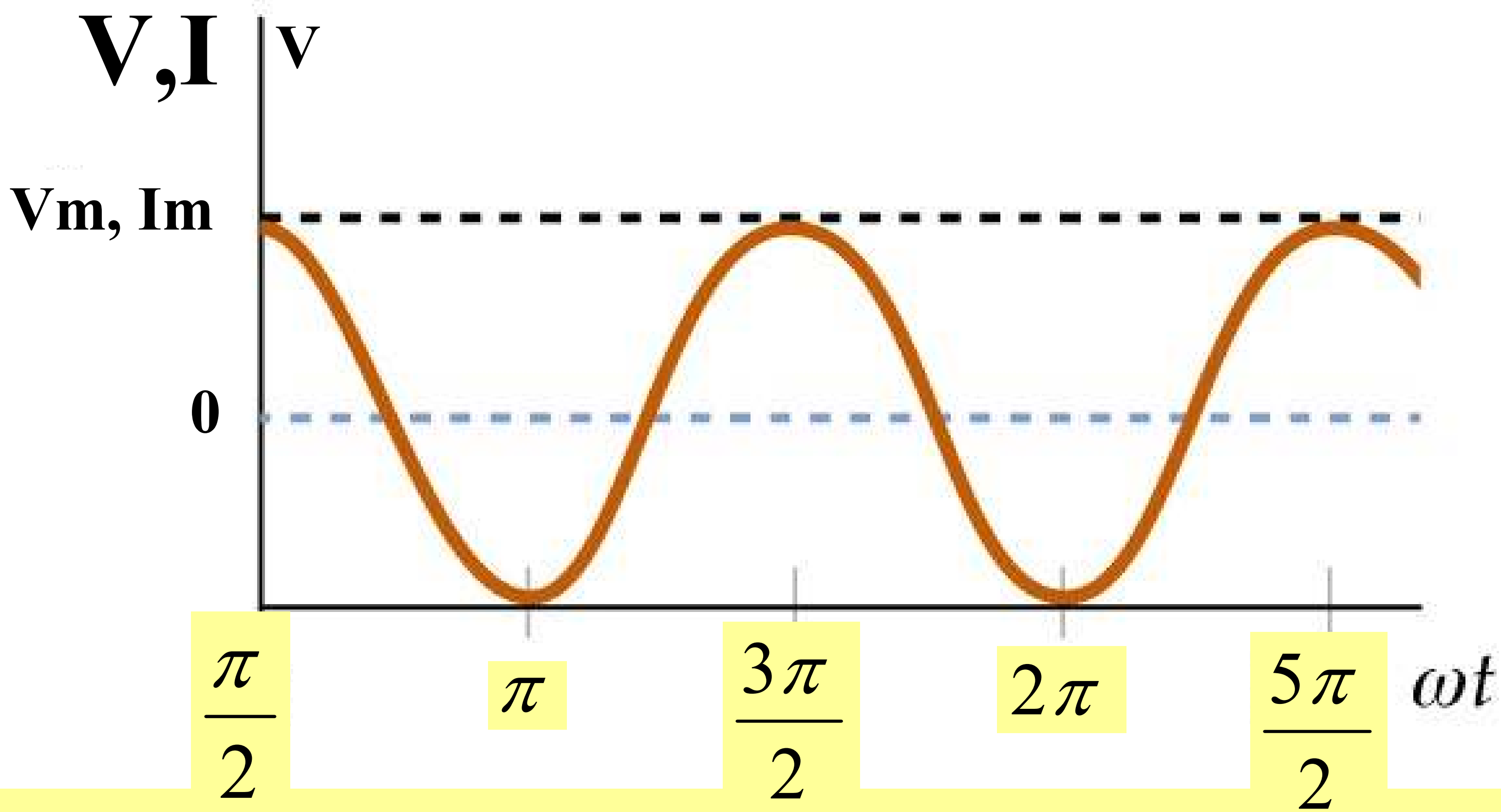
・交流回路の素子の役割と
　　　　電流と電圧波形
・インピーダンス
（・共振現象）


I
+
+
ℰ
R
−
−


S
− Q
+ Q
−
+
+
−
L $\frac{dI}{dt}$
I



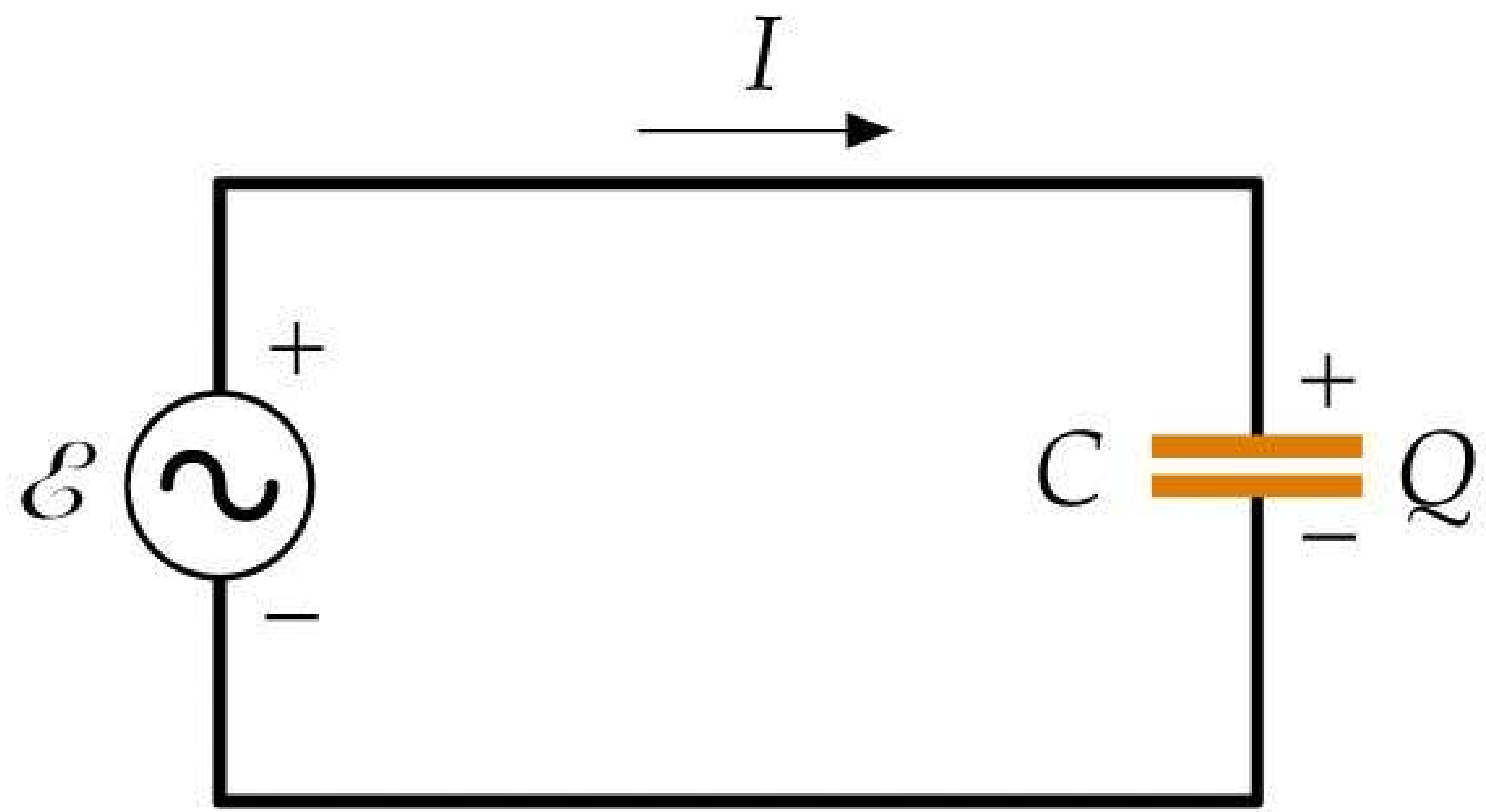
$$\varepsilon = Vm \cdot \sin \omega t$$

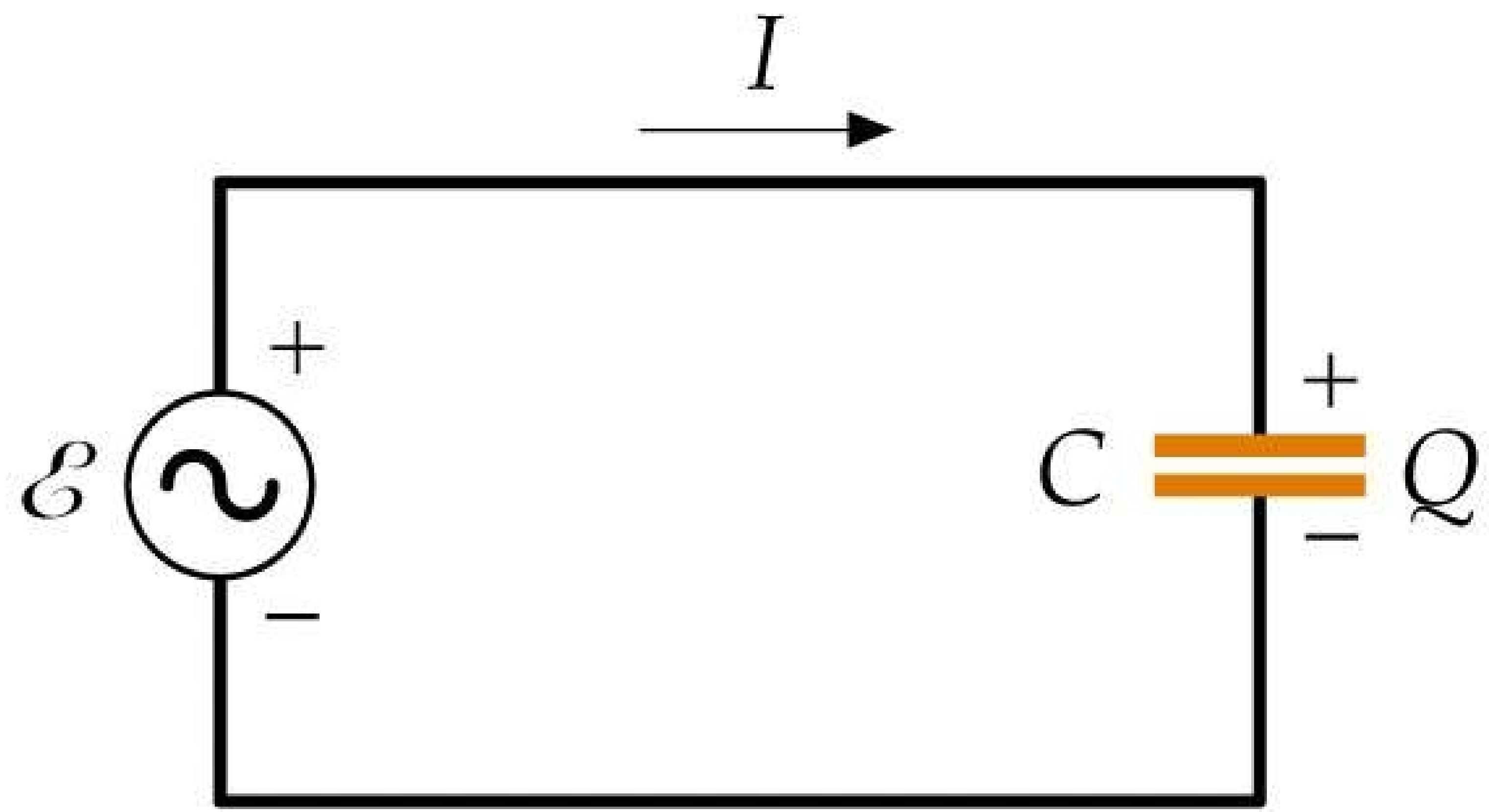


$$V = Vm \cdot \sin \omega t$$

$$I = Im \cdot \sin \omega t$$

では電気容量を接続した回路では電圧･電流の関係はどうだろう？





$$\varepsilon = Vm \cdot \sin \omega t$$

$$V = Vm \cdot \sin \omega t$$

$$Q = CV$$

$$\frac{d}{dt} Q = I$$

$$I = \frac{dQ}{dt} =$$

$$\omega C \frac{d}{dt}(Vm \sin \omega t)$$

$$= \omega C Vm \cos \omega t$$

電流は電圧を
時間微分した形になる。

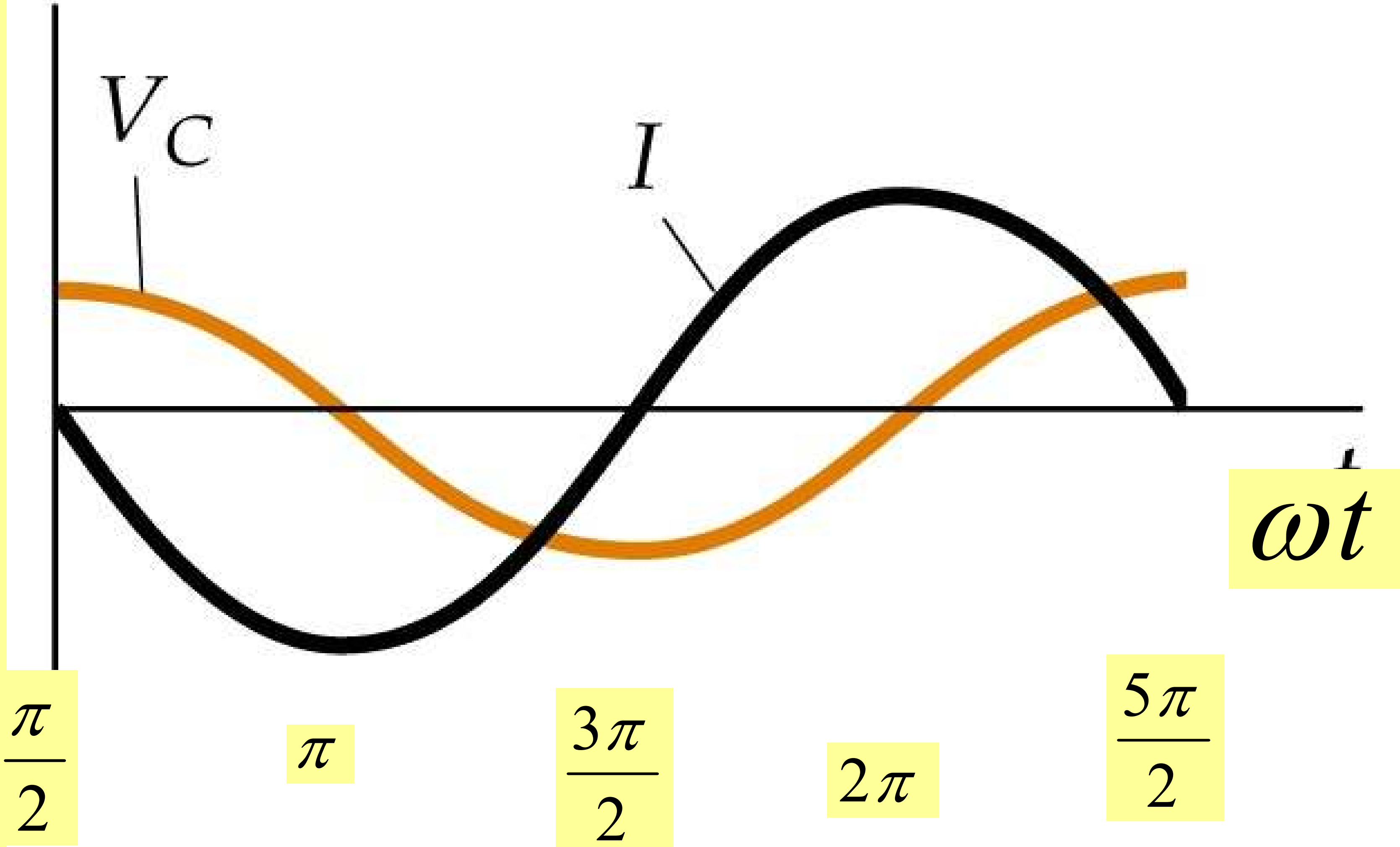
$$V = Vm \cdot \sin \omega t$$

$$I = \omega C Vm \cdot \sin(\omega t + \frac{\pi}{2})$$

$$I = \omega C V m \ \cos \omega t$$

$$= \omega C V m \ \sin(\omega t + \frac{\pi}{2})$$

電流は電圧に対してπ/2だけ位相がずれて進んだ波になっている。

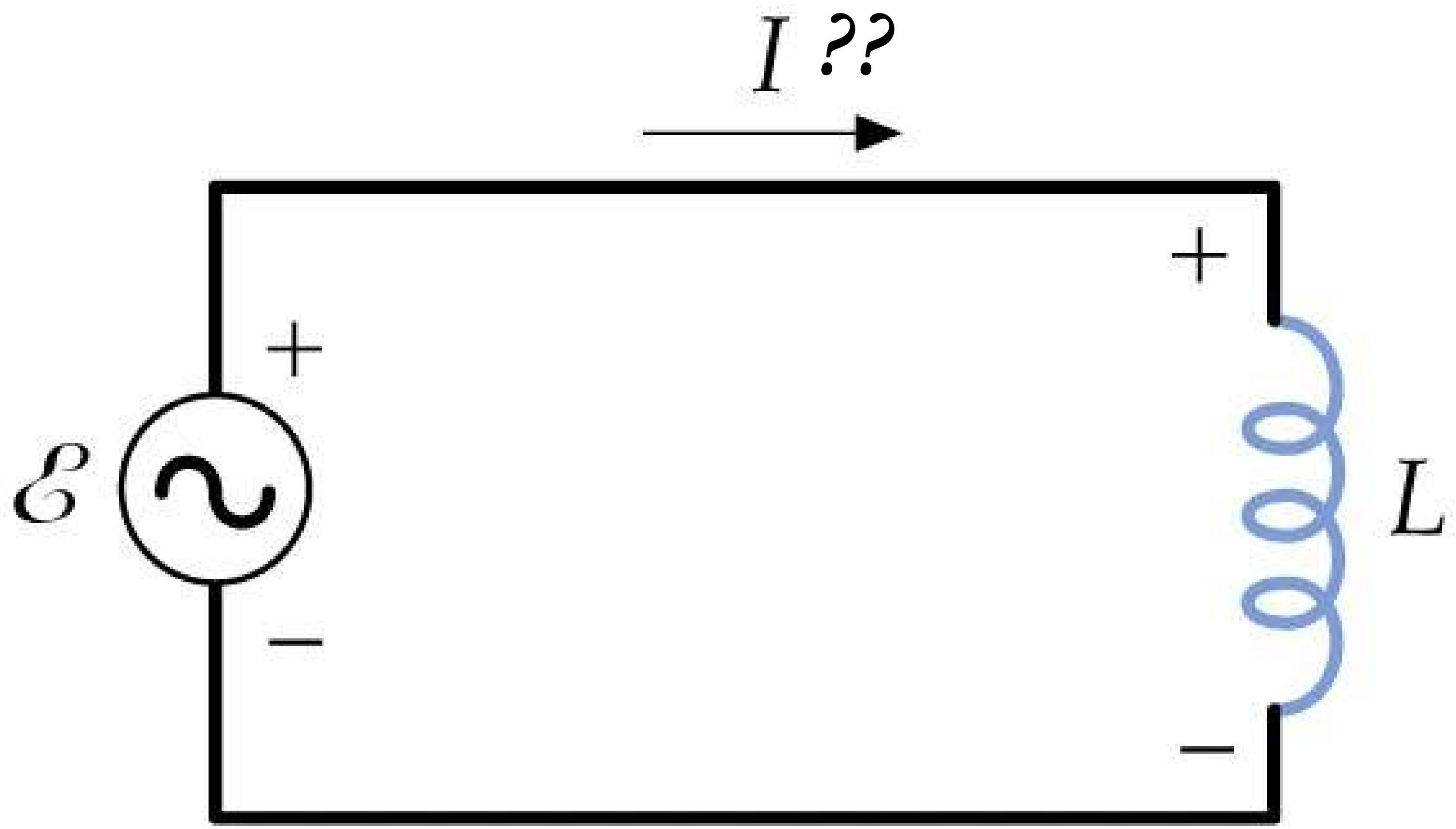

V_C
I
ωt
$\frac{\pi}{2}$
π
$\frac{3\pi}{2}$
2π
$\frac{5\pi}{2}$

$$Im = \omega CVm \qquad I = \frac{V}{R}$$

オームの法則に対してRの対応を見ると

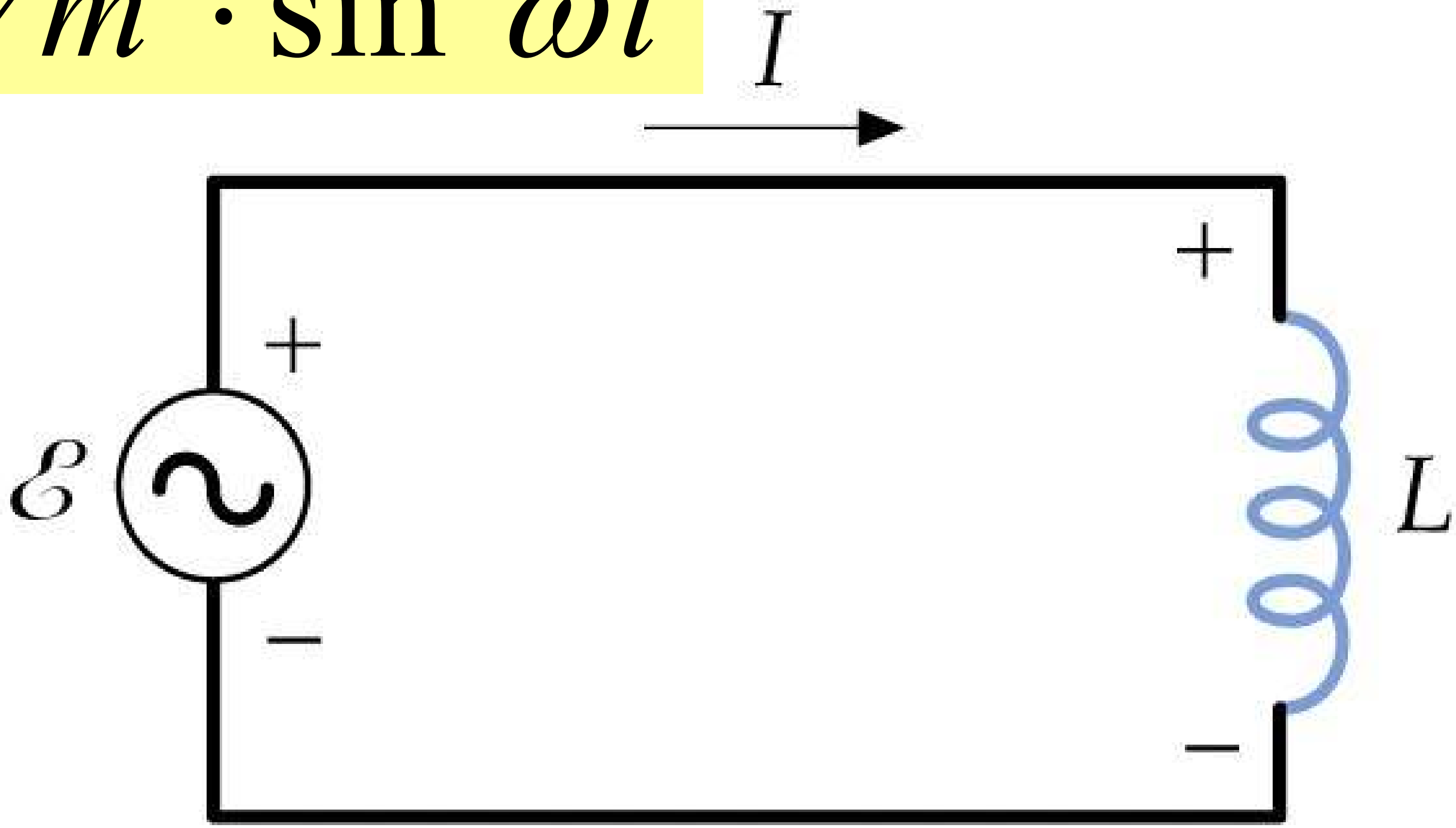
$$R \rightarrow 1/\omega C \qquad X_C = 1/\omega C$$

誘導素子を接続するとどのようか？



$$\varepsilon = Vm \cdot \sin \omega t$$

$$\varepsilon = Vm \cdot \sin \omega t$$



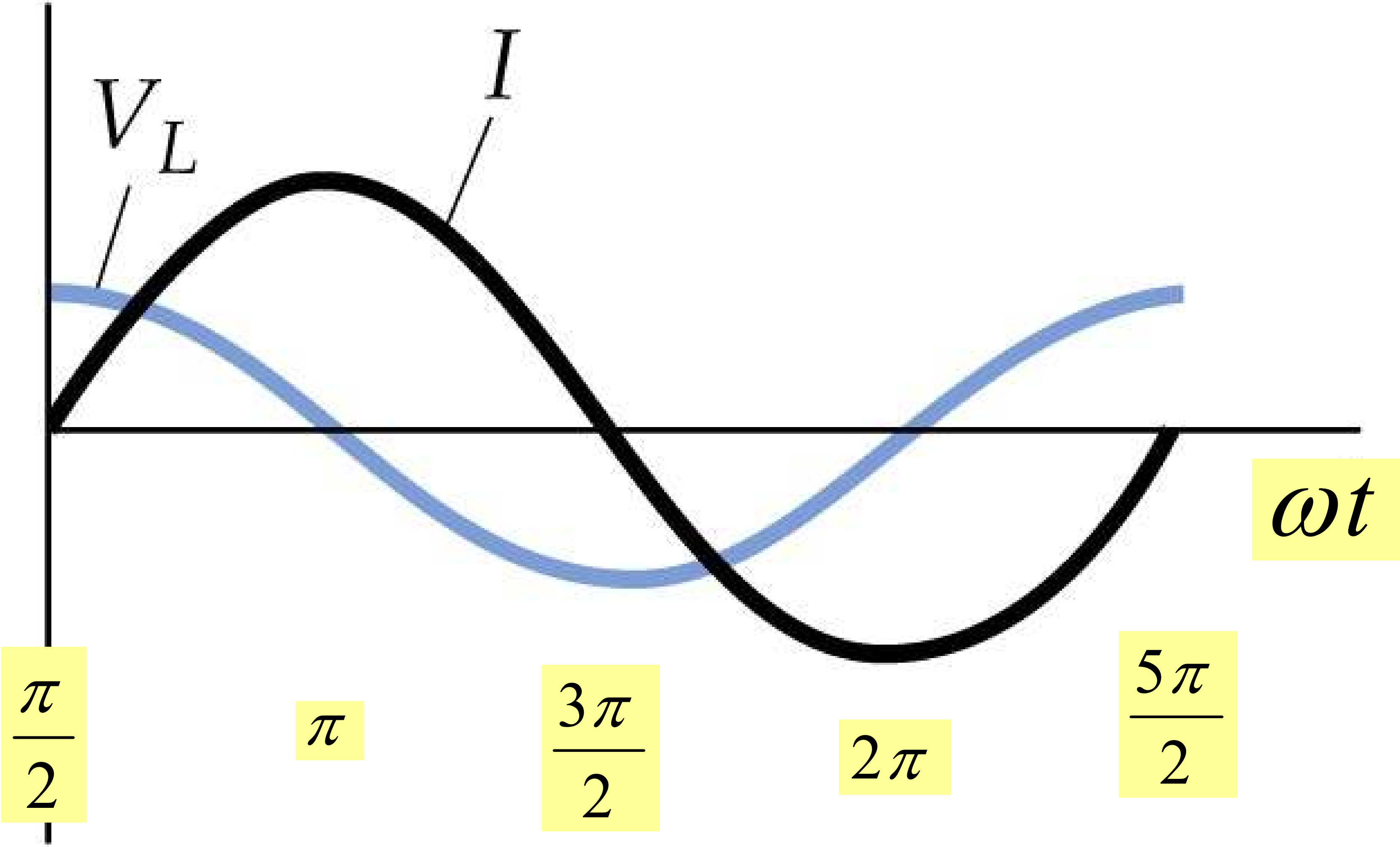
$$\varepsilon - L\frac{dI}{dt} = 0$$

$$L\frac{dI}{dt} = Vm \cdot \sin \omega t$$

$$I = \frac{Vm}{L}\int \sin \omega t \cdot dt$$

$$= -\frac{Vm}{\omega L}\cos \omega t$$

電流は電圧を
時間積分した形になる。



V_L
I
ωt
$\frac{\pi}{2}$
π
$\frac{3\pi}{2}$
2π
$\frac{5\pi}{2}$

$$I = -\frac{Vm}{\omega L}\cos \omega t$$

$$= \frac{Vm}{\omega L}\sin(\omega t - \frac{\pi}{2})$$

電流は電圧に対してπ/2だけ位相がずれて遅れた波になっている。

$$V = Vm \cdot \sin \omega t$$

$$I = \frac{Vm}{\omega L} \sin(\omega t - \frac{\pi}{2})$$

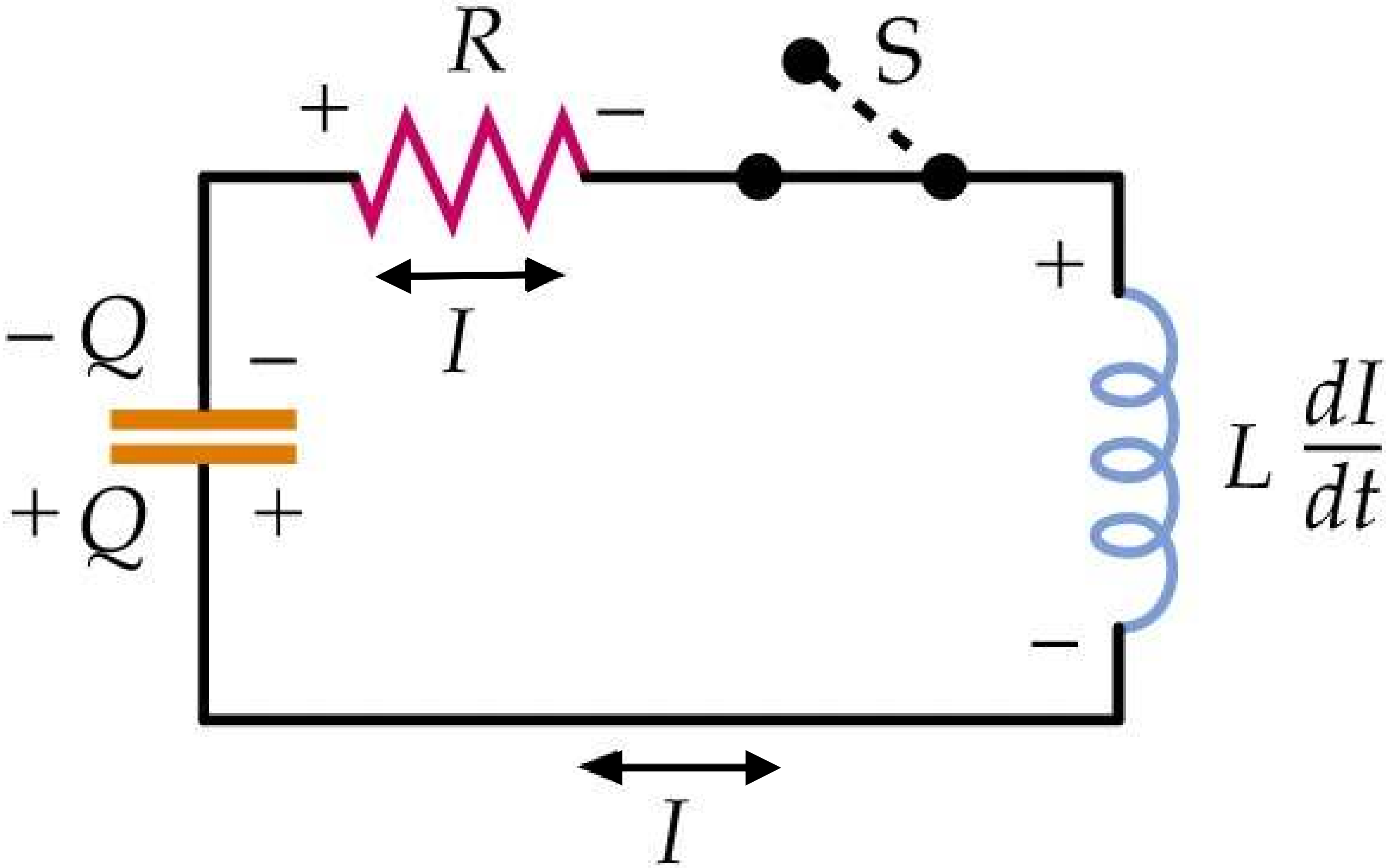
$$Im = \frac{Vm}{\omega L}$$

$$I = \frac{V}{R}$$

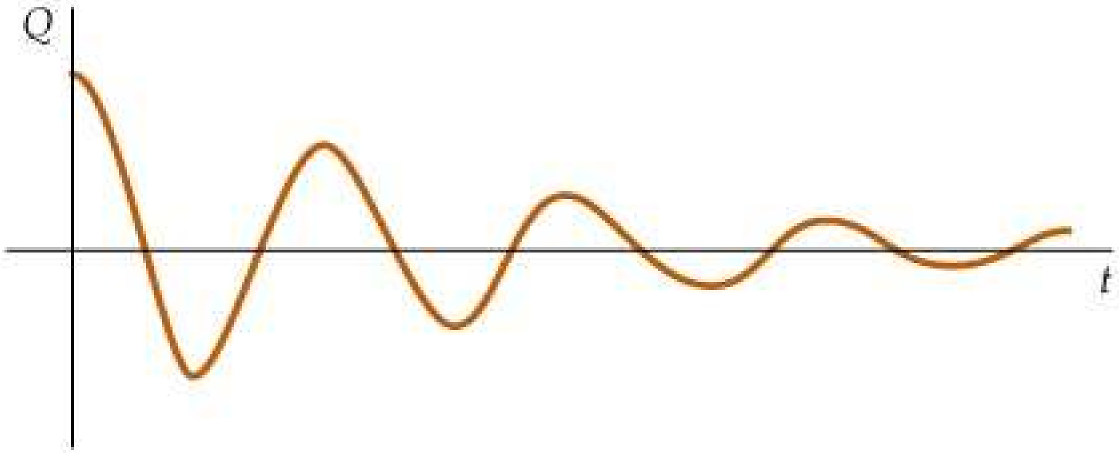
オームの法則に対してRの対応を見ると

$$R \rightarrow \omega L$$

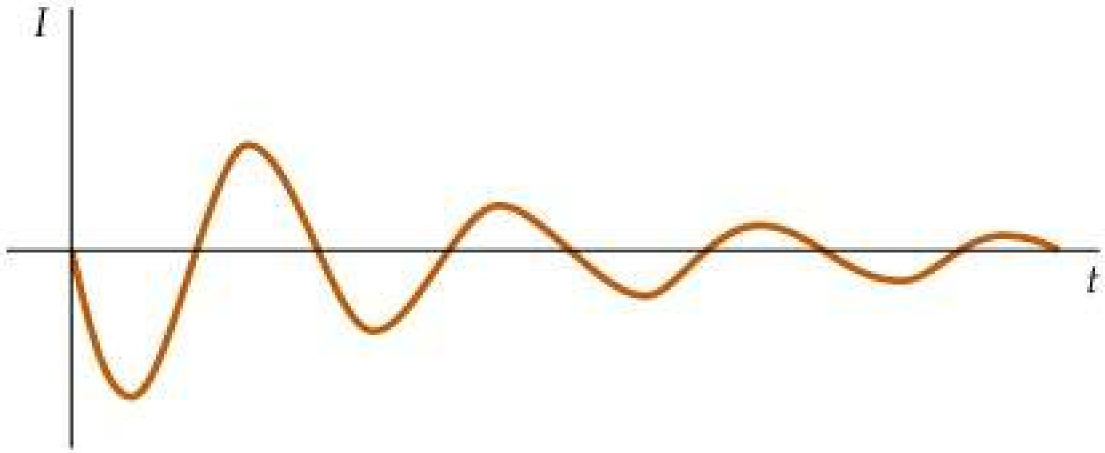
$$X_L = \omega L$$



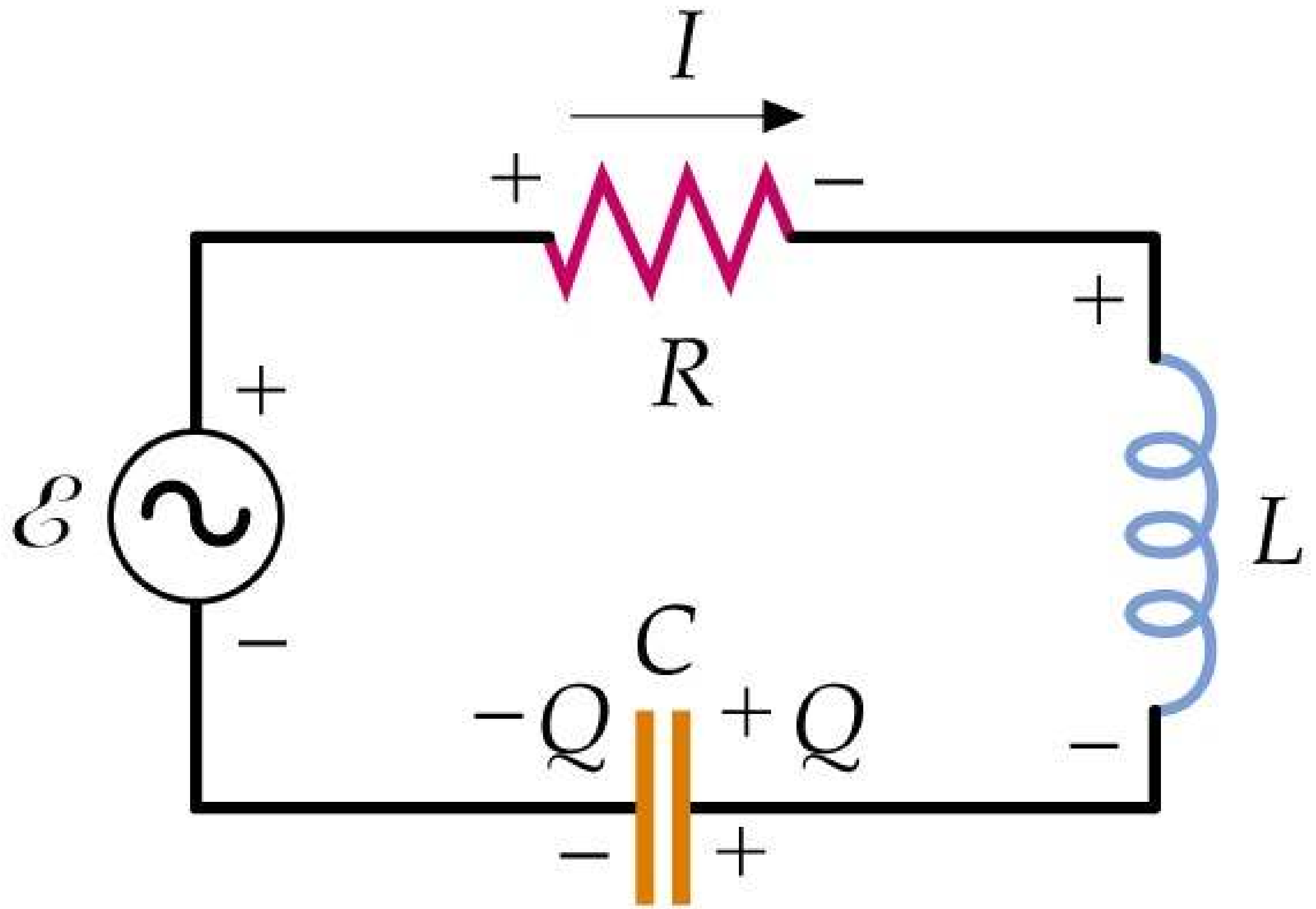
R
S
+
−
I
− Q
−
+ Q
+
+
L $\frac{dI}{dt}$
−
I



(a)



(b)



$$V = Vm \cdot \sin \omega t$$
$$I = \mathrm{Im} \cdot \sin(\omega t - \delta)$$

電圧の時間変化（瞬間の値）

$$\mathrm{v}_R = \mathrm{Im} \cdot R \cdot \sin \omega t = V_R \sin \omega t$$

$$\mathrm{v}_L = \mathrm{Im} \cdot X_L \cdot \sin(\omega t + \frac{\pi}{2})$$

$$= V_L \cdot \cos \omega t$$

$$\mathrm{v}_C = \mathrm{Im} \cdot X_C \cdot \sin(\omega t - \frac{\pi}{2})$$

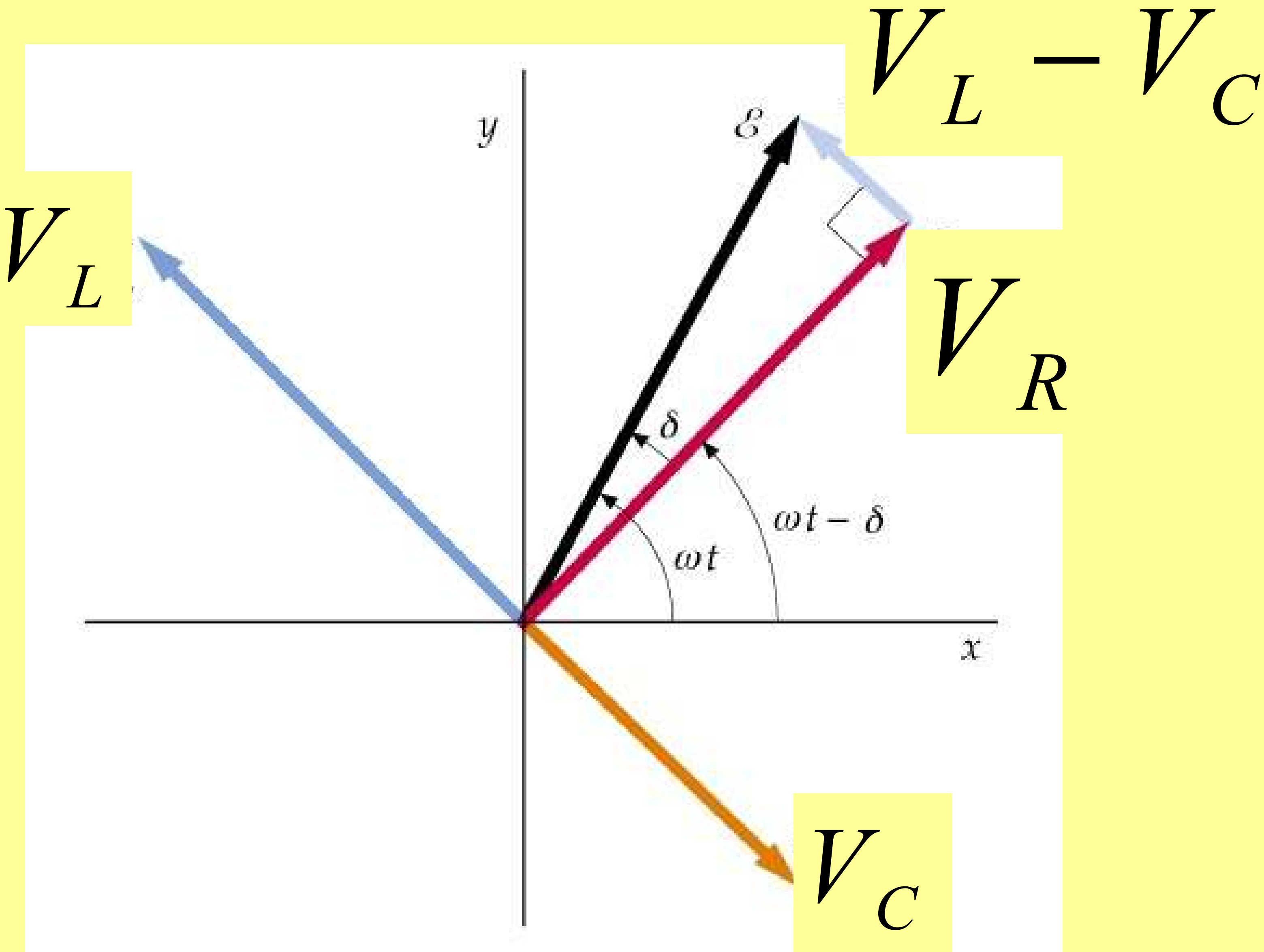
$$= -V_C \cdot \cos \omega t$$

電圧のピーク値

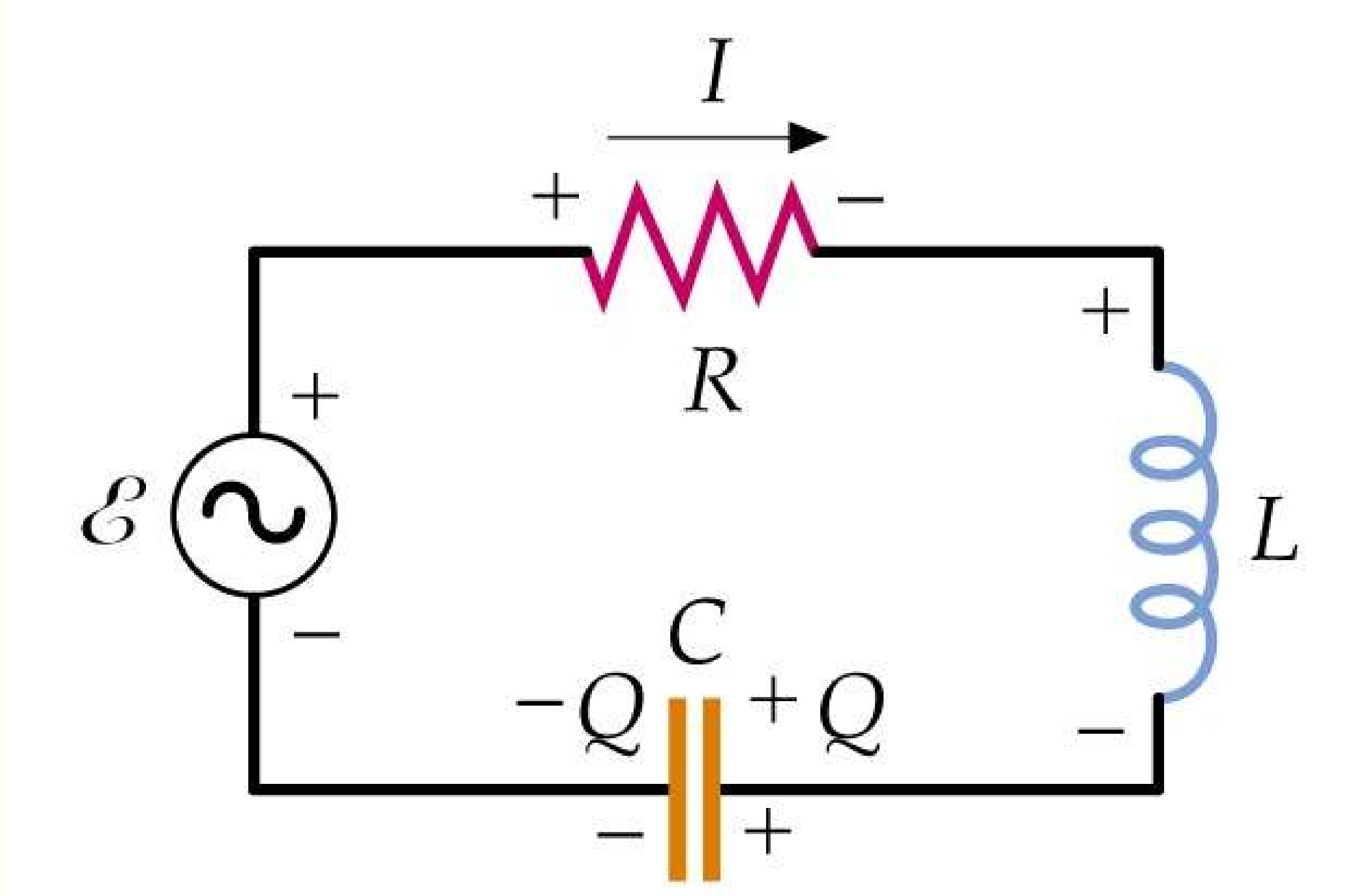
$$V_R = \mathrm{Im} \cdot R$$

$$V_L = \mathrm{Im} \cdot X_L$$

$$V_C = \mathrm{Im} \cdot X_C$$



$V_L - V_C$
$\mathcal{E}$
y
V_L
V_R
δ
$\omega t - \delta$
ωt
x
V_C

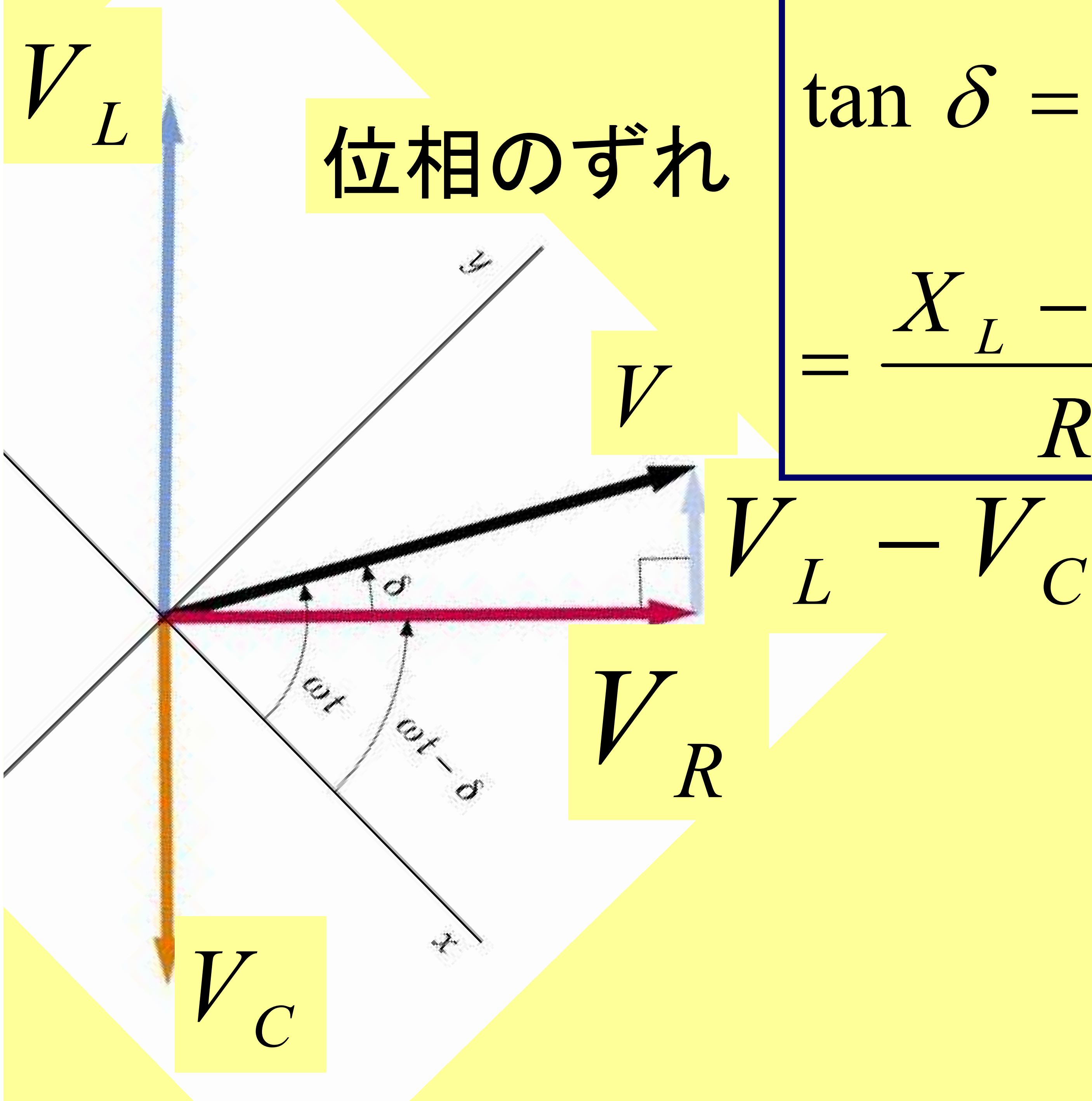


抵抗での電流・電圧は同位相！
抵抗の電圧変化（電流）基準に考えてみる。

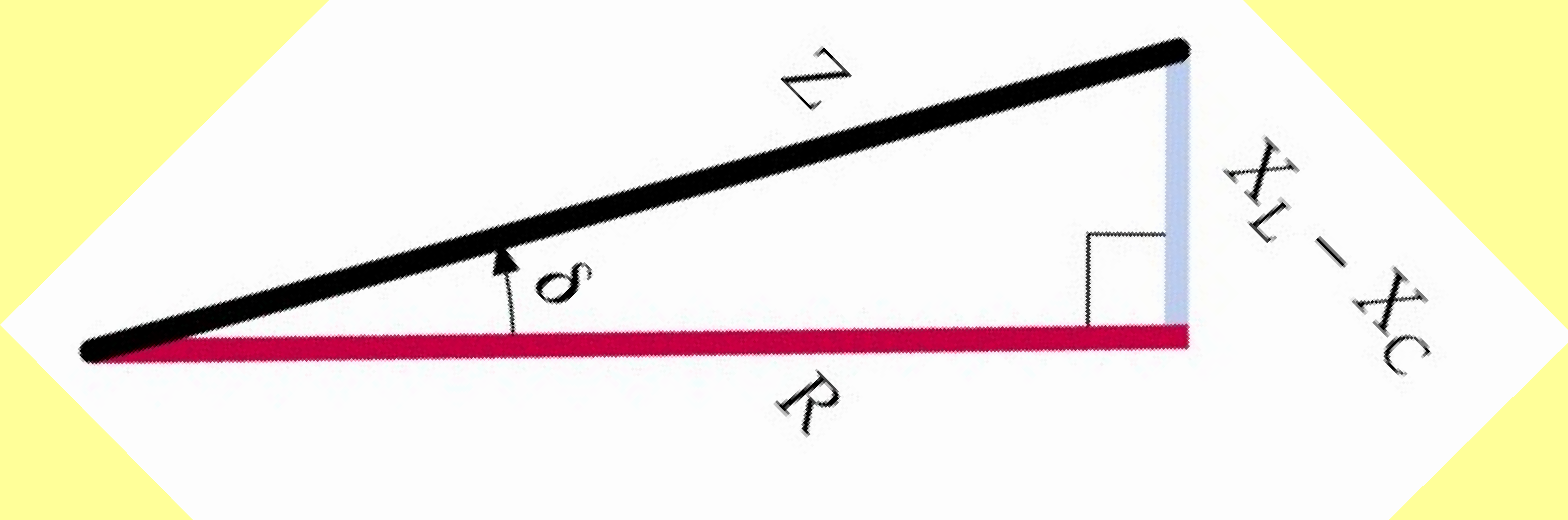
位相のずれ

$$\tan\delta = \frac{V_L - V_C}{V_R}$$

$$= \frac{X_L - X_C}{R}$$



電流の最大値と電圧の最大値の関係



$$V_m = \sqrt{(V_R)^2 + (V_L - V_C)^2}$$

$$= \sqrt{(\mathrm{Im} \cdot R)^2 + (\mathrm{Im} \cdot X_L - \mathrm{Im} \cdot X_C)^2}$$

$$= \mathrm{Im} \cdot \sqrt{(R)^2 + (X_L - X_C)^2}$$

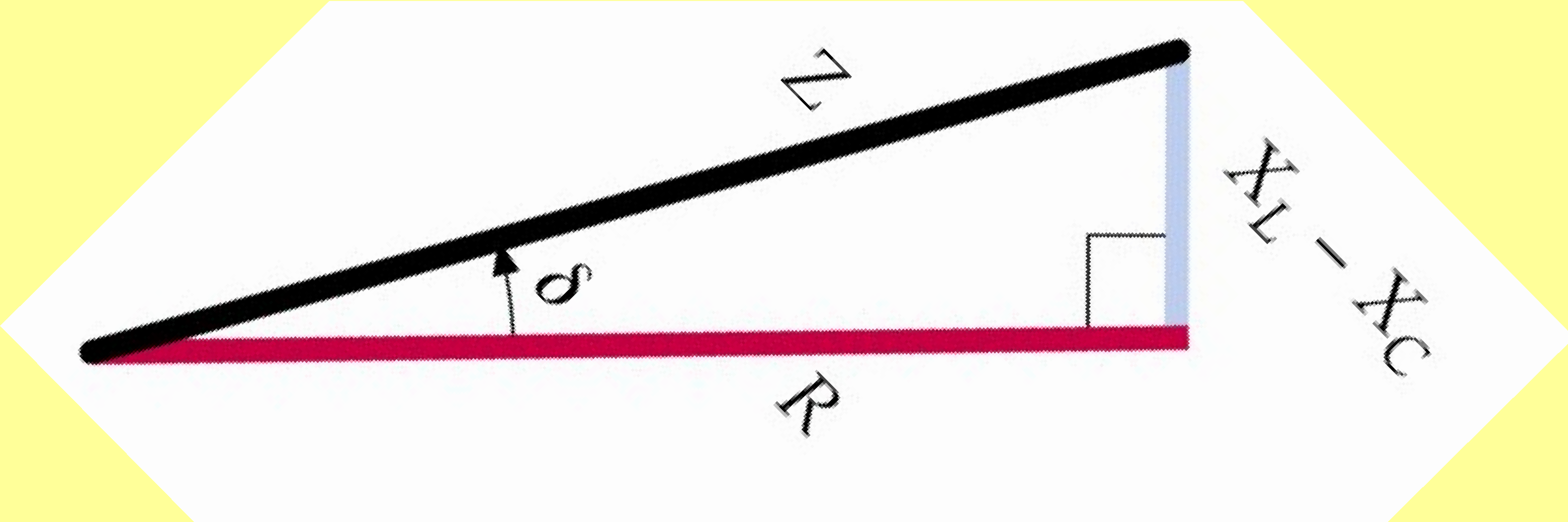
インピーダンス
（交流回路のオームの法則）

$$V_m = \mathrm{Im} \cdot \sqrt{(R)^2 + (X_L - X_C)^2}$$

$$Z = \sqrt{(R)^2 + (X_L - X_C)^2}$$

インピーダンス

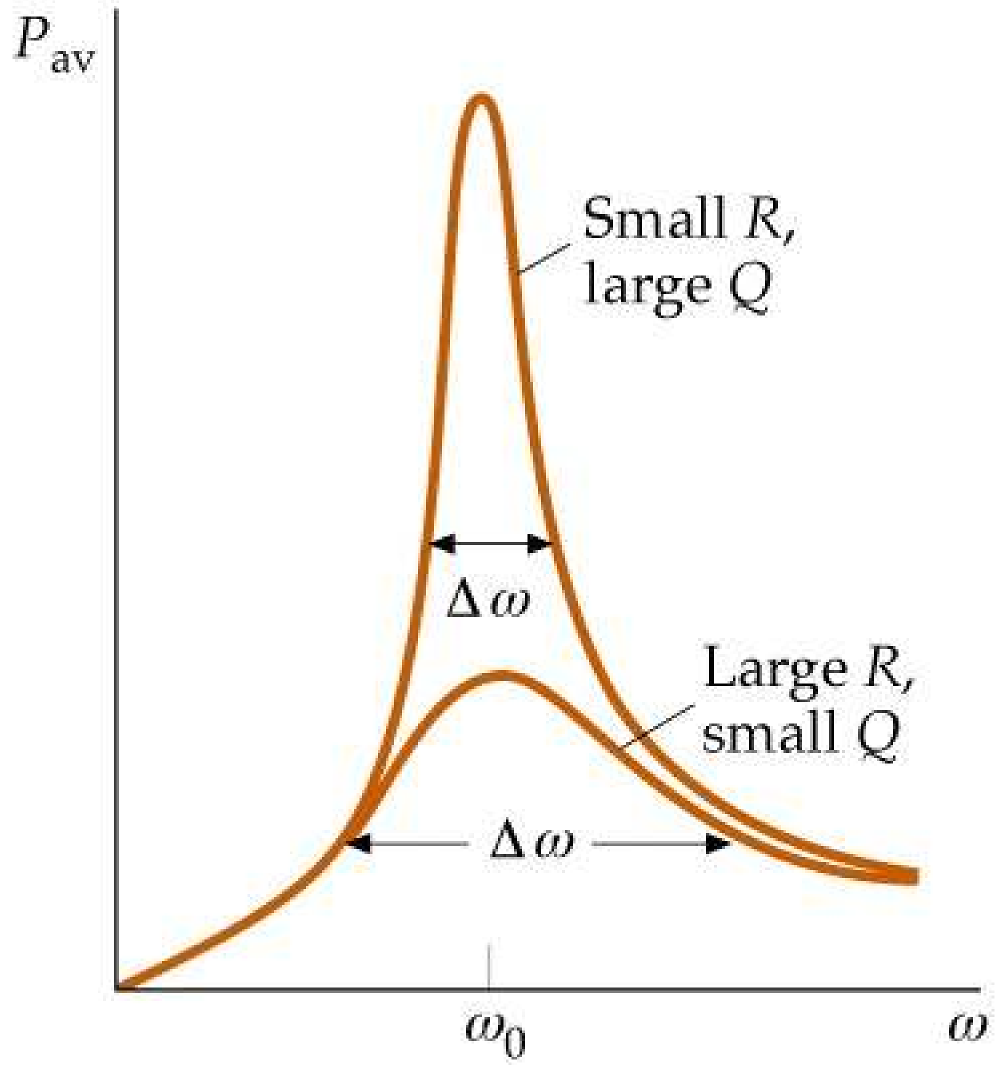
電流の最大値と電圧の最大値の関係



$$V_m = \sqrt{(V_R)^2 + (V_L - V_C)^2}$$

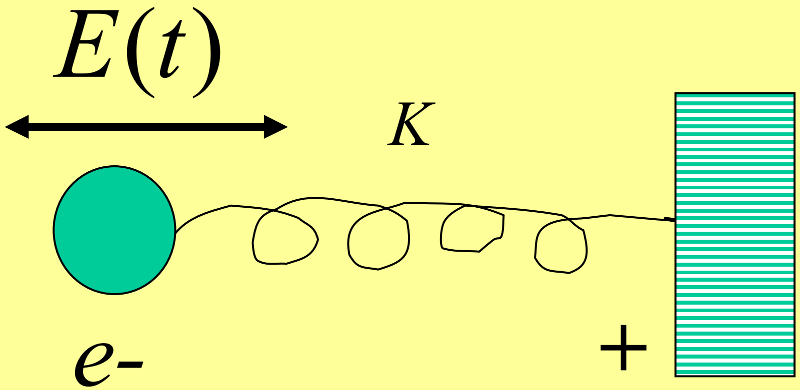
$$= \sqrt{(\mathrm{Im}\cdot R)^2 + (\mathrm{Im}\cdot X_L - \mathrm{Im}\cdot X_C)^2}$$

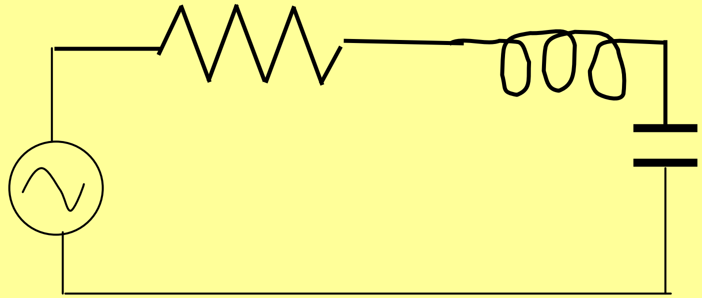
$$= \mathrm{Im}\cdot\sqrt{(R)^2 + (X_L - X_C)^2}$$



P_{av}
Small R, large Q
$\Delta\omega$
Large R, small Q
$\Delta\omega$
ω_0
ω

2階の微分方程式の相似と解





$$m\frac{d^2x}{dt^2}+\gamma\frac{dx}{dt}+Kx(t)=-eE(t)$$

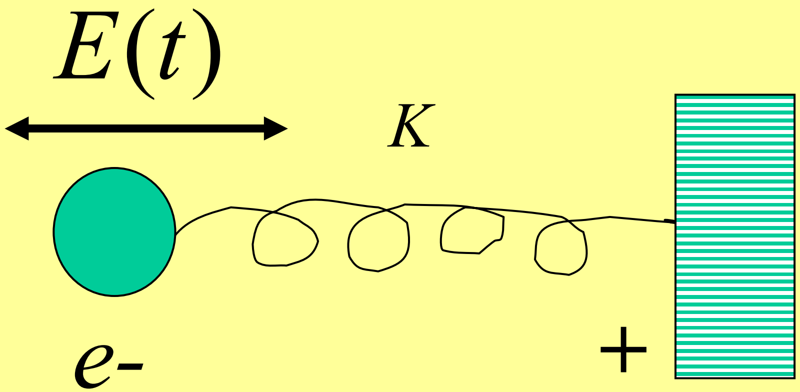
$$L\frac{d^2q}{dt^2}+R\frac{dq}{dt}+\frac{1}{C}q=E$$

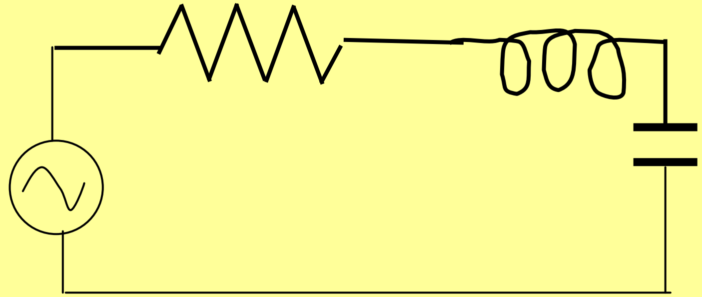
質量	m	L
減衰	γ	R
ばね定数	K	$1/C$

一般的な定数係数の微分方程式の解

$$x = A\exp[\lambda t]$$

ばねの振動とRLC回路





$$m\frac{d^2x}{dt^2}+\gamma\frac{dx}{dt}+Kx(t)=-eE(t)$$

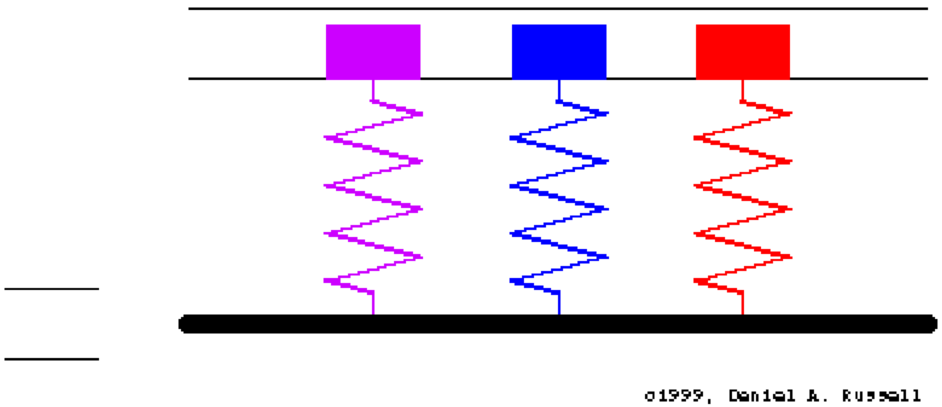
$$L\frac{d^2q}{dt^2}+R\frac{dq}{dt}+\frac{1}{C}q=E$$

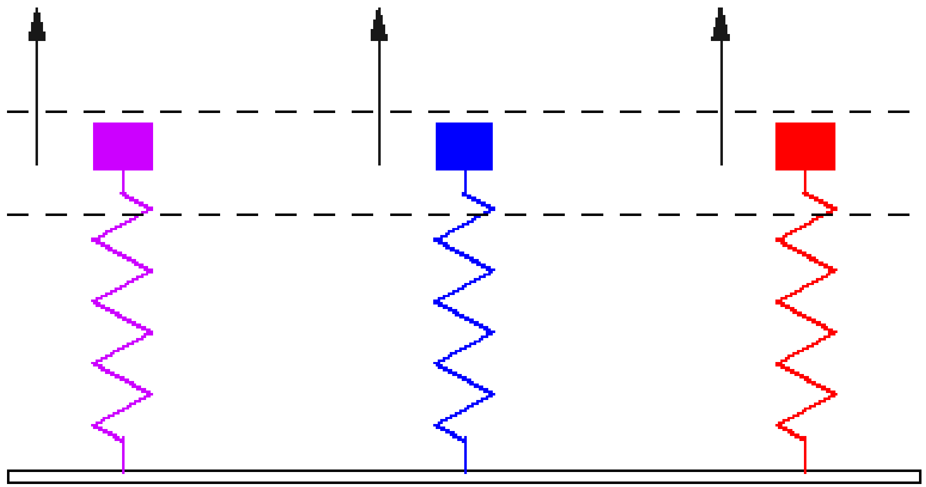
振動系の共鳴角周波数

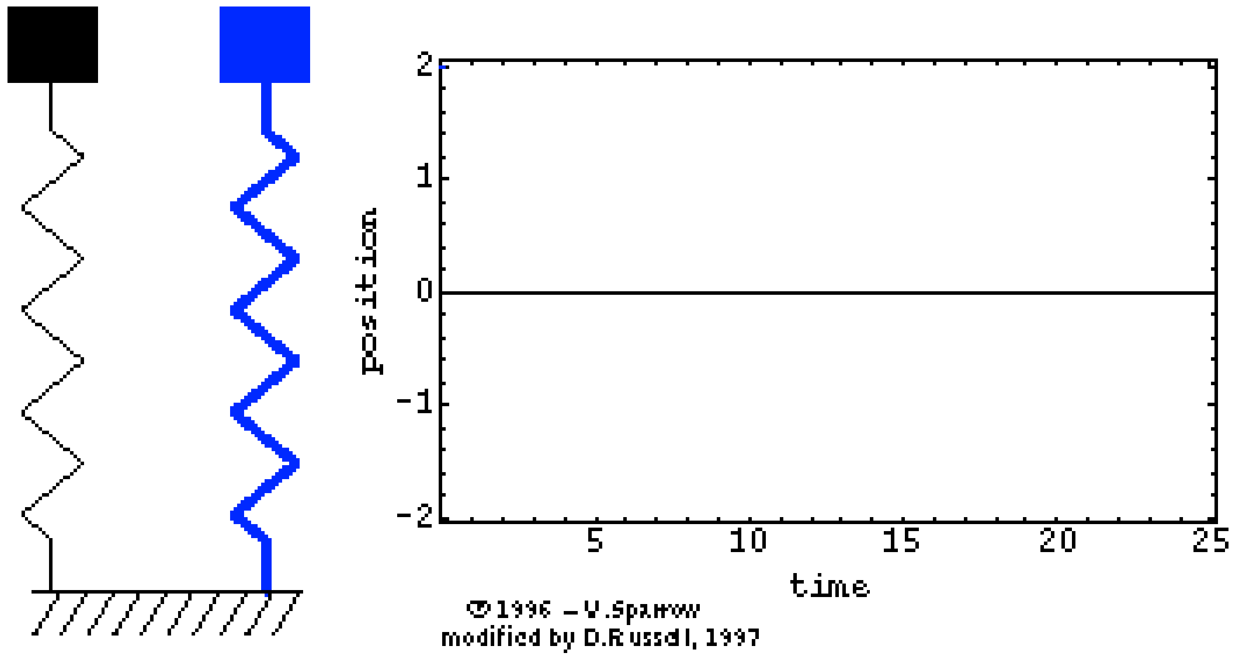
$$\omega_0=\sqrt{K/m} \quad << \quad \omega_0=1/\sqrt{LC}$$

～kHz　　　　～GHz

共　振　現　象







position
time
2
1
0
-1
-2
5
10
15
20
25
©1996 – V.Sparrow
modified by D.Russell, 1997

$$L\frac{dI}{dt} + RI + \frac{1}{C}q = V_0 \sin \omega t$$

$$L\frac{d^2 I}{dt^2} + R\frac{dI}{dt} + \frac{1}{C}I = V_0 \omega \sin \omega t$$

$$L\frac{d^2 I}{dt^2} + R\frac{dI}{dt} + \frac{1}{C}I = V_0 \omega \sin \omega t$$

$$I = I_0 \sin(\omega t - \phi)$$

$$I_0 \left\{ R\omega \sin\phi - (L\omega^2 \cos\phi - \frac{1}{C}\cos\phi) \right\} \sin \omega t$$

$$+ \left\{ I_0 \left(R\omega \cos\phi + L\omega^2 \sin\phi - \frac{1}{C}\sin\phi \right) - \omega V_0 \right\} \cos \omega t = 0$$

$$R\sin\phi-(\omega L-\frac{1}{\omega C})\cos\phi=0$$

$$R^2\sin^2\phi+(\omega L-\frac{1}{\omega C})^2\cos^2\phi$$

$$-2R(\omega L-\frac{1}{\omega C})\sin\phi\cos\phi=0$$

$$R\cos\phi+(\omega L-\frac{1}{\omega C})\sin\phi=\frac{V_0}{I_0}$$

$$R^2\cos^2\phi+(\omega L-\frac{1}{\omega C})^2\sin^2\phi$$

$$+2R(\omega L-\frac{1}{\omega C})\sin\phi\cos\phi=\left(\frac{V_0}{I_0}\right)^2$$

$$R^2 \sin^2\phi + (\omega L - \frac{1}{\omega C})^2 \cos^2\phi - 2R(\omega L - \frac{1}{\omega C})\sin\phi\cos\phi = 0$$

$$R^2 \cos^2\phi + (\omega L - \frac{1}{\omega C})^2 \sin^2\phi + 2R(\omega L - \frac{1}{\omega C})\sin\phi\cos\phi = \left(\frac{V_0}{I_0}\right)^2$$

$$R^2 + (\omega L - \frac{1}{\omega C})^2 = \left(\frac{V_0}{I_0}\right)^2$$

$$\left(\frac{V_0}{I_0}\right) = \sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2} = Z$$